DEMONSTRATION OF THE RIEMANN HYPOTHESIS

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Summary: 158 years ago that in the complex analysis a hypothesis was raised, which was used in principle to demonstrate a theory about prime numbers, but, without any proof; with the passing Over the years, this hypothesis has become very important, since it has multiple applications to physics, to number theory, statistics, among others In this article I present a demonstration that I consider is the one that has been dodging all this time.

1. INTRODUCTION

In mathematics and in all sciences (although they have passed 2017 years after Christ), there are still some problems that have not been solved but that have provided knowledge, endless solid foundations that have allowed to create many inventions that, in the past were very needed, but due to the circumstances, it was not possible to use the elements or instruments for what is needed.

This is the case of the Riemann hypothesis, which resulted in demonstrating a exercise on the prime numbers, but its truthfulness (at the time), not was important, however, with the passage of time this statement was Acquiring greater strength.

This hypothesis says: "In pure mathematics, the hypothesis of Riemann, formulated for the first time by Bernhard Riemann in 1859, is a conjecture about the zeros distribution of the zeta function of Riemann"; this conjecture in his moment was not very important because it was used to talk about the numbers cousins, but with the passage of time has taken quite a lot of importance due to its without number of applications, but it is a mathematical plan that takes 158 years without being shown and I personally hope to know your demonstrated some day.

At the time that corresponded to me to do the thesis to obtain my title of graduated in mathematics, I was interested in this subject and made a general approach to what she is in reality and its applications; with the Over the years this theme continued in my mind and I started working hard in his demonstration.

On this occasion, I propose a possible demonstration, which has already been revised by a mathematical and physical teacher, who approved it and who is now raised to those who have greater knowledge in the field of mathematics pure.

2. PRELIMINARIESThe Riemann zeta function

2.1 THE ZEAL FUNCTION OF RIEMANN IS ANALYTICAL

The Riemann zeta function ($\zeta - function$), is given by an analytical prolongation of the p-series as follows:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s},$$

where, $s > 1$
Taking $s = x + iy$,and replacing we have:

$$\zeta(x+iy) = \sum_{n=1}^{\infty} \frac{1}{n^{x+iy}}$$

By properties of exponents all exponent can be expressed in the form, $\frac{1}{x^n} = x^{-n}$, having:

$$\zeta(x+iy) = \sum_{n=1}^{\infty} n^{-x} n^{-iy}$$

Since the exponential function and the logarithmic function are inverse, every number can be expressed in the form, $e^{\ln x} = x$, with $x \in \mathbb{R}$; applying this we have:

$$\zeta(x+iy) = \sum_{n=1}^{\infty} n^{-x} e^{\ln\left(n^{-iy}\right)}$$

By properties of the logarithm, all exponent can come out as a factor like this:

$$\zeta(x+iy) = \sum_{n=1}^{\infty} n^{-x} e^{-iy\ln(n)}$$

By definition, a complex exponent is expressed as the sum of sine and cosine:

$$\zeta(x+iy) = \sum_{n=1}^{\infty} n^{-x} \left[\cos\left(y\ln\left(n\right)\right) - i\sin\left(y\ln\left(n\right)\right)\right]$$

Here we have an expression in which a complex number is evidently reflected, in which the real part (u) and the complex part (v) can be taken as follows:of residue

$$u = n^{-x}\cos\left(y\ln\left(n\right)\right) \quad v = -n^{-x}\sin\left(y\ln\left(n\right)\right)$$

Applying the Cauchy-Riemann equations, we have:

$$\begin{array}{ll} u_x = -n^{-x}\ln(n)\cos\left(y\ln\left(n\right)\right) & v_x = n^{-x}\ln(n)\sin\left(y\ln\left(n\right)\right) \\ u_y = -n^{-x}\ln(n)\sin\left(y\ln\left(n\right)\right) & v_y = -n^{-x}\ln(n)\cos\left(y\ln\left(n\right)\right) \end{array}$$

Here we have that is effectively met:

$$\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}$$

which are the Cauchy-Riemann equations.

