An improved method of generating Z-number based on OWA weights and maximum entropy

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Abstract

How to generate Z-number is an important and open issue in the uncertain information processing of Z-number. In [1], a method of generating Z-number using OWA weight and maximum entropy is investigated. However, the meaning of the method in [1] is not clear enough according to the definition of Z-number. Inspired by the methodology in [1], we improve the method of determining Z-number based on OWA weights and maximum entropy, which is more clear about the meaning of Z-number. Some numerical examples are used to illustrate the effectiveness of the proposed method.

Keywords: Z-number, OWA, Maximum entropy, Reliability, Decision making

1. Introduction

Z-numbers, is proposed by Zadeh in 2011 to model uncertain information [2], which is different from the notion of Z-numbers proposed by Mahler [3].

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The inherent meaning or definition of Z-number in [2] is denoted as bellow,

$$Z = (A, B) = Z^{+}\left(A, \mu_A \cdot p_{X_A} \ is \ B\right)$$

which indicates if the variable X_A for A is a random variable, and Z^+ is a indicator of a Z^+-number[2], where p_{X_A} is the probability of random variable X_A for fuzzy set A, the membership function of fuzzy set A is denoted as $\mu_A(x)$, $x \in X_A$, $x \in R$, R is the real value domain, the membership function of fuzzy set B is denoted as $\mu_B(x)$, $x \in X_B$, $x \in R$, R is the real value domain. In addition, $\mu_A \cdot p_{X_A} \ is \ B$ indicates $\mu_B\left(\int \mu_A(x) \cdot p_{X_A}(x)\, dx\right)$. If X_A is a random variable, the meaning of Z-number is explained by

$$Z = Z^{+}\left(A, \mu_A \cdot p_{X_A} \ is \ B\right) \tag{1}$$

A simple Z-number is shown as Figure 1.

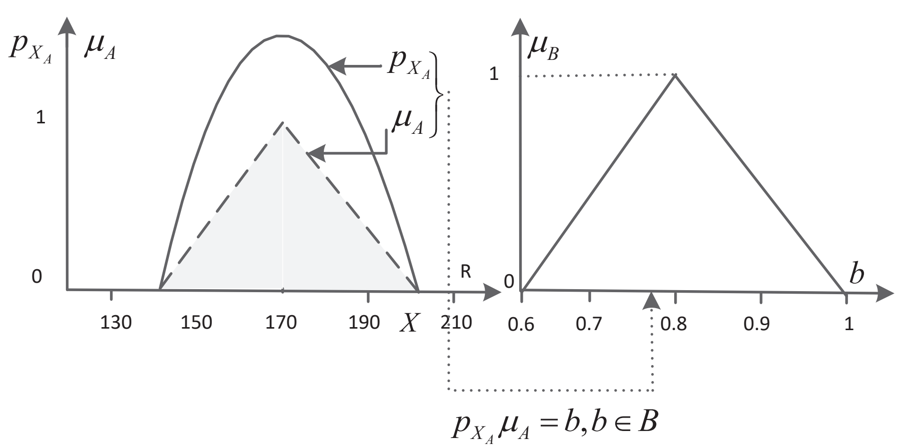

Figure 1: A simple Z-number

Based on the variable X_A for fuzzy set A is a random variable, Aliev et. al. established arithmetic of discrete Z-numbers [4], arithmetic of continuous

Z-numbers [5], approximate reasoning on Z-numbers[6], and functions on Z-numbers[7], etc[8].

Some other work on Z-number have been investigated, such as (1) theoretical researches of Z-number: arithmetic of discrete Z-numbers [4], arithmetic of continuous Z-numbers [5], Z*-numbers [9], modeling Z-number [10], and (2) applications of Z-number: Z-evaluations [11], computing with words (CWW) [12], modeling in psychological research [13], Z-number-based linear programming [14], sensor data fusion [15], dynamic plant control [16], data envelopment analysis [17], failure modes risk assessment [18, 19], medical diagnosis [20], stable strategies analysis [21], and other decision making models with Z-number [22, 23, 24, 25, 26, 27].

From the previous study of Z-number, the information of Z-number is given directly by the domain experts subjectively. In [1], a method of generating Z-number based on OWA weight using maximum entropy is initially proposed to guide the determination of Z-number to reduce the subjectiveness from the experts. The main idea of generating Z-number in [1] is summarized as two steps: (1) Determine the OWA weights using maximum entropy according the the orness measure; (2) Generating Z-number using the generated probabilities (weights) of the fuzzy random variable. Especially, in the second step, the authors in [1] assume that $B = \int_R \mu_A(x)\, p_X(x)\, dx$ and $\mu_B(x) = p_X(x)\, \mu_A(x)$. According to the suggestion of Zadeh [2] and Aliev [4, 5], the membership function of the reliability B should be better explained by $\mu_B\left(\int \mu_A(x) \cdot p_{X_A}(x)\, dx\right)$, which indicates that the value of the part of reliability B should be $b = \int \mu_A(x) \cdot p_{X_A}(x)\, dx$, where $b \in B$. In addition, the membership function should be related to decision attitude of the expert.

