Proving Fermat's Last Theorem Using Circles

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Introduction

Famously Fermat claimed to have a proof that

$$x^n + y^n = z^n$$

has no solutions in positive integers (x, y, z) for $n \geq 3$ [1]. The proof he had in mind was too long to fit into the margins of a book he was reading and hence he, as the legend has it, wasn't able to show the proof. Given that it almost could have fit what might it have been?

Here we give a geometric argument that can almost fit into the white space, the margins of a typical page, of a say a modern undergraduate calculus textbook. The idea almost fits; it's a single paragraph.

Marginal proof

Let A, B, and C be three concentric circles of radii $\frac{3}{\sqrt{\pi}}$, $\frac{4}{\sqrt{\pi}}$, and $\frac{(3+4=7)}{\sqrt{\pi}}$, respectively. In this case a fraction of the area of C gives the sum of the areas of A and B, but this is only possible when

$$\pi \left(\frac{n}{\sqrt{\pi}} \right)^2 = a^2.$$

That is when the left hand exponent is 2. This follows from the transcendence of π.

Conclusion

The proof likely can't be the one Fermat had in his mind. Fermat (1601-1665) pre-dates Lambert (irrationality of π) [2] and Lindemann (transcendence of π) [3] by a century or more. But Fermat would have an approximation of π and could have sensed an elaboration, the consideration of a finite number of cases, would not fit.

I don't think the proof is correct, but it is tantalizingly simple: it approaches the problem by way of showing one implication of one case, a squared case that exists: $3^2 + 4^2 = 5^2$. It is true that solutions to

$$\pi \left(\frac{a}{\sqrt{\pi}} \right)^n = a^n,$$

a and n positive integers, are only possible when $n = 2$.

References

[1] R. Blitzer, *Algebra and Trigonometry*, 4th ed., Upper Saddle, NJ, 2010.

[2] J. Lambert, Mémoire sur quelques propriétés remarquables des quantitiés transcendentes circulaires et logarithmiques, *Historie de l'Académie Royale des Sciences et des Belles-Lettres der Berlin* **17** (1761) 265–276.

[3] F. Lindemann, Über die Zahl π, *Math. Ann.* **20** (1882) 213–225.