question 453 : An Interesting Radical

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abstract
This note presents some formulas related with the real root of the equation:
$$x^{11} - x^{10} - 1 = 0$$

Formulas

- Let

$$R = \sqrt[11]{1 + \sqrt[11/10]{1 + \sqrt[11/10]{1 + \sqrt[11/10]{1 + \ldots}}}}$$

then

$$R^{11} - R^{10} - 1 = 0$$

$$R^9 - R^7 - R^6 + R^4 + R^3 - R - 1 = 0$$

$$\frac{1}{R} = \frac{1}{11} \sum_{n=0}^{\infty} \frac{(-1)^n \Gamma\left(\frac{n+1}{11}\right)}{\Gamma\left(\frac{12 - 10n}{11}\right) n!}$$

$$\frac{\pi \phi}{5} + R^{-2} = \int_R^{\infty} \left(\frac{x^8}{1 + x^{10}} + \frac{\sqrt[10]{x - 1}}{x^2} \right) dx$$

$$\frac{\pi \phi}{5} + R^{-2} = \frac{1}{R} \, {}_2F_1\left(1, \frac{1}{10}, \frac{11}{10}, -R^{-10}\right) + \frac{10}{9} R^{-9/10} \, {}_2F_1\left(-\frac{1}{10}, \frac{9}{10}, \frac{19}{10}, \frac{1}{R}\right)$$

Remarks:

- The number pi: $\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.
- Golden mean: $\phi = \frac{1 + \sqrt{5}}{2}$.
- Hypergeometric function: ${}_2F_1(a, b, c, x) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} x^n$, $|x| < 1$.

• Gamma function: $\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} \, dx$, $x > 0$.

