SunQM-3: Solving Schrodinger equation for Solar quantum mechanics {N,n} structure

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Abstract

In this paper, Schrodinger equation is established for the non-spin Solar QM {N,n} structure model. Solving the Schrodinger equation gives the orbit energy E_n = -3.81E+11 *(m / n^2), where m is the orbital moving object's mass (in kg), and n is the total n of the orbit using Sun core {0,1} as r_1. Combining r^2*$|R_{nl}|$^2 to the deduced Sun's interior mass density formula D=1.26E+23/r^2.33, a plot of {N,n} QM calculated (that is close to the true) mass density vs. r (within Sun ball {0,2}) has been made.

Introduction

In memory of Erwin Schrödinger (who proposed Schrodinger equation in 1925), Max Born (who proposed $|\Psi|^2 \propto$ probability in 1926), and Wolfgang Pauli (who proposed the exclusion principle in 1925).

A series of papers [1] are used to present my discovery of Solar system quantum mechanics {N,n} structure: Paper-1, how to quantize the orbits of Solar system solely based on the quantum orbit relationship of $r_n = r_1*n^2$, which was first obtained from Bohr model, and had also been proved to be correct in Schrodinger equation solution. Paper-2, how to quantize the orbit energy of Solar system using E = hf, and how it led to the discovery of a new constant, a generalized Planck constant (named H-C unit). Paper-3, how to seamlessly transform Schrodinger's equation/solution into Bohr-kind simple model for Solar system. Paper-5, how to use a new QM method (C-QM, based on interior {N,n} QM, multiplier n', |R(n,l)|^2 |Y(l,m)|^2 guided mass occupancy, and RF) to analyze our world in scales from string to universe. Paper-6, relativistic C-QM. Each of these main papers has its own supplementary papers to present my additional results/thoughts that related to the main paper. Papers are abbreviated as: Main paper: SunQM-1, SunQM-2, SunQM-3, SunQM-5, etc. Supplementary paper: SunQM-1s1, SunQM-1s2, SunQM-1s3, SunQM-2s1, SunQM-3s1, SunQM-3s2, SunQM-3s3, SunQM-3s6, SunQM-5s1, etc.

In paper SunQM-1, I successfully extended $r_n = r_1*n^2$ relationship from Bohr model to Solar system's {N,n} QM structure. In paper SunQM-2, I also successfully extended E = hf relationship from micro-world's QM to Solar system and macro-world QM. In {N,n} QM model, a series of regular quantum number n (from n=1 to n=6) is grouped by a higher level quantum number N (named N-period, or N super-shell). In current paper (SunQM-3), I try to use Schrodinger equation to model the n states (from 1 to 5) within each N super-shell, so that I can use Schrodinger equation and solution to describe the Solar {N,n} QM structure. Therefore I can seamlessly transform a pre-Sun's ball-like structure into a Bohr-kind simple structure for Solar system (see paper SunQM-3s1). Furthermore, I can use the same method to calculate out a series of snapshot pictures of pre-Sun ball disk-lization process (see paper SunQM-3s2). Note: for {N,n} QM nomenclature as well as the general notes for {N,n} QM model, please see my paper SnQM-p1 section VII and VIII. Note: Microsoft Excel's number format is often used in this paper, for example: x^2 = x^2, 3.4E+12 = $3.4*10^{12}$, 5.6E-9 = $5.6*10^{-9}$.

I. Schrodinger equation and solution for a central G-force (non-spin) pre-Sun ball structure

A pre-solar nebula, here I named it as "pre-Sun" or "pre-Sun ball", can be simplified as a ball-like structure, with the mass density decreases as the r increases (from the reduced mass center) inside the ball. According to paper SunQM-1, the Solar system has a {N,n} QM structure, where n=1, 2, ..6, and N from 0, 1, 2, .. 5, or even higher. What I am going to do here is to build (and then to solve) Schrodinger equation for a spherical shell space between {N,1} to {N,6}, with quantum number n from 1 to 6, but only for a single N super shell. Then extend the same pattern from N to N-1, N-2, …, to the whole pre-Sun ball.

I-a. A pre-Sun ball model for QM calculation

Let us set a pre-Sun ball model of {N+1,1}RF, which is just collapsed from {N+2,1}RF, and is going to further collapse into {N,1}RF in the future. The mass density =0 at r > r of {N+1,1}RF ball, and mass density >0 at r ≤ r of {N+1,1}RF ball, so this pre-Sun ball has a edge at {N+1,1}. We can further set {N+1,1} = {6,1}, with r = 2.99E+17 meters, or 2.0E+6 AU, or 36× larger in r than the most outer edge of Oort cloud. Now let us study an object moving in space between {N+1,1}={N,6} and {N,1}, or between {6,1}={5,6} and {5,1}.

From paper SunQM-1 we know that mass density in the shell space between {5,1} and {6,1} was much lower than that in {5,1}RF pre-Sun ball. Inside the spherical shell space between {5,1} and {6,1}, all objects (gas, solid fragments) stayed in theirs circular (or eccentric) RF orbits driven by Newton's second low: the force for the circular movement $F=ma=m\ v_n^2/r_n$, was provided and balanced by Newton's low of universal gravitation $F=G*Mm/r_n^2$, (from Douglas Giancoli's book: Physics for scientists & Engineers with modern physics, 4th edit., pp146. eq 6-5). Now let us build and simplify a non-spin pre-Sun ball QM {N,n} model:

1) Although each object in space between {5,1} and {6,1} had a random eccentric orbit (meaning random in both eccentricity and rotation direction), we can average out the randomness of all objects' eccentricity, and represent them in a way that all of objects have circular moving orbits with the original randomness of rotation direction. The dynamics of collapse and disk-lyzation of pre-Sun ball is an averaged effect of all objects, so that if we use these averaged-represented objects (circular orbit, random direction of rotation), it will produce the same dynamics of pre-Sun ball's collapse and disk-lyzation as that of the original objects. This averaged-represented objects model can be extended to every shell of the pre-Sun ball model.