Now, it is necessary to emphasize that these equations are fulfilled if the series converges and, this does it only if $|s| > 1$.

In the Cauchy-Riemann equations, it can be seen that the zeta function is continuously derivable throughout its domain $|s| > 1$, therefore, the Riemann zeta function is holomorphic in the domain $|s| > 1$.

This function, since it is holomorphic, then we can extend it analytically to the whole complex plane, this is done through the functional equation:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s),$$

where it has a simple pole in $s = 1$ of residue 1.

2.2 PRINCIPLE OF REFLECTION OF SCHWARZ

Having the holomorphic zeta function, we can talk about the Schwarz reflection principle, which states: "*Sean ω an open $\mathbb{C}$ which is symmetric about $\mathbb{R}$, and either f a holomorphic function in ω^+ that extends with continuity to I and it takes real values in I. Then there is $F : \omega \to \mathbb{C}$ holomorfa such that $F = f$* in *$\omega^+ \cup I$*."

In other words:

$$F(\overline{z}) = \overline{F(z)}$$

Let's see if this is fulfilled in the Riemann zeta function.

Be

$$\zeta(\overline{z}) = \zeta(\overline{x+iy})$$

here we have the Riemann zeta function evaluated in a conjugate, then we have:

$$\zeta(\overline{x+iy}) = \zeta(x-iy)$$

Here, we express the Riemann zeta function as the series that it represents:

$$\zeta(x-iy) = \sum_{n=1}^{\infty} \frac{1}{n^{x-iy}}$$

By expanding the series we have:

$$= \frac{1}{1^{x-iy}} + \frac{1}{2^{x-iy}} + \cdots + \frac{1}{n^{x-iy}} + \cdots$$

As properties of the exponents, one has to: $\frac{1}{x^{a+b}} = x^{-a}x^{-b}$ and, applying this in the series we have

$$= 1^{-x}1^{iy} + 2^{-x}2^{iy} + \cdots + n^{-x}n^{iy} + \cdots$$

Applying the fact that the exponential function and the logarithmic function are inverse, we have

$$= 1^{-x}e^{\ln 1^{iy}} + 2^{-x}e^{\ln 2^{iy}} + \cdots + n^{-x}e^{\ln n^{iy}} + \cdots$$

As a property of logarithms, an exponent that it possesses can come out as a factor like this:

$$= 1^{-x}e^{iy\ln 1} + 2^{-x}e^{iy\ln 2} + \cdots + n^{-x}e^{iy\ln n} + \cdots$$

By definition, a complex exponent is expressed as the sum of sine and cosine like this:

$$= 1^{-x}\left[\cos(y\ln 1) + i\sin(y\ln herewecangroupsinesandcosinesthathavethesameargument, likethis: 1)\right] + 2^{-x}\left[\cos(y\ln 2\right.$$

separating the real part of the complex part, we have:

$$= 1^{-x}\cos(y\ln 1) + 2^{-x}\cos(y\ln 2) + \cdots + i\left[1^{-x}\sin(y\ln 1) + 2^{-x}\sin(y\ln 2) + \cdots\right]$$

here we can express $+ = - \cdot -$, as follows:

$$= 1^{-x}\cos(y\ln 1) + 2^{-x}\cos(y\ln 2) + \cdots - i\left[-1^{-x}\sin(y\ln 1) - 2^{-x}\sin(y\ln 2) - \cdots\right]$$

here we take advantage of the fact that the sine function is an odd function ($\sin(-x) = -\sin x$):

$$= 1^{-x}\cos(y\ln 1) + 2^{-x}\cos(y\ln 2) + \cdots - i\left[1^{-x}\sin(-y\ln 1) + 2^{-x}\sin(-y\ln 2) + \cdots\right]$$

here by factoring the minus sign into the complex part, we actually see that we have a conjugate like this:

$$= \overline{1^{-x}\cos(y\ln 1) + 2^{-x}\cos(y\ln 2) + \cdots + i\left[1^{-x}\sin(-y\ln 1) + 2^{-x}\sin(-y\ln 2) + \cdots\right]}$$

here we can group sines and cosines that have the same argument, so:

$$= \overline{1^{-x}\left[\cos(y\ln 1) + i\sin(-y\ln 1)\right] + 2^{-x}\left[\cos(y\ln 2) + i\sin(-y\ln 2)\right] + \cdots}$$

here we can use the hollow of the sine function is odd, to express these complexes as conjugated numbers like this:

$$= \overline{1^{-x}\left[\cos(y\ln 1) - i\sin(y\ln 1)\right] + 2^{-x}\left[\cos(y\ln 2) - i\sin(y\ln 2)\right] + \cdots}$$

this sums of sines and cosines can be expressed as an exponential function with a complex exponent like this:

$$= \overline{1^{-x}e^{-iy\ln 1} + 2^{-x}e^{-iy\ln 2} + \cdots + n^{-x}e^{-iy\ln n} + \cdots}$$

we can use the exponent factors of the exponentials as exponents of the arguments of these exponentials like this:

$$= \overline{1^{-x}e^{\ln 1^{-iy}} + 2^{-x}e^{\ln 2^{-iy}} + \cdots + n^{-x}e^{\ln n^{-iy}} + \cdots}$$

Since the exponential function and the logarithmic function are inverse functions, we have:

$$= \overline{1^{-x}1^{-iy} + 2^{-x}2^{-iy} + \cdots + n^{-x}n^{-iy} + \cdots}$$

By properties of exponents, negative exponents can be expressed positively from the inverse of the base, like this:

$$= \overline{\frac{1}{1^x 1^{iy}} + \frac{1}{2^x 2^{iy}} + \cdots + \frac{1}{n^x n^{iy}} + \cdots}$$

by properties of the exponents, the product of two equal bases with different exponents can be expressed as the common base elevated to the sauma of the exponents that are different like this:

$$= \overline{\frac{1}{1^{x+iy}} + \frac{1}{2^{x+iy}} + \cdots + \frac{1}{n^{x+iy}} + \cdots}$$

All these values that are being added have a pattern, therefore they can be expressed as a summation like this:

$$= \overline{\sum_{n=1}^{\infty} \frac{1}{n^{x+iy}}}$$

this sum defines the $p-$series like this:

$$\overline{\sum_{n=1}^{\infty} \frac{1}{n^{x+iy}}} = \overline{\zeta(x+iy)}$$

Which defines the zeta function as follows:

$$\zeta(\overline{z}) = \overline{\zeta(z)}$$

Then, the Riemann zeta function does satisfy the Schwarz reflection principle.

3. DEMONSTRATION OF THE RIEMANN HYPOTHESIS

3.1 HYPOTHESIS.
The real part of all non-trivial zero of the Riemann zeta function is *1/2.*

3.2. DEMONSTRATION

Let's suppose $z = r + it$ is a non-trivial zero of the Riemann zeta function, let's see if the conjugates of these zeros $(r - it)$, they are also roots of $\zeta-$function. Applying the Schwarz reflection principle (which we have just shown that if it satisfies it), we have

$$\zeta(\overline{r+it}) = \overline{\zeta(r+it)}$$

As $r + it$ is a zero of the function, then you have:

$$\zeta(\overline{r+it}) = \overline{0}$$

By definition of the conjugates, we know that $\overline{0} = 0$, por lo tanto

$$\zeta(\overline{r+it}) = 0$$

Then, the conjugates of the non-trivial zeros of the $\zeta-$function, they are also zeros of this.

Knowing that the real part of any complex number and its conjugate are equal, therefore, in the functional equation we have $\zeta(r + it)$ and $\zeta(1 - (r + it))$, so

$$r = 1 - r$$

adding up r on both sides we have

$$r + r = 1$$

here we have $r + r$, adding up you have

$$2r = 1$$

clearing r finally we have

$$r = \frac{1}{2}$$

therefore, the real part of the non-trivial zeros of the zeta function is $\frac{1}{2}$.