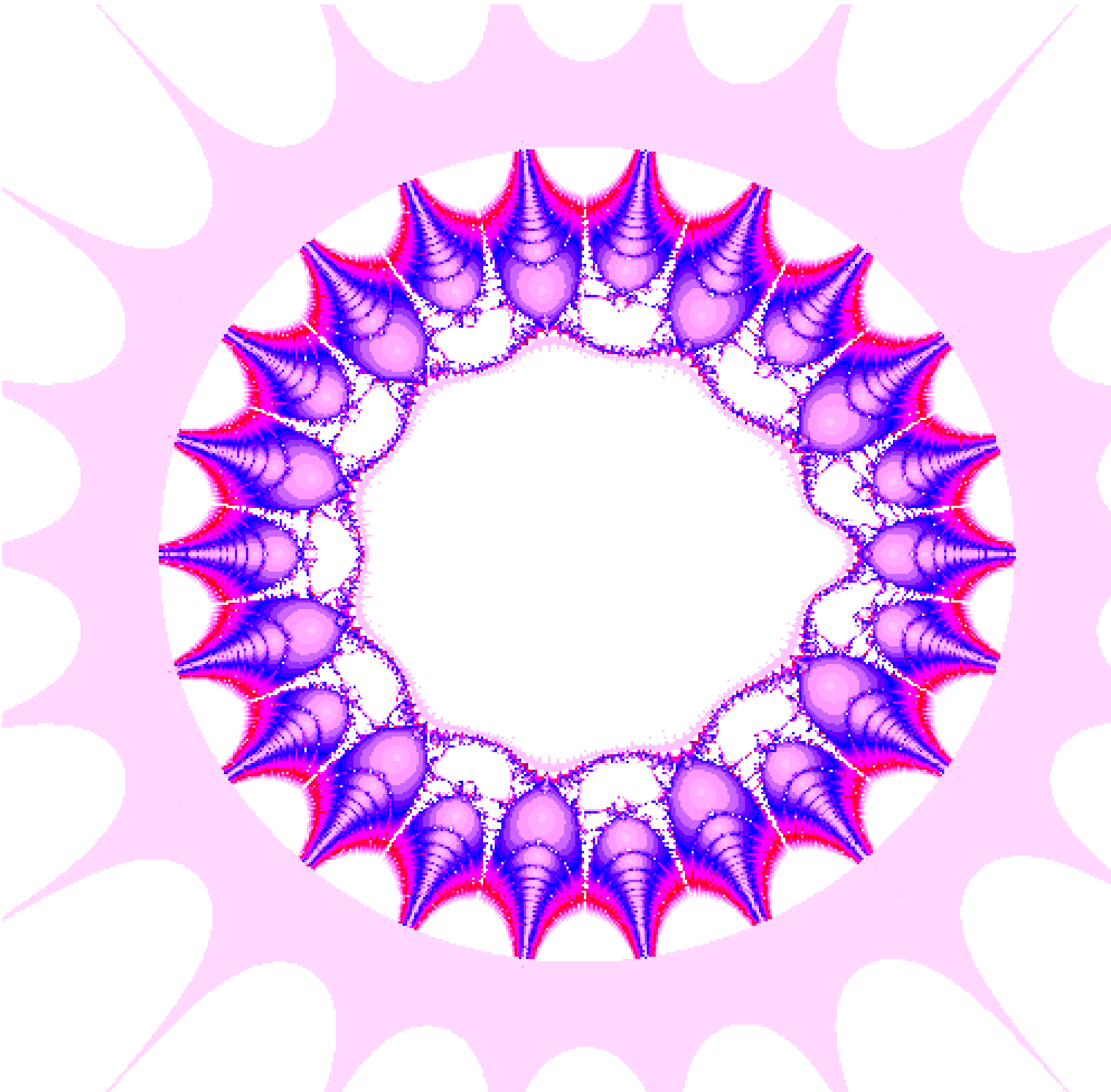

Fig.1 Newton - Julia set for: $f(z) = \sin(z^{11} - z^{10} - 1)$

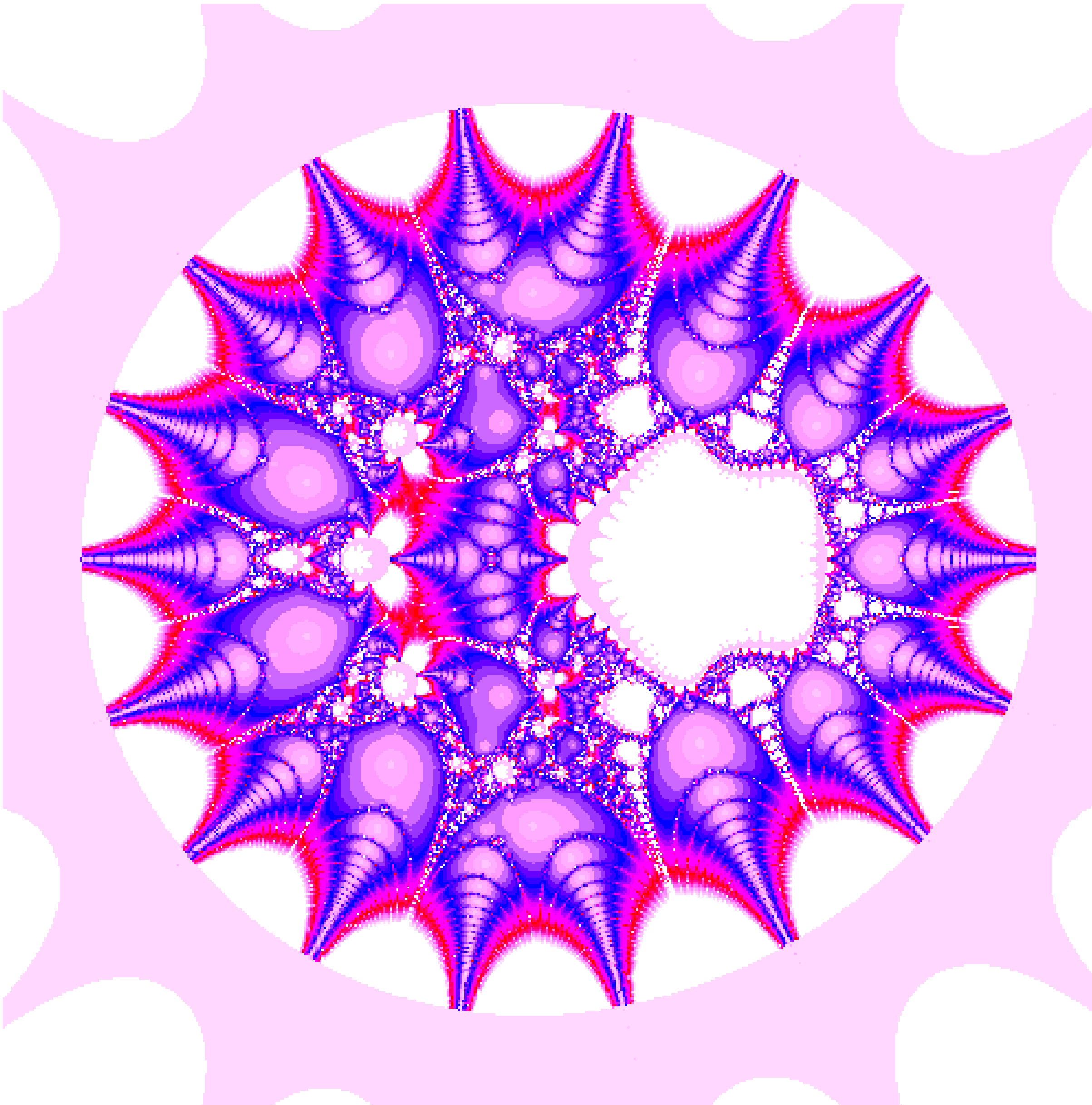

Fig. 2 Newton - Julia set for: $f(z) = \sin\left(z^9 - z^7 - z^6 + z^4 + z^3 - z - 1\right)$

References

• S. Ramanujan: Collected Papers , Chelsea , New York , 1962 .
• E. Valdebenito: Collected Papers , viXra.org .