2) Let us define the sum of mass of all objects within {5,1}RF ball as M. To a single averaged-represented object (m) in space between {5,1} and {6,1}, all mass within {5,1}RF ball can be treated as a point mass M at the origin point (see Figure 1a).

3) Let us define the sum of mass of all objects in space between {5,1} and {6,1} as Σm . To a single averaged-represented object (m) in shell space between {5,1} and {6,1}, the gravitational interaction of all rest objects in the same shell with this object m is equivalent to the interaction of a point object with mass ≈ Σm at origin to this object m (see Figure 1b).

4) Because M >> Σm, so that in the simplified model, the gravitational interaction of this object m with all rest objects in the same shell space between {5,1} and {6,1} can be omitted. In other word, comparing to the interaction in item-2, the interaction in item-3 cab be omitted. (This is in r-dimension. In θφ-2D-dimention, Σm exert the same value but opposite interaction on m, so they can be cancelled out as zero).

5) To a single averaged-represented object (m) in shell space between {5,1} and {6,1}, the gravitational interaction of all rest objects outside the shell of {6,1} is equivalent to zero (see Figure 1c).

6) For this single object, we can use Schrodinger equation to describe its QM movement in the N super-shell space (from n=1 to n=5). Solving the equation gives the 3D probability density distribution for this single object in the {N,n=1..5}o super-shell space. Since this object is any one of all objects in N super-shell, so the all objects' 3D mass density distribution is directly proportional to the single object's 3D probability density distribution.

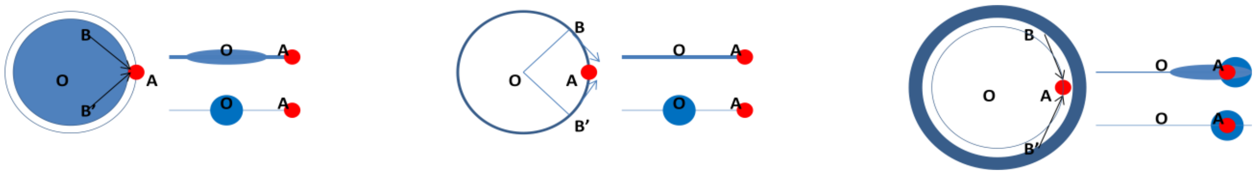


Figure 1a. To a single averaged-represented object (A) in space between {5,1} and {6,1}, all mass (B, B') within {5,1}RF ball can be treated as a point mass O at the origin point.
Figure 1b. All rest objects in the same shell (B, B') interacting with object A is equivalent to the interaction of a point object with mass ≈ Σm at origin O to this object A (I derived a calculation for this result, but not shown here).
Figure 1c. To a single averaged-represented object (A) in shell space between {5,1} and {6,1}, the gravitational interaction of all rest objects outside the shell of {6,1} (B, B') is equivalent to zero. Note: I know this is correct from my physics sense. I also believe it can be proved mathematically, but sorry I don't know (and don't have time to figure out) how to do it.

So now the pre-Sun ball model has been simplified as: a center object (with mass M) gravitationally interact with a circular orbit moving object (with mass m, at a distance r, in shell space between {5,1} and {6,1}). In this model, if consider it as a QM system, its r_1 is at {5,1}RF ball surface. Also notice that in this model, M, r (which is equivalent to r_n), r_1, are required parameters, m is not required (any m value is fine as long as m<<M so that we do NOT have to use the reduced mass).

Now compare this model to a Hydrogen atom model, both have a central force attracting an orbital moving small object, the only difference is G-force vs. EM-force. From QM text books, we know the QM solution for Hydrogen atom. From my previous paper SunQM-2, we know that QM parameters between Hydrogen atom model and Solar {N,n} QM are exchangeable. So for a pre-Sun ball's QM, we just need to take Hydrogen atom's QM formula, then replace: $a_0 \to r_1$, $h \to h_{gen} = (h/m_e)m_{planet}$, $m_e \to m_{planet}$, $(Ze^2/4\pi\varepsilon_0) \to GMm_{planet}$, that is it!

Following is a summary of the simplified non-spin pre-Sun ball QM {N,n} model:

1) Center mass M is >> orbit moving mass m, so that the reduced mass is omitted
2) After averaging out all eccentricities, the orbits of all objects are averaged and represented in circular RF movement.
3) To any single mass m that moving in orbit {N',n}, the G-force interaction with only the mass M inside the pre-Sun {N,1}RF ball need to be considered, and it can be treated as a point mass M at the center when r of {N,1} < r of {N',n}. The G-force interaction with other matters Σm in the same shell (or outside the shell) is omitted.
4) For this single object, we can use Schrodinger equation to describe its QM movement in the N super-shell space (from n=1 to n=5). The 3D probability density distribution for this single object in the {N,n=1..5}o super-shell space is directly proportional to all object's 3D mass density distribution in the same space.
5) After collapse of a {N+1,1} ball to {N,1} ball, > 99.9% of mass in shell space between {N+1,1} and {N,1} fall into {N,1} ball, so that approximations 1) through 4) are always valid in the new collapsed pre-Sun ball {N,n} structure.

