Using Scientific Notation To Express Fermat's Last Theorem

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Abstract

A number base uses any whole number greater than one. Scientific notation can be used to express any whole number in any base. As Fermat's Last Theorem concerns whole numbers greater than one to powers of n, we can express it using scientific notation.

Introduction

Famously Fermat claimed to have a proof that

$$x^n + y^n = z^n$$

has no solutions in positive integers (x, y, z) for $n \geq 3$ [1]. The proof he had in mind was too long to fit into the margins of a book he was reading and hence he, as the legend has it, wasn't able to show the proof. Here we give some reasoning that suggests that the theorem is true.

Scientific notation

Fermat's Last Theorem can be expressed with

$$1.0E(n, a) + 1.0E(n, b) = E(n, c)$$

where $1.0E(n, a) = 1.0 \times a^n$. If $a = 10$, we have the usual scientific notation. For example, $1.0E(3, 10) = 1.0 \times 10^3 = 1000$.

For concreteness, suppose $n = 3$ and consider any c, c a positive integer, say $c = 5$. Then let C be a circle of radius $\frac{\sqrt{625}}{\sqrt{\pi}}$. This circle will consist of 5 sectors of area $5^3 = 125$ each. Is it possible that there exists a and b such that $a^4 < 625$ and $a^5 < 625$ with $a^3 + b^3 = c^3$? Clearly, $a = 3$ and $b = 4$ work. But how likely is it that you dial a 27 and then another 64 that you will land on 125? How likely is it that the sum of two, let's call them easy n-exponents in a base, is another easy n-exponent in a different base?

Imagine a circular slide rules in a base of an unusual kind. One starts with a disc with a movable radius on it. As one moves the radius sector areas are generated. One full revolution gives say x^n and then magically as one continues to revolve the radius the disc, the radius extends and a disc of area $x^n + y^n$ is formed. It's radius can't generate another area of z^n value. One dials the first and then adds the second to get the third, but given the mantissas are the same the exponents will have to vary, but that it is precisely what is required to stay the same. The squared case is an exemption; it is a circle.

References

[1] R. Blitzer, *Algebra and Trigonometry*, 4th ed., Upper Saddle, NJ, 2010.