Hence, we improve the method of generating Z-number inspired in [1] and the thought of Zadeh [2] and Aliev [4, 5] as the following three steps: (1) Determine the OWA weights using maximum entropy according the the orness measure; (2) Generate the value of reliability part of Z-number; (3) Generate the membership function of the reliability part of Z-number. The proposed method is more clear about the meaning of Z-number than previous study to deal with the reliability of Z-information.

The paper is organized as follows. The preliminaries of OWA operator, orness measure are briefly introduced in Section 2. In Section 3, The method of generating Z-number based on OWA weights and maximum entropy is improved and proposed . Section 3, some numerical examples are used to illustrate the effectiveness of the proposed method. Finally, this paper is concluded in Section 4.

2. Preliminaries

Definition 2.1. *OWA operator [28, 29, 30]: An OWA operator of dimension n is a function $F : R^n \to R$ that has an associated n vector W, $W = [w_1, w_2, \ldots, w_n]^T$ such that $w_i \in [0,1], \sum_i w_i = 1$. Furthermore,*

$$F(a_1, a_2, \ldots, a_n) = \sum_j w_j b_j \tag{2}$$

where b_j is the jth largest of the a_i. OWA operator is a simple and effective aggregating formulation in the data fusion, which has been applied in many *applications, such as aggregating probability distribution [31], multi-criteria decision making [32, 33].*

Definition 2.2. *Orness measure [28]: orness measure is proposed by Yager to evaluate the decision maker's attitude, which is denoted by*

$$orness\,(W) = \frac{1}{n-1} \sum\nolimits_{i=1}^{n} \left[(n-1)\, w_i \right] \tag{3}$$

(1) Pessimistic attitude. If we select W, where $W = W_* = [0, 0, \ldots, 1]^T$, this is the aggregation rule used in the pessimistic strategy.(2) Optimistic attitude. If we select W, where $W = W^* = [1, 0, \ldots, 0]^T$, this is used in the optimistic strategy.(3) Normative approach.If we select W,where $W = W_{AVG}[1/n, 1/n, ..., 1/n]^T$, essentially, this function is the normative strategy.

Definition 2.3. *The maximum entropy to evaluate the weight distribution according to orness measure (decision maker's attitude) [34], which is denoted by*

$$Maximize: \quad H\,(W) = -\sum w_i \ln\,(w_i) \tag{4}$$

$$s.t \begin{cases} \alpha = \frac{\sum_{i=1}^{n} [(n-i) w_i]}{n-1} \\ \sum_i w_i = 1 \\ 0 \leq w_i \leq 1 \end{cases} \tag{5}$$

In the next section, we improve the method of generating Z-number based on OWA weights and maximum entropy. The improved method considers additional value of the reliability part B and membership function of the reliability part B comparing with the work in [1].

3. Improved method of generating Z-number

3.1. Determine the OWA weights using maximum entropy according the the orness measure

Assume the constraint part of fuzzy A and orness value α are given, the membership function of A is denoted by $\mu_A(x)$, $x \in X$, and X is a random variable, we can evaluate the OWA weights by solving the following optimization problem.

$$Maximize: \quad H(W) = -\sum w_i \ln(w_i) \tag{6}$$

$$s.t \begin{cases} \alpha = \frac{\sum_{i=1}^{n}[(n-i)w_i]}{n-1} \\ \sum_i w_i = 1 \\ 0 \leq w_i \leq 1 \end{cases} \tag{7}$$

where w_i is the weight of the i-th criteria, α is the orness measure (Eq. (3)). Then the obtained probability of the variable X for the constraint part A is defined as

$$p_X(x_i) = w_{n-i+1} \tag{8}$$

3.2. Generate the value of reliability part of Z-number

After evaluation of the probability $p_X(x_i)$ of the variable X for the constraint part A, we obtain the value of the reliability part B, i.e. b = p(A), which id denoted by

$$b = p(A) = \int_R \mu_A(x)\, p_X(x)\, dx, \; b \in B \tag{9}$$

where R is domain of the real number.

3.3. Generate the membership function of the reliability part of Z-number

The membership function of the reliability part B is relate to the attitude of the decision maker, i.e. orness measure, which is denoted by

$$\mu_B(x) = \mu_{p_A}(p_A) = \alpha \tag{10}$$

The relation between the orness measure and membership function of B is shown in Figure 2.