I-b. Schrodinger equation for a non-spin pre-Sun ball model

Following are the Schrodinger equations for a single (non-relativistic) particle orbiting around an attractive force center, valid for both Hydrogen atom and pre-Sun ball models. The time-dependent form (from wiki "Schrödinger equation"):

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r},t)\right]\Psi(\boldsymbol{r},t) \qquad \text{eq-1}$$

and then the time-independent form:

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r})\right]\psi(\boldsymbol{r}) = E\,\psi(\boldsymbol{r}) \qquad \text{eq-2}$$

(Note: start from here, down to sections I-c and I-d, all major equations are copied from Davis J Griffiths' book "Introduction to Quantum mechanics, 2nd ed. 2005". The "eq-4.xx" is the equation number in Griffiths' book). The time-independent form in spherical coordinates (pp135, eq-4.14) is

$$\frac{-\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\,sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\,\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\,sin^2\theta}\left(\frac{\partial^2\psi}{\partial\varphi^2}\right)\right]+V\psi=E\psi \qquad \text{eq-3}$$

The eigenstate of this equation is

$$\psi(r,\theta,\varphi)=R(r)\,\Theta(\theta)\,\Phi(\varphi) \qquad \text{eq-4}$$

The last two factors of ψ are often grouped together as spherical harmonics, so the eigenfunction become

$$\psi(r,\theta,\varphi)=R(r)\,Y_{lm}(\theta,\varphi) \qquad \text{eq-5}$$

I-c. Solving Schrodinger equation (of the pre-Sun ball model) for wave function

Similar to what is shown in Griffiths' book (eq-4.16, and eq-4.17), the Schrodinger equation of pre-Sun ball model can also be separated into two equations, the first one is only in r-dimension (1D), and the second one is only in θ and φ 2D-dimension.

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)-\frac{2mr^2}{\hbar^2}[\,V(r)-\;E\,]=l(l+1) \qquad \text{eq-6}$$

$$\frac{1}{Y}\left[\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\,\frac{\partial Y}{\partial\theta}\right)+\frac{1}{sin^2\theta}\left(\frac{\partial^2 Y}{\partial\varphi^2}\right)\right]=-l(l+1) \qquad \text{eq-7}$$

Solving the Schrodinger equation in r-dimension give the radial wave function $R_{nl}(r)$ for the pre-Sun ball model, which is the same as that for hydrogen's (shown in Table 4.7 in Griffiths' book), except the a_0 in formula need to be changed into r_1.

Solving the Schrodinger equation in θφ-dimension give the spherical harmonics wave function $Y_{lm}(\theta\varphi)$ for the pre-Sun ball model, which is the same as that for hydrogen's (shown in eq-4.32 and Table 4.3 in Griffiths' book).

$$Y_l^m(\theta,\varphi)=\;\varepsilon\sqrt{\frac{(2l+1)}{4\pi}\,\frac{(l-|m|)!}{(l+|m|)!}}\;e^{im\varphi}\,P_l^m(cos\,\theta) \qquad \text{eq-8}$$

Where $\varepsilon = (-1)^m$ for $m \geq 0$ and $\varepsilon = 1$ for $m \leq 0$, and P_l^m is the associated Legendre function. Its orthogonal relationship is shown in eq-4.33

$$\int_0^{2\pi}\int_0^{\pi}[Y_l^m(\theta,\varphi)]^*\,[Y_{l'}^{m'}(\theta,\varphi)]\;sin\theta\;d\theta\;d\varphi=\;\delta_{ll'}\,\delta_{mm'} \qquad \text{eq-9}$$

The combined 3D (r, φ, θ) wave function for the Schrodinger equation of the pre-Sun ball model is the same as that for hydrogen's (shown in eq-4.89), except the "a" should be replaced by r_1 .

$$\psi_{nlm}=\sqrt{\left(\frac{2}{na}\right)^3\frac{(n-l-1)!}{2n[(n+1)!]^3}}\;e^{-r/na}\left(\frac{2r}{na}\right)^l\left[L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right)\right]Y_l^m(\theta,\varphi) \qquad \text{eq-10}$$

Where L is the associated Laguerre polynomial. Its orthogonal relationship is shown in eq-4.90

$$\int \psi^*_{nlm}\, \psi_{n'l'm'}\, r^2\, sin\theta\; dr\; d\theta\; d\varphi = \delta_{nn'}\, \delta_{ll'}\, \delta_{mm'} \qquad \text{eq-11}$$

I-d. Solving Schrodinger equation (of the pre-Sun ball model) for orbit energy E_{nl}

Solving Schrodinger equation of the pre-Sun ball model give the same orbit energy formula as that for Hydrogen atom (eq-4.70),

$$E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}\,, \qquad n = 1, 2, 3, \ldots \qquad \text{eq-12}$$

except we need to do some replacement here: m (was m_e) now is the mass of the object moving in circular orbit, h → h_{gen} =H*m, $(Ze^2/4\pi\varepsilon_0)$ → GMm. So we have E_n = - m*(2π)^2 /2/(H*m)^2 *(GMm)^2 /n^2 = -2m(πGM/Hn)^2, or

$$E_n = -\left[2m\left(\frac{\pi GM}{H}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}\,, \qquad n = 1, 2, 3, \ldots \qquad \text{eq-13}$$

where H=h/m' is a quasi-constant (see paper SunQM-2).

Now let us calculate the H value for each r_1, from r_1={0,1} up to r_1={5,1}, as
H = (πGM/n) * sqrt(-2m/E_n).
From paper SunQM-2, we know that planet's circular orbit energy also equals to $E_n = -(1/2)mv_{n\text{-orbit}}^2$, So
H = (πGM/n) * sqrt(-2m/(-(1/2)m$v_{n\text{-orbit}}^2$))
or
H = 2πGM/(n$v_{n\text{-orbit}}$)
We know n*$v_{n\text{-orbit}}$ = $v_{1\text{-oribt}}$, so

$$\mathbf{H = 2\pi GM/v_{1-orbit}} \qquad \text{eq-14}$$

which means, in Solar system (or in any center massed G-force system), once we chosen an orbit from its {N,1} as r_1, the quasi-Planck constant H is determined only by the orbit velocity of this orbit.