Having found the real part of the nontrivial zero of $\zeta-$function, we have (replacing in the analytic extension):

$$\zeta\left(\frac{1}{2}+it\right) = 2^{\frac{1}{2}+it}\pi^{\frac{1}{2}+it-1}\sin\left(\frac{\pi\left(\frac{1}{2}+it\right)}{2}\right)\Gamma\left(1-\left(\frac{1}{2}+it\right)\right)\zeta\left(1-\left(\frac{1}{2}+it\right)\right)$$

Clearing, we finally get

$$\zeta\left(\frac{1}{2}+it\right) = 2^{\frac{1}{2}+it}\pi^{-\frac{1}{2}+it}\sin\left(\frac{\pi}{4}+i\frac{\pi t}{2}\right)\Gamma\left(\frac{1}{2}-it\right)\zeta\left(\frac{1}{2}-it\right)$$

Now, in this equation, we must find the value of t.

To begin, let's consider which of the factors in this equation is zero, since with only one of them being, the whole expression will also be zero, since all expressions are linked by a product. $2^{\frac{1}{2}+it}\pi^{\frac{1}{2}+it-1} \neq 0$, Thus

$$\sin\left(\tfrac{\pi}{4}+i\tfrac{\pi t}{2}\right) = 0 \quad \vee \quad \Gamma\left(\tfrac{1}{2}-it\right) = 0$$

Let's start looking $\sin\left(\frac{\pi}{4}+i\frac{\pi t}{2}\right)$:

As is known, the function $\sin(s)$ is zero, only if $s = k\pi$, with $k \in \mathbb{Z}$ and s is in the field of complex numbers ($\mathbb{C}$).Clearing finally we have:

Let's now see the value it takes $\Gamma\left(\frac{1}{2}-it\right)$.

Replacing $\frac{1}{2}+it$ in $\Gamma(1-s)$ we have:

$$\Gamma\left(1-\left(\frac{1}{2}+it\right)\right) = \Gamma\left(\frac{1}{2}-it\right)$$

Applying the integral definition of the Gamma function, we have

$$\Gamma\left(\frac{1}{2}-it\right) = \int_0^\infty T^{\frac{1}{2}-it-1}e^{-T}d$$

Despejando finalmente tenemos:

$$\Gamma\left(\frac{1}{2}-it\right) = \int_0^\infty T^{-\frac{1}{2}-it}e^{-T}dT$$

By changing the variable (substitution), we have

$$\begin{array}{ccc} T = x^{a+bi} & si\ T = 0 & x = 0 \\ dT = (a+bi)x^{a-1+bi}dx & si\ T = \infty & x = \infty \end{array}$$

Replacing

$$\int_0^\infty T^{-\frac{1}{2}-it}e^{-T}dT = \int_0^\infty \left(x^{a+bi}\right)^{-\frac{1}{2}-it} e^{-x^{a+bi}}(a+bi)x^{a-1+bi}dx$$

The constant $a+bi$ can leave the integral

$$\int_0^\infty \left(x^{a+bi}\right)^{-\frac{1}{2}-it} e^{-x^{a+bi}}(a+bi)x^{a-1+bi}dx = (a+bi)\int_0^\infty x^{\left(-\frac{a}{2}+bt\right)+i\left(-at-\frac{b}{2}\right)}e^{-x^{a+bi}}x^{a-1+bi}dx$$

Doing the operations presented with the constants we finally have:

$$\int_0^\infty T^{-\frac{1}{2}-it}e^{-T}dT = (a+bi)\int_0^\infty x^{\left(-\frac{a}{2}+bt+a-1\right)+i\left(-at-\frac{b}{2}+b\right)}e^{-x^{a+bi}}dx$$

Here results a system of equations of 2×2, with the variables a and b; you have:

$$\begin{aligned} a+2bt &= 2 \qquad (1)\\ -2at+b &= 0 \qquad (2)\end{aligned}$$

Using the equalization method, we have:
* Multiplying $(1)*2t$ and adding (2)