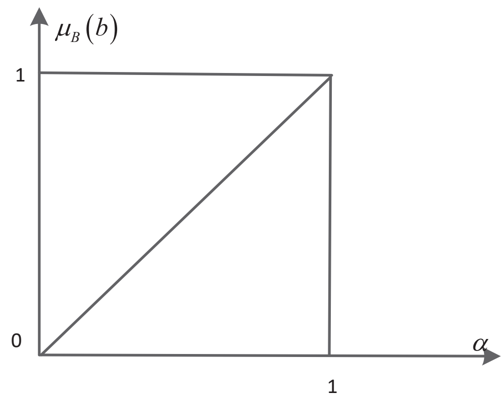

Figure 2: Relation between the orness measure and membership function of B

At last the generated Z-number is $Z = (A, \alpha/b)$, α is the orness value and b is given in Eq. (9)

In next section, we use the examples in [1] to illustrate the effectiveness of the proposed method.

4. Numerical examples

Suppose an expert want to establish a fuzzy model to evaluate the quality of some certain goods. First, the expert gives a fuzzy number according to his special domain knowledge. Assume the fuzzy number is expressed as

$$A = \frac{\mu_A}{u} = \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{1}{4} + \frac{1}{5} + \frac{0.7}{6} + \frac{0.5}{7} + \frac{0}{8} \tag{11}$$

$$\tag{12}$$

Assume the quality of the goods increase better from 1 to 8. His attitude for this issue may be different, possible with pessimistic attitude, possible with optimistic attitude, possible with neutral attitude, and possible with other attitude. His attitude can be calculated using the orness measure given in Eq. (3). The following five cases in [1] with different orness meaning are considered including pessimistic attitude, optimistic attitude, normative attitude, and other two different attitude.

Case 4.1. *Assume the expert gives his opinion* $A = \frac{\mu_A}{x} = \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{1}{4} + \frac{1}{5} + \frac{0.7}{6} + \frac{0.5}{7} + \frac{0}{8}$*, his decision attitude (preference) for this evaluation is optimistic (orness measure* $\alpha = 1$*). This means* $W = W_* = [1, 0, \ldots, 0]^T$ *by the maximum entropy method using Eq. (6), which can be regarded as the probability of the random variable* X *given as* $p_X = [0, 0, \ldots, 1]$*.*

Then we can obtain the fuzzy probability distribution function of constraint part A, i.e. $p_X(x_i)\mu_A(x_i)/x_i$*,*

$$\begin{aligned}
&p_X(x_i)\mu_A(x_i)/x_i = \\
&= \frac{0 \times 0}{1} + \frac{0.5 \times 0}{2} + \frac{0.7 \times 0}{3} + \frac{1 \times 0}{4} + \frac{1 \times 0}{5} + \frac{0.7 \times 0}{6} + \frac{0.5 \times 0}{7} + \frac{0 \times 1}{8} \\
&= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0}{5} + \frac{0}{6} + \frac{0}{7} + \frac{0}{8}
\end{aligned}$$

Next, we obtain the fuzzy probability of constraint part A, i.e. $P(A)$*.*

According to Eq. (9), we can get the value of reliability part B, i.e. $b = P(A)$,

$$
\begin{aligned}
& b = P(A) \\
& = 0 \times 0 + 0.5 \times 0 + 0.7 \times 0 + 1 \times 0 + 1 \times 0 + 0.7 \times 0 + 0.5 \times 0 + 0 \times 1 \\
& = 0
\end{aligned}
$$

According to Eq. (10), we can get the membership function of reliability part B, i.e. $\mu_B(x)$,

$$\mu_B(x) = \mu_{p_A}(p_A) = \alpha = 1$$

Hence, the reliability part $B = \mu_B/b = 1/0$, the generated Z-number $Z = (A, B)$ is shown as Figure 3 and Figure 4.