Table 1a. Using $E_n = -2m[\pi GM/(Hn)]^2$ to calculate out H value for each r_1, from r_1={5,1} down to r_1={0,1}.

				classical	QM						
NASA's data of planets				particle	{N,n} QM model calc-ed data						particle
	mass	orbit-r	planet orbit-v	orbit E=-(1/2)mv²	{N,n} model, N=	{N,n} model, n=	period factor =	{N,n} model, total n=	{N,n} model calc-ed $r_n=r_1n^2$	v_n = sqrt(GM/r_n)	$E_n=-(1/2)mv_n^2$
unit	kg	m	m/s	J					m	m/s	J
Sun core		1.74E+08			0	1	6	1	1.74E+08	8.7E+05	
Sun	1.99E+30	6.96E+08			0	2	6	2	6.96E+08	4.4E+05	
					1	1	6	6	6.26E+09	1.5E+05	
Mercury	3.3E+23	5.79E+10	47400	**-3.71E+32**	1	3	6	18	5.64E+10	48536	**-3.89E+32**
Venus	4.87E+24	1.08E+11	35000	**-2.98E+33**	1	4	6	24	1.00E+11	36402	**-3.23E+33**
Earth	5.97E+24	1.49E+11	29800	**-2.65E+33**	1	5	6	30	1.57E+11	29122	**-2.53E+33**
Mars	6.42E+23	2.28E+11	24100	**-1.86E+32**	1	6	6	36	2.25E+11	24268	**-1.89E+32**
Jupiter	1.9E+27	7.78E+11	13100	**-1.63E+35**	2	2	5.33	64.0	7.12E+11	13659	**-1.77E+35**
Saturn	5.68E+26	1.43E+12	9700	**-2.67E+34**	2	3	5.33	95.9	1.60E+12	9106	**-2.35E+34**
Uranus	8.68E+25	2.97E+12	6800	**-2.01E+33**	2	4	5.33	127.9	2.85E+12	6830	**-2.02E+33**
Neptune	1.02E+26	4.51E+12	5400	**-1.49E+33**	2	5	5.33	159.9	4.45E+12	5464	**-1.52E+33**
Pluto	1.46E+22	5.91E+12	4700	**-1.61E+29**	2	6	5.33	191.9	6.40E+12	4553	**-1.51E+29**
object-34	1				3	4	6	767.5	1.02E+14	1138	**-6.48E+05**
object-44	1				4	4	6	4605.1	3.69E+15	190	**-1.80E+04**
object-54	1				5	4	6	27631	1.33E+17	32	**-5.00E+02**
object-64	1				6	4	6	165784	4.78E+18	5	**-1.39E+01**

	set r1={0,1}				set r1={1,1}			
	calc total n from {N.n}	H = 2πGM/(n $v_{n\text{-orbit}}$)	Hn=	E_n = -2m[πGM/(Hn)]^2	calc total n from {N.n}	H = 2πGM/(n $v_{n\text{-orbit}}$)	Hn=	E_n = -2m[πGM/(Hn)]^2
unit		J.s/kg	J.s/kg	J		J.s/kg	J.s/kg	J
Sun core	1							
Sun	2							
	6							
Mercury	18	9.55E+14	1.72E+16	**-3.89E+32**	3	5.73E+15	1.72E+16	**-3.89E+32**
Venus	24	9.55E+14	2.29E+16	**-3.23E+33**	4	5.73E+15	2.29E+16	**-3.23E+33**
Earth	30	9.55E+14	2.86E+16	**-2.53E+33**	5	5.73E+15	2.86E+16	**-2.53E+33**
Mars	36	9.55E+14	3.44E+16	**-1.89E+32**	6	5.73E+15	3.44E+16	**-1.89E+32**
Jupiter	64.0	9.55E+14	6.11E+16	**-1.77E+35**	10.7	5.73E+15	6.11E+16	**-1.77E+35**
Saturn	95.9	9.55E+14	9.16E+16	**-2.35E+34**	16.0	5.73E+15	9.16E+16	**-2.35E+34**
Uranus	127.9	9.55E+14	1.22E+17	**-2.02E+33**	21.3	5.73E+15	1.22E+17	**-2.02E+33**
Neptune	159.9	9.55E+14	1.53E+17	**-1.52E+33**	26.7	5.73E+15	1.53E+17	**-1.52E+33**
Pluto	191.9	9.55E+14	1.83E+17	**-1.51E+29**	32.0	5.73E+15	1.83E+17	**-1.51E+29**
object-34	767.5	9.55E+14	7.33E+17	**-6.48E+05**	127.9	5.73E+15	7.33E+17	**-6.48E+05**
object-44	4605.1	9.55E+14	4.40E+18	**-1.80E+04**	767.5	5.73E+15	4.40E+18	**-1.80E+04**
object-54	27630.7	9.55E+14	2.64E+19	**-5.00E+02**	4605.1	5.73E+15	2.64E+19	**-5.00E+02**
object-64	165784	9.55E+14	1.58E+20	**-1.39E+01**	27631	5.73E+15	1.58E+20	**-1.39E+01**