$2at+4bt^2 = 4t$
$-2at+b = 0$

$b(4t^2+1) = 4t$

Clearing b, finally we have:

$$b = \frac{4t}{4t^2+1}$$

b replacing in (1) and clear a,you have:

$$\begin{aligned} a+2t\left(\tfrac{4t}{4t^2+1}\right) &= 2\\ a &= 2-\tfrac{8t^2}{4t^2+1}\\ a &= \tfrac{8t^2+2-8t^2}{4t^2+1}\\ a &= \tfrac{2}{4t^2+1}\end{aligned}$$

Replacing the obtained values, we have:

$$(a+bi)\int_0^\infty x^{\left(\frac{a}{2}+bt-1\right)+i\left(-at+\frac{b}{2}\right)}e^{-x^{a+bi}}dx = \left(\frac{2+4it}{4t^2+1}\right)\int_0^\infty x^{\left(\frac{1}{4t^2+1}+\frac{4t^2}{4t^2+1}-1\right)+i\left(-\frac{2t}{4t^2+1}+\frac{2t}{4t^2+1}\right)}e^{-x^{\frac{2+4it}{4t^2+1}}}dx$$

Replacing a and b, we have:

$$\left(\frac{2+4it}{4t^2+1}\right)\int_0^\infty x^{\left(\frac{1}{4t^2+1}+\frac{4t^2}{4t^2+1}-1\right)+i\left(-\frac{2t}{4t^2+1}+\frac{2t}{4t^2+1}\right)}e^{-x^{\frac{2+4it}{4t^2+1}}}dx = \left(\frac{2+4it}{4t^2+1}\right)\int_0^\infty x^{\left(\frac{1+4t^2-4t^2-1}{4t^2+1}\right)+i0}e^{-x^{\frac{2+4it}{4t^2+1}}}dx$$

adding up the complex values we have:

$$\left(\frac{2+4it}{4t^2+1}\right)\int_0^\infty x^{\left(\frac{1+4t^2-4t^2-1}{4t^2+1}\right)+i0} e^{-x^{\frac{2+4it}{4t^2+1}}}dx = \left(\frac{2+4it}{4t^2+1}\right)\int_0^\infty x^{(0)+i0} e^{-x^{\frac{2+4it}{4t^2+1}}}dx$$

Finally we have:

$$(a+bi)\int_0^\infty x^{\left(\frac{a}{2}+bt-1\right)+i\left(-at+\frac{b}{2}\right)} e^{-x^{a+bi}}dx = \left(\frac{2+4it}{4t^2+1}\right)\int_0^\infty e^{-x^{\frac{2+4it}{4t^2+1}}}dx$$

Once again changing the variable (substitution), we have:

$$\begin{array}{ll} x = z^{c+mi} & si\ x=0 \quad \rightarrow \quad z=0 \\ dx = (c+mi)z^{a-1+bi}dz & si\ x=\infty \quad \rightarrow \quad z=\infty \end{array}$$

$$\begin{aligned} \left(\frac{2+4it}{4t^2+1}\right)\int_0^\infty e^{-x^{\frac{2+4it}{4t^2+1}}}dx &= \left(\frac{2+4it}{4t^2+1}\right)\int_0^\infty e^{-\left(z^{c+mi}\right)^{\frac{2+4it}{4t^2+1}}}(a+bi)z^{a-1+bi}dz \\ &= \left(\frac{2+4it}{4t^2+1}\right)(c+mi)\int_0^\infty e^{-z^{\frac{2c-4mt}{4t^2+1}+i\frac{4ct+2m}{4t^2+1}}} z^{c-1+mi}dz \end{aligned}$$

again raising a system of equations 2×2 :

$$\begin{array}{llll} \frac{2c-4mt}{4t^2+1} = 1 & \rightarrow & 2c-4mt = 4t^2+1 & (1) \\ \frac{4ct+2m}{4t^2+1} = 0 & \rightarrow & 4ct+2m = 0 & (2) \end{array}$$