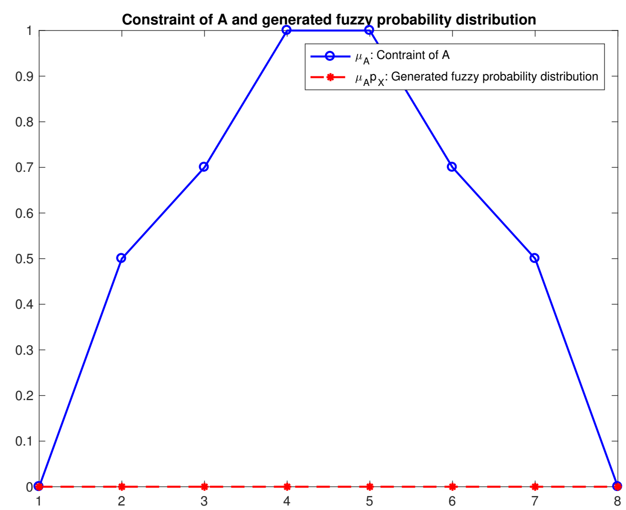

Figure 3: Membership function of constraint part A and the fuzzy probability distribution function of A

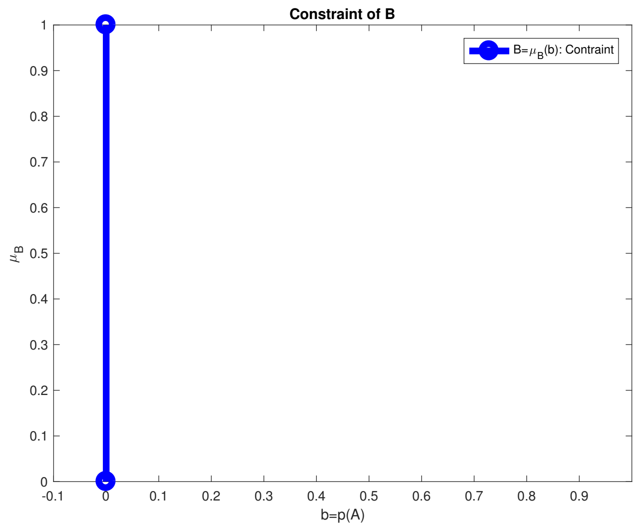

Figure 4: Membership function of reliability part B

Result analysis in Case 1: The attitude of the expert is optimistic (i.e. orness measure $\alpha = 1$), which indicates he want to give a high evaluation, i.e., maybe the index of the quality of the goods is 8. However, the fuzzy evaluation of A given by himself is not consistent with the real attitude of himself. Hence, the evaluation of A is not reliable for the optimistic attitude, the proposed method can cover this situation, i.e. the obtained reliability part $B = \mu_B/b = 1/0$.

Case 4.2. *Assume the expert gives his opinion* $A = \frac{\mu_A}{x} = \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{1}{4} + \frac{1}{5} + \frac{0.7}{6} + \frac{0.5}{7} + \frac{0}{8}$, *his decision attitude (preference) for this evaluation is pessimistic (orness measure* $\alpha = 0$*). This means* $W = W_* = [0, 0, \ldots, 1]^T$ *by the maximum entropy method using Eq. (6), which can be regarded as the probability of the random variable* X *given as* $p_X = [1, 0, \ldots, 0]$.

Then we can obtain the fuzzy probability distribution function of constraint part A, i.e. $p_X(x_i)\mu_A(x_i)/x_i$,

$$
\begin{aligned}
&p_X(x_i)\mu_A(x_i)/x_i = \\
&= \frac{0\times 1}{1} + \frac{0.5\times 0}{2} + \frac{0.7\times 0}{3} + \frac{1\times 0}{4} + \frac{1\times 0}{5} + \frac{0.7\times 0}{6} + \frac{0.5\times 0}{7} + \frac{0\times 0}{8} \\
&= \frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0}{5} + \frac{0}{6} + \frac{0}{7} + \frac{0}{8}
\end{aligned}
$$

Next, we obtain the fuzzy probability of constraint part A, i.e. $P(A)$. *According to Eq. (9), we can get the value of reliability part* B*, i.e.* $b = P(A)$,

$$
\begin{aligned}
b &= P(A) \\
&= 0\times 1 + 0.5\times 0 + 0.7\times 0 + 1\times 0 + 1\times 0 + 0.7\times 0 + 0.5\times 0 + 0\times 0 \\
&= 0
\end{aligned}
$$

According to Eq. (10), we can get the membership function of reliability part B*, i.e.* $\mu_B(x)$,

$$\mu_B(x) = \mu_{p_A}(p_A) = \alpha = 0$$

Hence, the reliability part $B = \mu_B/b = 0/0$*, the generated Z-number* $Z = (A, B)$ *is shown as Figure 5 and Figure 6.*

Result analysis in Case 2: The attitude of the expert is pessimistic (i.e. orness measure $\alpha = 0$), which indicates he want to give a low evaluation, i.e., maybe the index of the quality of the goods is 1. However, the fuzzy evaluation of A given by himself is not consistent with the real attitude of himself. Hence, the evaluation of A is not reliable for the optimistic attitude, the proposed method can cover this situation, i.e. the obtained reliability part $B = \mu_B/b = 0/0$.