	set r1={2,1}				set r1={3,1}			
	calc total n from {N.n}	H = 2πGM/(n $v_{n\text{-orbit}}$)	Hn=	E_n = -2m[πGM/(Hn)]^2	calc total n from {N.n}	H = 2πGM/(n $v_{n\text{-orbit}}$)	Hn=	E_n = -2m[πGM/(Hn)]^2
unit		J.s/kg	J.s/kg	J		J.s/kg	J.s/kg	J
Sun core								
Sun								
Mercury	3/6^1	3.44E+16	1.72E+16	**-3.89E+32**	3/6^2	2.06E+17	1.72E+16	**-3.89E+32**
Venus	4/6^1	3.44E+16	2.29E+16	**-3.23E+33**	4/6^2	2.06E+17	2.29E+16	**-3.23E+33**
Earth	5/6^1	3.44E+16	2.86E+16	**-2.53E+33**	5/6^2	2.06E+17	2.86E+16	**-2.53E+33**
Mars	1.00	3.44E+16	3.44E+16	**-1.89E+32**	1/6	2.06E+17	3.44E+16	**-1.89E+32**
Jupiter	1.78	3.44E+16	6.11E+16	**-1.77E+35**	0.30	2.06E+17	6.11E+16	**-1.77E+35**
Saturn	2.67	3.44E+16	9.16E+16	**-2.35E+34**	0.44	2.06E+17	9.16E+16	**-2.35E+34**
Uranus	3.55	3.44E+16	1.22E+17	**-2.02E+33**	0.59	2.06E+17	1.22E+17	**-2.02E+33**
Neptune	4.44	3.44E+16	1.53E+17	**-1.52E+33**	0.74	2.06E+17	1.53E+17	**-1.52E+33**
Pluto	5.33	3.44E+16	1.83E+17	**-1.51E+29**	0.89	2.06E+17	1.83E+17	**-1.51E+29**
object-34	21.3	3.44E+16	7.33E+17	**-6.48E+05**	3.55	2.06E+17	7.33E+17	**-6.48E+05**
object-44	127.9	3.44E+16	4.40E+18	**-1.80E+04**	21.32	2.06E+17	4.40E+18	**-1.80E+04**
object-54	767.5	3.44E+16	2.64E+19	**-5.00E+02**	127.92	2.06E+17	2.64E+19	**-5.00E+02**
object-64	4605.1	3.44E+16	1.58E+20	**-1.39E+01**	767.52	2.06E+17	1.58E+20	**-1.39E+01**

	set r1={4,1}				set r1={5,1}			
	calc total n from {N.n}	H = 2πGM/(n $v_{n\text{-orbit}}$)	Hn=	E_n = -2m[πGM/(Hn)]^2	calc total n from {N.n}	H = 2πGM/(n $v_{n\text{-orbit}}$)	Hn=	E_n = -2m[πGM/(Hn)]^2
unit		J.s/kg	J.s/kg	J		J.s/kg	J.s/kg	J
Sun core								
Sun								
Mercury	3/6^3	1.24E+18	1.72E+16	**-3.89E+32**	3/6^4	7.42E+18	1.72E+16	**-3.89E+32**
Venus	1/54	1.24E+18	2.29E+16	**-3.23E+33**	1/324	7.42E+18	2.29E+16	**-3.23E+33**
Earth	5/216	1.24E+18	2.86E+16	**-2.53E+33**	1/259	7.42E+18	2.86E+16	**-2.53E+33**
Mars	1/36	1.24E+18	3.44E+16	**-1.89E+32**	1/216	7.42E+18	3.44E+16	**-1.89E+32**
Jupiter	0.049	1.24E+18	6.11E+16	**-1.77E+35**	0.0082	7.42E+18	6.11E+16	**-1.77E+35**
Saturn	0.074	1.24E+18	9.16E+16	**-2.35E+34**	0.0123	7.42E+18	9.16E+16	**-2.35E+34**
Uranus	0.099	1.24E+18	1.22E+17	**-2.02E+33**	0.0165	7.42E+18	1.22E+17	**-2.02E+33**
Neptune	0.123	1.24E+18	1.53E+17	**-1.52E+33**	0.0206	7.42E+18	1.53E+17	**-1.52E+33**
Pluto	0.148	1.24E+18	1.83E+17	**-1.51E+29**	0.0247	7.42E+18	1.83E+17	**-1.51E+29**
object-34	0.59	1.24E+18	7.33E+17	**-6.48E+05**	0.10	7.42E+18	7.33E+17	**-6.48E+05**
object-44	3.55	1.24E+18	4.40E+18	**-1.80E+04**	0.59	7.42E+18	4.40E+18	**-1.80E+04**
object-54	21.32	1.24E+18	2.64E+19	**-5.00E+02**	3.55	7.42E+18	2.64E+19	**-5.00E+02**
object-64	127.92	1.24E+18	1.58E+20	**-1.39E+01**	21.32	7.42E+18	1.58E+20	**-1.39E+01**

Note: To calculate total n from {N,n}, when N< 2, total n= n*6^N, when N ≥ 2, total n= n*6^(N-1)*5.33 (start from {2,2} and up). Refer to paper SunQM-1 section I.

In Table 1a, I constructed the H for Solar system's eight known planets using the same Solar QM {N,n} model in the paper SunQM-2. Four pretended objects, each with assumed 1 kg in mass, are put in this model at {3,4}, {4,4}, {5,4}, and {6,4} orbit, and theirs H are also calculated. In the pre-Sun ball model, we always assuming that after collapse of a {N+1,1}RF ball to form {N,1}RF ball, > 99.9% of mass in shell space between {N+1,1} and {N,1} falls into {N,1}RF ball, so that after each collapse (from {6,1}, down to {1,1}), the new smaller {N,1}RF ball always contains ≥ 99.9% of mass of the Solar system. Consider current Sun's core contains only part of Sun's mass (≈34% at 0.2 of Sun's radius, from wiki "Solar core"), this assumption is valid from {0,2} to ≥ {6,1} . Table 1b listed all H values (unit = J*s/kg) extracted from Table 1a.