Using the substitution method:
Clearing m of (2), we have:

$4ct + 2m = 0$
$2m = -4ct$
$m = -2ct$

Replacing in (1)

$2c - 4(-2ct)t = 4t^2 + 1$
$2c + 8ct^2 = 4t^2 + 1$
$c(2 + 8t^2) = 4t^2 + 1$
$c = \frac{4t^2+1}{2+8t^2}$
$c = \frac{4t^2+1}{2(4t^2+1)}$
$c = \frac{1}{2}$

Replacing in (2)

$4\left(\frac{1}{2}\right)t + 2m = 0$
$2t + 2m = 0$
$2m = -2t$
$m = -t$

By replacing the obtained values, we have:

$$\left(\frac{2+4it}{4t^2+1}\right)(c+mi)\int_0^\infty e^{-z^{\frac{2c-4mt}{4t^2+1}+i\frac{4ct+2m}{4t^2+1}}} z^{c-1+mi}dz = \left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}+(-t)i\right)\int_0^\infty e^{-z^{\frac{2\left(\frac{1}{2}\right)_{-4(-t)t}}{4t^2+1}+i\frac{4\left(\frac{1}{2}\right)t+2(-t)}{4t^2+1}}} z^{\frac{1}{2}-1+i(-}$$

Adding and subtracting you have

$$\left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}+(-t)i\right)\int_0^\infty e^{-z^{\frac{2\left(\frac{1}{2}\right)_{-4(-t)t}}{4t^2+1}+i\frac{4\left(\frac{1}{2}\right)t+2(-t)}{4t^2+1}}} z^{\frac{1}{2}-1+i(-t)}dz = \left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\int_0^\infty e^{-z^{\frac{1+4t^2}{4t^2+1}+i\frac{2t-2t}{4t^2+1}}} z^{-\frac{1}{2}-i}$$

The complex part disappears

$$\left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\int_0^\infty e^{-z^{\frac{1+4t^2}{4t^2+1}+i\frac{2t-2t}{4t^2+1}}} z^{-\frac{1}{2}-it}dz = \left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\int_0^\infty e^{-z^{1+i0}} z^{-\frac{1}{2}-it}dz$$

having:

$$\left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\int_0^\infty e^{-z^{1+i0}} z^{-\frac{1}{2}-it}dz = \left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\int_0^\infty e^{-z} z^{\frac{1}{2}-it-1}dz$$

applying the integral definition of the Gamma function we have:

$$\left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\int_0^\infty e^{-z} z^{\frac{1}{2}-it-1}dz = \left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\Gamma\left(\frac{1}{2}-it\right)$$

With these results, we finally have:

$$\left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\Gamma\left(\frac{1}{2}-it\right) = \Gamma\left(\frac{1}{2}-it\right)$$

Leaving the Gamma function to one side only we have:

$$\left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)\Gamma\left(\frac{1}{2}-it\right) - \Gamma\left(\frac{1}{2}-it\right) = 0$$

Factoring the Gamma function:

$$\Gamma\left(\frac{1}{2}-it\right)\left[\left(\frac{2+4it}{4t^2+1}\right)\left(\frac{1}{2}-it\right)-1\right] = 0$$

Factoring 2:

$$\Gamma\left(\frac{1}{2}-it\right)\left[\left(\frac{2(1+2it)}{4t^2+1}\right)\left(\frac{1+2it}{2}\right)-1\right] = 0$$

Realizing the product:

$$\Gamma\left(\frac{1}{2}-it\right)\left[\frac{(1+2it)^2}{4t^2+1}-1\right] = 0$$

Adding:

$$\Gamma\left(\frac{1}{2}-it\right)\left(\frac{1+4it-4t^2-4t^2-1}{4t^2+1}\right)=0$$

Simplifying:

$$\Gamma\left(\frac{1}{2}-it\right)\left(\frac{4it-8t^2}{4t^2+1}\right)=0$$

Clearing the Gamma function we have:

$$\Gamma\left(\frac{1}{2}-it\right)=0\left(\frac{4t^2+1}{4t(i-2t)}\right)$$

As $t\in\mathbb{R}$, this fractional that is multiplied by zero (it should be noted that the t that we are driving is real), it is not indeterminate in any value that pertenesca to this set, therefore:

$$\Gamma\left(\frac{1}{2}-it\right)=0$$

Something to emphasize in this obtained value, is that, the result was zero without needing to change the value of t in complex.