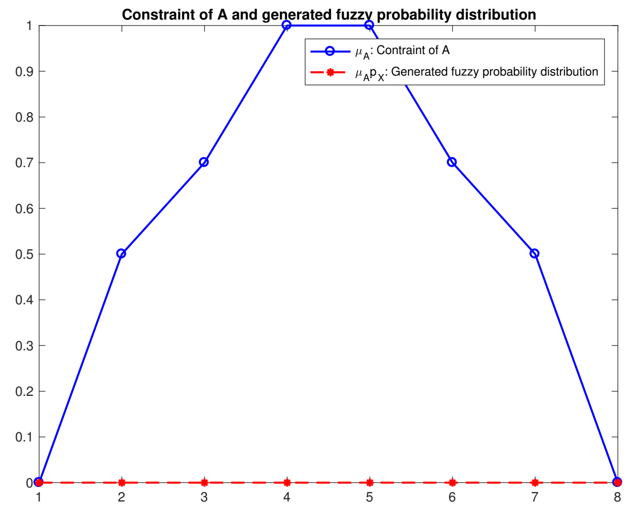

Figure 5: Membership function of constraint part A and the fuzzy probability distribution function of A

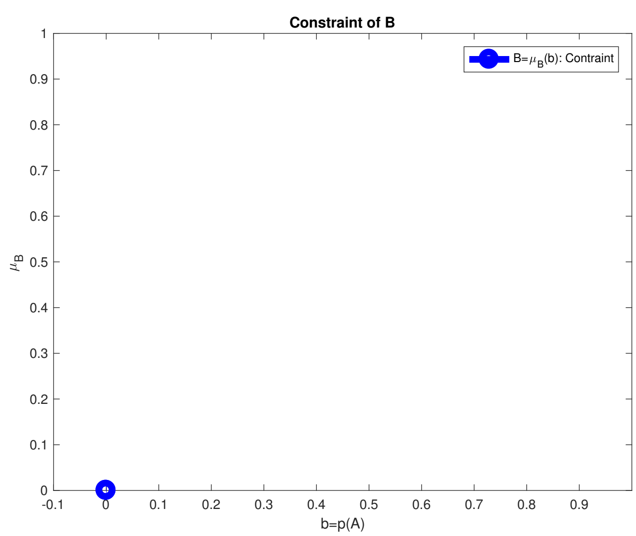

Figure 6: Membership function of reliability part B

Case 4.3. *Assume the expert gives his opinion* $A = \frac{\mu_A}{x} = \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{1}{4} + \frac{1}{5} + \frac{0.7}{6} + \frac{0.5}{7} + \frac{0}{8}$*, his decision attitude (preference) for this evaluation is normative (orness measure* $\alpha = 0.5$*). This means* $W = W_* = [1/8, 1/8, \ldots, 1/8]^T$ *by the maximum entropy method using Eq. (6), which can be regarded as the probability of the random variable* X *given as* $p_X = [1/8, 1/8, \ldots, 1/8]$*.*

Then we can obtain the fuzzy probability distribution function of constraint part A*, i.e.* $p_X(x_i)\mu_A(x_i)/x_i$*,*

$$
\begin{aligned}
&p_X(x_i)\mu_A(x_i)/x_i = \\
&= \frac{0 \times 1/8}{1} + \frac{0.5 \times 1/8}{2} + \frac{0.7 \times 1/8}{3} + \frac{1 \times 1/8}{4} + \frac{1 \times 1/8}{5} + \frac{0.7 \times 1/8}{6} \\
&+ \frac{0.5 \times 1/8}{7} + \frac{0 \times 1/8}{8} \\
&= \frac{0}{1} + \frac{0.0625}{2} + \frac{0.0875}{3} + \frac{0.1250}{4} + \frac{0.1250}{5} + \frac{0.0875}{6} + \frac{0.0625}{7} + \frac{0}{8}
\end{aligned}
$$

Next, we obtain the fuzzy probability of constraint part A*, i.e.* $P(A)$*. According to Eq. (9), we can get the value of reliability part* B*, i.e.* $b = P(A)$*,*

$$
\begin{aligned}
&b = P(A) \\
&= 0 + 0.0625 + 0.0875 + 0.1250 + 0.1250 + 0.0875 + 0.0625 + 0 \\
&= 0.55
\end{aligned}
$$

According to Eq. (10), we can get the membership function of reliability part B*, i.e.* $\mu_B(x)$*,*

$$\mu_B(x) = \mu_{p_A}(p_A) = \alpha = 0.5 \tag{13}$$

Hence, the reliability part $B = \mu_B/b = 0.5/0.55$*, the generated Z-number* $Z = (A, B)$ *is shown as Figure 7 and Figure 8.*