Table 1b. H values at each r_1.

r_1 at meter	H= J*s/kg
{0,1}	9.55E+14
{1,1}	5.73E+15
{2,1}	3.44E+16
{3,1}	2.06E+17
{4,1}	1.24E+18
{5,1}	7.42E+18

From the calculated results in Table 1a, we can see: for a 1 kg object moving in orbit {5,4}, if using {5,1} as r_1, then its total n≈3.55, its H≈7.42E+18 (J.s/kg), its orbit E_n = -2m[πGM/(Hn)]^2 = -500 (J). Or, for an object with m kg moving in orbit {5,4}, if using {5,1} as r_1, then its total n≈3.55, its H≈7.42E+18 (J.s/kg), its orbit E_n = -2m[πGM/(Hn)]^2 = -500 *m (J). For the same object, we can use any {N,1} as r_1, e.g., set {2,1} as r_1, then its total n≈767.5, its H≈3.44E+16 (J.s/kg), its orbit E_n = -2m[πGM/(Hn)]^2 = -500 *m (J). So even with different r_1, the E_n of the same object keeps the same (which make physics sense).

This calculation is valid for any object (planet, rock, gas molecule, etc.) that doing circular orbital movement in the Solar system. For example, let us take Earth on {1,5} orbit, if use {1,1} as r_1, then its total n=5, its H≈5.73E+15 (J.s/kg), its orbit E_n = -2m[πGM/(Hn)]^2 = -2.53E+33 (J). Then if we switch to use {5,1} as r_1, now its total n≈0.00386, its H≈7.42E+18 (J.s/kg), its orbit E_n = -2m[πGM/(Hn)]^2 = -2.53E+33 (J), still the same. This is because in formula of E_n, Hn = constant. So whenever we choose s different r as r_1, the n changes, but the H also changes correspondingly.

To mimic hydrogen atom model's formula E_n=E_1/n^2, and E_1= -13.6eV, let us use {0,1} as r_1, then H≈9.55E+14 (J.s/kg). E_n = -2m[πGM/(Hn)]^2, E_n = E_1 /n^2, E_1= -2m[πGM/(H)]^2= -3.81E+11 *m, or

$$\mathbf{E_n = -3.81 * 10^{11}} * \left(\frac{\mathbf{m}}{\mathbf{n^2}}\right) \quad \text{(Joule)} \qquad \text{eq-15}$$

where m is the orbit moving object's mass (in kg), and n is the total n of the orbit using Sun core {0,1} as r_1 (Note: it has to use {0,1} as r_1 for value -3.81E+11), and E_n in unit J. So now we can calculate all planets' orbit energy by using this simple formula (see Table 1c). Note: this formula not only suitable for planet, but also suitable for any object (e.g., Hydrogen atom) inside the Sun (see SunQM-2s1).

In columns 6 & 7 of Table 1c, the orbit E_n is calculated by using semi-QM method. That is, using planet's real r to directly calculate out its n (=sqrt(r_n/r_1)), then using E_n = -3.81E+11 *(m/n^2) to calculate its orbit E. In columns 8 through 16, the E_n is calculated by using a complete QM {N,n} model, so it does not need to know planet's orbit r, only use r_1 at {0,1}, then using E_n = -3.81E+11 *(m/n^2) to calculate out every orbit's E. The resulted E_n (in columns 7 and 16, comparing to columns 5 and 14) are the same within error range.

Table 1c. Calculate all planets' orbit energy by using E_n = -3.81E+11 *(m/n^2).

				classical	semi-QM		QM									
NASA's data of planets				particle			{N,n} QM model calc-ed data						particle	set r1={0,1}	
	mass	Sun's radius or planet orbit-r	planet orbit-v	orbit E=-(1/2)mv^2	n= sqrt(rn/r1)	**E_n = -3.81E+11 *(m/n^2)**	{N,n} model, N=	{N,n} model, n=	period factor=	{N,n} model, total n=	{N,n} model calc-ed $r_n=r_1n^2$	v_n = sqrt(GM/r_n)	E_n=-(1/2)mv_n^2	calc total n from {N.n}	**E_n = -3.81E+11 *(m/n^2)**
unit	kg	m	m/s	J		J					m	m/s	J		**J**
Sun core		1.74E+08					0	1	6	1	1.74E+08	873647.0		1	
SUN	1.99E+30	6.96E+08					0	2	6	2	6.96E+08	436824		2	
							1	1	6	6	6.26E+09	145608		6	
Mercury	3.3E+23	5.79E+10	47400	-3.71E+32	18.2	**-3.78E+32**	1	3	6	18	5.64E+10	48536	-3.89E+32	18	**-3.88E+32**
Venus	4.87E+24	1.08E+11	35000	-2.98E+33	24.9	**-2.99E+33**	1	4	6	24	1.00E+11	36402	-3.23E+33	24	**-3.22E+33**
Earth	5.97E+24	1.49E+11	29800	-2.65E+33	29.3	**-2.66E+33**	1	5	6	30	1.57E+11	29122	-2.53E+33	30	**-2.53E+33**
Mars	6.42E+23	2.28E+11	24100	-1.86E+32	36.2	**-1.87E+32**	1	6	6	36	2.25E+11	24268	-1.89E+32	36	**-1.89E+32**
Jupiter	1.9E+27	7.78E+11	13100	-1.63E+35	66.9	**-1.62E+35**	2	2	5.33	63.96	7.12E+11	13659	-1.77E+35	64.0	**-1.77E+35**
Saturn	5.68E+26	1.43E+12	9700	-2.67E+34	90.6	**-2.64E+34**	2	3	5.33	95.94	1.60E+12	9106	-2.35E+34	95.9	**-2.35E+34**
Uranus	8.68E+25	2.97E+12	6800	-2.01E+33	130.8	**-1.93E+33**	2	4	5.33	127.92	2.85E+12	6830	-2.02E+33	127.9	**-2.02E+33**
Neptune	1.02E+26	4.51E+12	5400	-1.49E+33	161.0	**-1.50E+33**	2	5	5.33	159.9	4.45E+12	5464	-1.52E+33	159.9	**-1.52E+33**
Pluto	1.46E+22	5.91E+12	4700	-1.61E+29	184.4	**-1.64E+29**	2	6	5.33	191.88	6.40E+12	4553	-1.51E+29	191.9	**-1.51E+29**