Now, this value of t we found it driving the value $\frac{1}{2}+it$, so this zero is obtained when $t\in(0,\infty)$, ¿qué pasa con los conjugados?. Veamos:

$$\zeta\left(\frac{1}{2}-it\right)=2^{\frac{1}{2}-it}\pi^{\frac{1}{2}-it-1}\sin\left(\frac{\pi\left(\frac{1}{2}-it\right)}{2}\right)\Gamma\left(1-\left(\frac{1}{2}-it\right)\right)\zeta\left(1-\left(\frac{1}{2}-it\right)\right)$$

Replacing the conjugate

$$\zeta\left(\frac{1}{2}-it\right)=2^{\frac{1}{2}-it}\pi^{-\frac{1}{2}-it}\sin\left(\frac{\pi}{4}-i\frac{\pi t}{2}\right)\Gamma\left(1-\frac{1}{2}+it\right)\zeta\left(1-\frac{1}{2}+it\right)$$

Clearing finally you get

$$\zeta\left(\frac{1}{2}-it\right)=2^{\frac{1}{2}-it}\pi^{-\frac{1}{2}-it}\sin\left(\frac{\pi}{4}-i\frac{\pi t}{2}\right)\Gamma\left(\frac{1}{2}+it\right)\zeta\left(\frac{1}{2}+it\right)$$

In this case we see that it appears $\zeta\left(\frac{1}{2}+it\right)$, which we just proved to be zero, therefore, if $t<0$, $\zeta\left(\frac{1}{2}+it\right)$ becomes $\zeta\left(\frac{1}{2}-it\right)$, which we just saw that is zero and vice versa, therefore, $\zeta\left(\frac{1}{2}+it\right)$ it is zero if $t\in(-\infty,0)\cup(0,\infty)$.

To finish, let's see what happens if $t=0$.
The functional equation has a particular expression if $0<t<1$, given by:

$$\zeta(s)=\frac{1}{1-2^{1-s}}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s}$$

Doing $s=\frac{1}{2}$, we have:

$$\zeta\left(\frac{1}{2}\right)=\frac{1}{1-2^{1/2}}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{1/2}}$$

For an approximate calculation of this series, we will use MATLAB with the following code, let's see:

```
clear
clc
format long
suma=0;
for n=1:100000000
suma=suma+((−1)^(n + 1))/(n^(1/2));
zeta(1/2)=1/(1 − 2^(1/2))∗suma
ans=zeta(1/2)=-1.460233798132537
```

Then, from here you have:

$$\zeta\left(\frac{1}{2}\right) \approx -1{,}460233798132537... \neq 0$$

so, we have:

$$\zeta\left(\frac{1}{2}+it\right) = 0 \quad \text{with} \quad t \in (-\infty, 0) \cup (0, \infty)$$

And, indeed, the nontrivial zeros of the Riemann zeta function are in the critical line $\frac{1}{2} + it$.

4. CONCLUSION

The Riemann hypothesis is a claim that Riemann enunciated to demonstrate the number of prime numbers less than a given quantity, but the fact that this claim was true was not important; the veracity of this hypothesis was acquiring greater value with the passage of time since it was appearing when making statements such as the Casimir effect or the sum of positive integers (this in the field of physics and number theory).

This function has been very useful in multiple fields, but here is the doubt. Is this hypothesis true? Today, 158 years after his application, I present to the world a demonstration of the Riemann hypothesis, which verifies the approaches in which it was used in previous years.

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