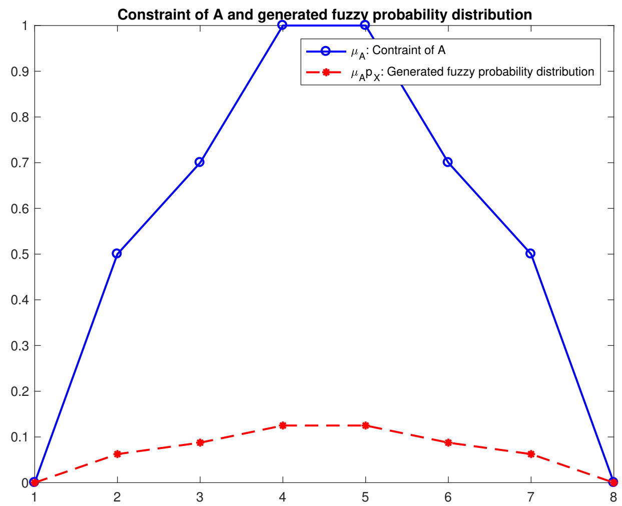

Figure 7: Membership function of constraint part A and the fuzzy probability distribution function of A

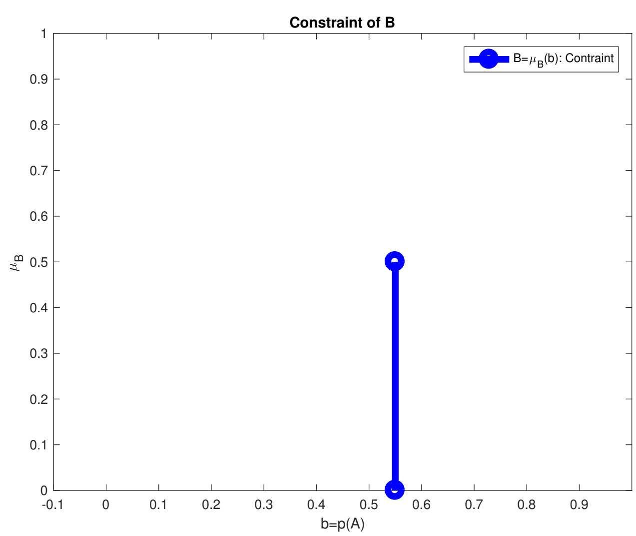

Figure 8: Membership function of reliability part B

Result analysis in Case 3: The attitude of the expert is normative (i.e. orness measure $\alpha = 0.5$), which indicates he want to give a medium evaluation, i.e., maybe the index of the quality of the goods is 4 or 5. However, the fuzzy evaluation of A given by himself is relative consistent with the real attitude of himself. Hence, the evaluation of A is relative reliable for the optimistic attitude, the proposed method can cover this situation, i.e. the obtained reliability part $B = \mu_B/b = 0.5/0.55$.

Case 4.4. *Assume the expert gives his opinion* $A = \frac{\mu_A}{x} = \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{1}{4} + \frac{1}{5} + \frac{0.7}{6} + \frac{0.5}{7} + \frac{0}{8}$*, his decision attitude (preference) for this evaluation is relative optimistic (orness measure* $\alpha = 0.7$*). This means* $W = W_{0.7} =$ *[0.2756, 0.2119, 0.1566, 0.1186, 0.0880, 0.0630, 0.0510, 0.0352]*^{T} *by the maximum entropy method using Eq. (6), which can be regarded as the probability of the random variable* X *given as* p_X*=[0.0352, 0.0510, 0.0630, 0.0880, 0.1186, 0.1566, 0.2119, 0.2756].*

Then we can obtain the fuzzy probability distribution function of constraint part A, i.e. $p_X(x_i)\mu_A(x_i)/x_i$,

$$
\begin{aligned}
&p_X(x_i)\mu_A(x_i)/x_i = \\
&= \frac{0 \times 0.0352}{1} + \frac{0.5 \times 0.0510}{2} + \frac{0.7 \times 0.0630}{3} + \frac{1 \times 0.0880}{4} \\
&+ \frac{1 \times 0.1186}{5} + \frac{0.7 \times 0.1566}{6} + \frac{0.5 \times 0.2119}{7} + \frac{0 \times 0.2756}{8} \\
&= \frac{0}{1} + \frac{0.0255}{2} + \frac{0.0441}{3} + \frac{0.0880}{4} + \frac{0.1186}{5} + \frac{0.1096}{6} + \frac{0.1060}{7} + \frac{0}{8}
\end{aligned}
$$

Next, we obtain the fuzzy probability of constraint part A, i.e. $P(A)$.

