

question 464: Two Integrals

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abstract

This note presents two integrals involving pi

1. Introduction

In this note we present two integrals that involve the constant pi :

$$\pi = \int_0^1 \frac{1}{x^{3/4} + x^{5/4}} dx = 3.14159265 \dots \quad (1)$$

2. Two Integrals

First Integral:

$$\frac{\pi}{2\sqrt{2}} = \int_0^1 \left(\sqrt{5-x^2 + \sqrt{(1-x^2)(9-x^2)}} - \sqrt{5-x^2 - \sqrt{(1-x^2)(9-x^2)}} \right) dx \quad (2)$$

Second Integral:

$$\frac{\pi}{2} - 1 = \int_0^u \ln\left(\frac{f(x)}{g(x)}\right) dx \quad (3)$$

where

$$u = \frac{(7-3\sqrt{5})\sqrt{2+\sqrt{5}}}{2} \quad (4)$$

$$f(x) = 1 - 2\sqrt{1-3x^2} \cos\left(\frac{2\pi}{3} + \frac{1}{3} \cos^{-1}\left(\frac{18x^2-1}{(1-3x^2)^{3/2}}\right)\right) \quad (5)$$

$$g(x) = 1 - 2\sqrt{1-3x^2} \cos\left(\frac{4\pi}{3} + \frac{1}{3} \cos^{-1}\left(\frac{18x^2-1}{(1-3x^2)^{3/2}}\right)\right) \quad (6)$$

References

- A. Boros, G. and Moll, V.H.: Irresistible Integrals, Cambridge, University Press,2004.
- B. Gradshteyn, I.S. and Ryzhik, I.M.: Table of Integrals, Series and Products. eds. A. Jeffrey and D. Zwillinger. Seventh Edition. Academic Press, 2007.