According to Eq. (9), we can get the value of reliability part B, i.e. $b = P(A)$,

$$\begin{aligned} b &= P(A) \\ &= 0 + 0.0255 + 0.0441 + 0.0880 + 0.1186 + 0.1096 + 0.1060 + 0 \\ &= 0.4918 \end{aligned}$$

According to Eq. (10), we can get the membership function of reliability part B, i.e. $\mu_B(x)$,

$$\mu_B(x) = \mu_{p_A}(p_A) = \alpha = 0.7$$

Hence, the reliability part $B = \mu_B/b = 0.7/0.4918$, the generated Z-number $Z = (A, B)$ is shown as Figure 9 and Figure 10.

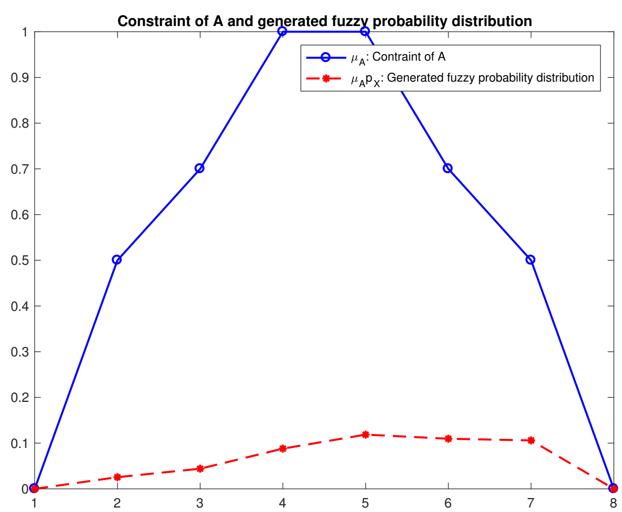

Figure 9: Membership function of constraint part A and the fuzzy probability distribution function of A

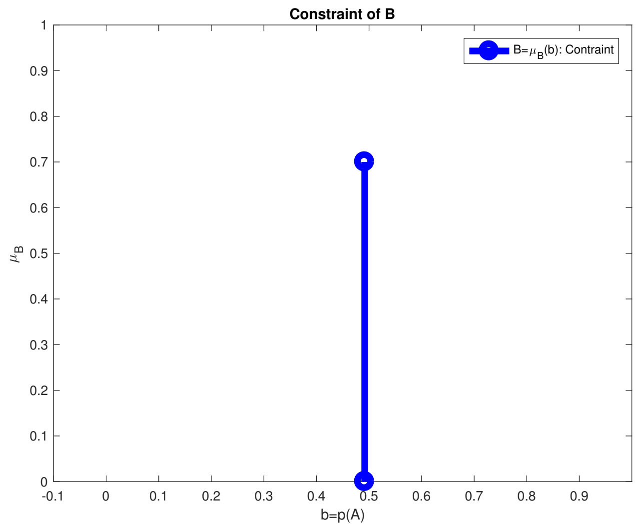

Figure 10: Membership function of reliability part B

Result analysis in Case 4: The attitude of the expert is relative optimistic (i.e. orness measure $\alpha = 0.7$), which indicates he want to give a relative high evaluation, i.e., maybe the index of the quality of the goods is 5, 6 or 7. However, the fuzzy evaluation of A given by himself is not consistent with the real attitude of himself. Hence, the evaluation of A is not reliable for the optimistic attitude, the proposed method can cover this situation, i.e. the obtained reliability part $B = \mu_B/b = 0.7/0.4918$.

Case 4.5. *Assume the expert gives his opinion* $A = \frac{\mu_A}{x} = \frac{0}{1} + \frac{0.5}{2} + \frac{0.7}{3} + \frac{1}{4} + \frac{1}{5} + \frac{0.7}{6} + \frac{0.5}{7} + \frac{0}{8}$*, his decision attitude (preference) for this evaluation is relative pessimistic (orness measure* $\alpha = 0.2$*). This means* $W = W_{0.2} =$ *[0.0129, 0.0207, 0.0338, 0.0548, 0.0898, 0.1445, 0.2387, 0.4047]*^{T} *by the maximum entropy method using Eq. (6), which can be regarded as the probability of the* *random variable X given as p_X=[0.4047, 0.2387, 0.1445, 0.0898, 0.0548, 0.0338, 0.0207, 0.0129].*

Then we can obtain the fuzzy probability distribution function of constraint part A, i.e. $p_X(x_i)\mu_A(x_i)/x_i$,

$$
\begin{aligned}
&p_X(x_i)\mu_A(x_i)/x_i = \\
&= \frac{0 \times 0.4047}{1} + \frac{0.5 \times 0.2387}{2} + \frac{0.7 \times 0.1445}{3} + \frac{1 \times 0.0898}{4} \\
&+ \frac{1 \times 0.0548}{5} + \frac{0.7 \times 0.0338}{6} + \frac{0.5 \times 0.0207}{7} + \frac{0 \times 0.0129}{8} \\
&= \frac{0}{1} + \frac{0.1194}{2} + \frac{0.1012}{3} + \frac{0.0898}{4} + \frac{0.0548}{5} + \frac{0.0237}{6} + \frac{0.0104}{7} + \frac{0}{8}
\end{aligned}
$$

Next, we obtain the fuzzy probability of constraint part A, i.e. $P(A)$. According to Eq. (9), we can get the value of reliability part B, i.e. $b = P(A)$,

$$
\begin{aligned}
&b = P(A) \\
&= 0 + 0.1194 + 0.1012 + 0.0898 + 0.0548 + 0.0237 + 0.0104 + 0 \\
&= 0.3993
\end{aligned}
$$

According to Eq. (10), we can get the membership function of reliability part B, i.e. $\mu_B(x)$,

$$\mu_B(x) = \mu_{p_A}(p_A) = \alpha = 0.2$$

Hence, the reliability part $B = \mu_B/b = 0.2/0.3993$, the generated Z-number $Z = (A, B)$ is shown as Figure 11 and Figure 12.

Result analysis in Case 5: The attitude of the expert is relative pessimistic (i.e. orness measure $\alpha = 0.2$), which indicates he want to give a relative

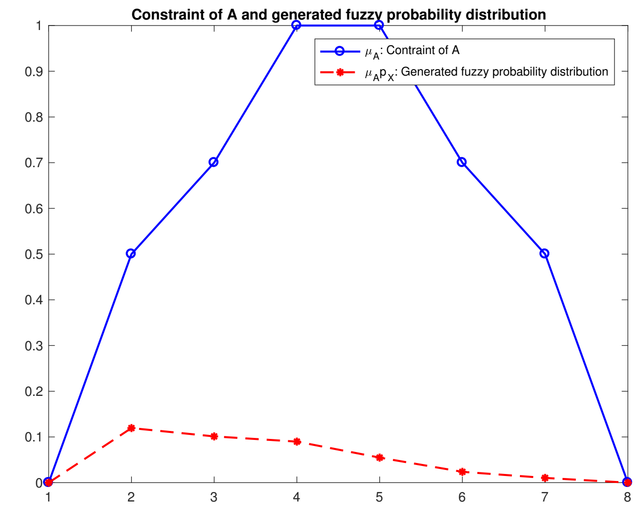

Figure 11: Membership function of constraint part A and the fuzzy probability distribution function of A

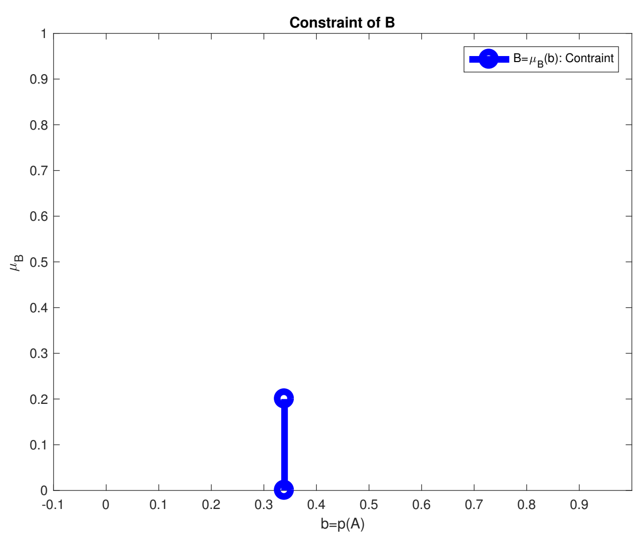

Figure 12: Membership function of reliability part B

high evaluation, i.e., maybe the index of the quality of the goods is 2, or 3. However, the fuzzy evaluation of A given by himself is not consistent with the real attitude of himself. Hence, the evaluation of A is not reliable for the optimistic attitude, the proposed method can cover this situation, i.e. the obtained reliability part $B = \mu_B/b = 0.2/0.3993$.

5. Conclusion

How to generate Z-number is an important and open issue in the uncertain information processing of Z-number. Inspired by the methodology in [1], we improve the method of determining Z-number based on OWA weights and maximum entropy, which is more clear about the meaning of Z-number. Some numerical examples are used to illustrated the effectiveness of the proposed method. Results show that the method can generate Z-number effectively to evaluate the reliability of the uncertain information.

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