Here I give some more explanation for Table 1a on how I calculated the total n (based on {N,n, Hot-G}) for super-shells of N=3, 4, 5 (which have {N,n, Cold-G}). As we learned from paper SunQM-1s1, due to the hydrogen fusion, the heated r_n is about 26% larger than the cold one, so the corresponding n is slightly bigger in value. We use {N,n,Cold-G} for the (cold) gravity-only {N,n}, and {N,n,Hot-G} for the Hot gravity {N,n}. In a pure {N,n,Cold-G} QM, for a {1,5//6} orbit, if use {0,1} as r_1, then its total n =5*6^1=30. If use {5,1} as r_1, its total n =5* 6^1 / 6^5=5/6^4. In a pure {N,n,Hot-G} QM, for a {1,5//6} orbit, if use {0,1} as r_1, then its total n is also =5*6^1=30. If use {5,1} as r_1, its total n is also =5* 6^1 / 6^5 = 5/6^4. However our Solar system has a {N,n,Hot-G} from {0,1} to {2,1} , a {N,n,Cold-G} from {3,1} to {6,1} or above. From {2,1} to {3,1} it is in hot-cold transition with period factor =5.33 ($\neq$6). Our total n is relative to {0,1} which is in a {N,n,Hot-G} QM system. So all total n has to be translated into a {N,n,Hot-G} system. The right method for total n calc is:

1) For {N,n} $\leq$ {2,1}, always use period factor =6, even when using $\geq$ {3,1} as r_1 (because it is from a hot-G-based {N,n} to a hot-G-based total n).
2) For {N,n} $\geq$ {2,2}, always include one 5.33, even when using $\leq$ {1,1} as r_1 (because it is a translation of a cold-G-based {N,n} into a hot-G-based total n).

Although the cold-G and hot-G complicated the total n calculation, it does have some good side: Just like the annual growth rings of a tree trunk recorded the climate history of the tree growth thousands years ago, or DNA's SNP and its distribution among people can tell us the history of early human migration, or, why mother, mom, 妈妈 (mama in Chinese), mamma (in Italian), sounds so similar (which made me curious when I was teenage), it actually tells us that we all come from a single tribe ~5000 generations ago, the {N,n} orbits in Solar system recorded the history of how pre-Sun ball quantumly collapsed to the current Sun, and {N,n,Cold-G} vs. {N,n,Hot-G} recorded the history of when the pre-Sun ball started the hydrogen fusion during a series of collapse.

I-e. *l* degeneracy of $E_{n,l}$ in a non-spin pre-Sun ball model

The solution of Schrodinger equation at r-dimension for the non-spin pre-Sun ball model shows that although the radial wave function R(n,*l*) depends on both n and *l* quantum number, the radial energy E_n is depend only on n, not on *l*. In other word, E_n is *l* degenerated. Before to explain what this means in the pre-Sun ball model, let us first understand more about the radial wave function R(n,*l*).

In the pre-Sun ball model, the mass density distribution along r is determined by the radial probability density $r^2|R_{n,l}(r)|^2$ (similar as that for Hydrogen atom QM). Since n is a base-5*6^ number in the Solar QM {N,n} system, we need to find the formulas of $R_{n,l}(r)$ from n=1 up to n=6, *l*=0,1,...5. I obtained formulas of R(n,l) up to n=4, *l*= 0,...4, from Davis J Griffiths 's book "Introduction to Quantum mechanics", 2nd ed. 2005. pp156, Table 4.7, or from book "A Modern Approach to quantum mechanics" by john Townsend, 2nd ed. Page 355. I obtained formulas of R(n,l) for n=5, *l*= 0,…4, (from an online R(n,l) calculator at: http://winter.group.shef.ac.uk/orbitron/AOs/5s/equations.html), shown as follows:

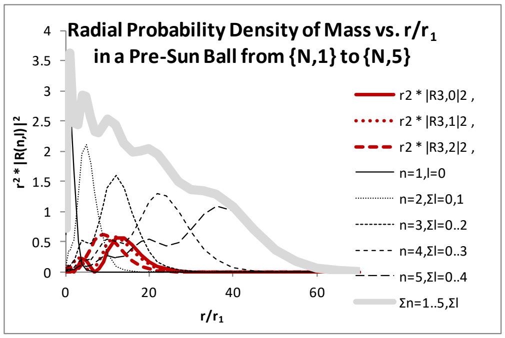
R(5,0) = 1/300 /sqrt(5) / [a^(3/2)] *[120 - 480/5 *(r/a) + 480/25 *(r/a)^2 -160 /125 *(r/a)^3 + 16/625 * (r/a)^4] *exp (-r/5/a)
R(5,1) = 1/150 /sqrt(30) / [a^(3/2)] *[240/5 * (r/a)- 360/25 *(r/a)^2 + 144/125 *(r/a)^3 - 16/625 *(r/a)^4] *exp (-r/5/a)
R(5,2) = 1/150 /sqrt(70) / [a^(3/2)] *[168/25 *(r/a)^2 - 112/125 *(r/a)^3 + 16/625 *(r/a)^4] *exp (-r/5/a)
R(5,3) = 1/300/sqrt(70) / [a^(3/2)] *[64/125 *(r/a)^3 - 16/625 *(r/a)^4] *exp (-r/5/a)
R(5,4) = 16 / 900 / 625 / sqrt(70) / [a^(3/2)] *(r/a)^4 *exp(-r/5/a)
I deduced R(6,5) myself according the same website:
R(6,5) = 1/12960/SQRT(77) / [a^(3/2)] *(2/6*r/a)^5 *exp(-r/6/a)

I was not able to obtain formula of R(n,l) for n=6, *l*= 0,…4, or for n>6. I searched it for over one year until in November 2017 when I discovered the rule that "all mass between r_n and r_{n+1} belong to orbit n (see paper SunQM-3s2)". Then I realized that for a {N,n//6} QM structure, the mass takes orbit of {N,n=1..5}o orbits. The mass in n=5 orbit will end at n=6. Therefore, to describe the mass density distribution of a N super-shell of {N,n=2..6}, I only need R(n, l) from n=1 to 5, I do not need R(n,l) for n=6.

Table 2. $r^2|R(n,l)|^2$ vs. r/r_1 for n=1 to 5

																						r cut-off=
r_1= (a=)	0.174	E+9 meters														{0,n=1}o	{0,n=2}o				{-2,n=1..5}o		36
r/r_1= (r/a=)	$r^2*\|R_{1,0}\|^2$,	$r^2*\|R_{2,0}\|^2$,	$r^2*\|R_{2,1}\|^2$,	$r^2*\|R_{3,0}\|^2$,	$r^2*\|R_{3,1}\|^2$,	$r^2*\|R_{3,2}\|^2$,	$r^2*\|R_{4,0}\|^2$,	$r^2*\|R_{4,1}\|^2$,	$r^2*\|R_{4,2}\|^2$,	$r^2*\|R_{4,3}\|^2$,	$r^2*\|R_{5,0}\|^2$,	$r^2*\|R_{5,1}\|^2$,	$r^2*\|R_{5,2}\|^2$,	$r^2*\|R_{5,3}\|^2$,	$r^2*\|R_{5,4}\|^2$,	n=1,l=0	n=2,Σl=0, 1	n=3,Σl=0.. 2	n=4,Σl=0.. 3	n=5,Σl=0.. 4	Σn=1..5,Σl	Σn=1,2	$\log(r/a_0)$=
0.2	0.616386	0.076227	0.000314	0.022541	0.00011	2.62E-08	0.009503	4.89E-05	1.57E-08	6.45E-13	0.004864	2.57E-05	9.23E-09	5.54E-13	6.28E-18	0.616386	0.07654	0.022651	0.009552	0.00489	0.730019	0.692927	0.2
0.4	1.652704	0.197244	0.004109	0.057941	0.001436	1.47E-06	0.02437	0.000638	8.79E-07	1.49E-10	0.01246	0.000334	5.16E-07	1.28E-10	5.94E-15	1.652704	0.201353	0.059378	0.025009	0.012795	1.951239	1.854057	0.4
0.6	2.492642	0.278191	0.017032	0.080687	0.005917	1.46E-05	0.033786	0.002623	8.75E-06	3.46E-09	0.017238	0.001373	5.13E-06	2.97E-09	3.16E-13	2.492642	0.295223	0.086619	0.036418	0.018616	2.929518	2.787865	0.6
0.8	2.970432	0.297487	0.044072	0.08448	0.015177	7.18E-05	0.03511	0.006707	4.3E-05	3.13E-08	0.017852	0.003505	2.52E-05	2.68E-08	5.18E-12	2.970432	0.341559	0.099729	0.04186	0.021382	3.474962	3.31199	0.8
1	3.111156	0.264281	0.088094	0.072556	0.029982	0.00024	0.029792	0.013194	0.000143	1.69E-07	0.015062	0.006883	8.37E-05	1.45E-07	4.45E-11	3.111156	0.352375	0.102778	0.04313	0.022029	3.631467	3.463531	1
2	1.684197	0	0.518526	0.001231	0.157627	0.007881	0.000918	0.066621	0.004588	2.62E-05	0.000581	0.034101	0.002655	2.23E-05	3.06E-08	1.684197	0.518526	0.16674	0.072153	0.03736	2.478975	2.202723	2
3	0.512845	0.321899	0.965697	0.115228	0.230456	0.046091	0.050894	0.088863	0.025676	0.000408	0.026538	0.043534	0.014556	0.000341	1.18E-06	0.512845	1.287597	0.391775	0.165841	0.08497	2.443028	1.800442	3
4	0.123389	0.842098	1.122798	0.219436	0.1662	0.13296	0.086421	0.051853	0.069137	0.002469	0.042687	0.022772	0.037935	0.002025	1.41E-05	0.123389	1.964896	0.518596	0.20988	0.105433	2.922193	2.088285	4
5	0.026092	1.089108	1.008433	0.176035	0.052081	0.260406	0.056167	0.007498	0.122472	0.008927	0.024889	0.001844	0.064001	0.007111	8.78E-05	0.026092	2.097541	0.488522	0.195065	0.097933	2.905152	2.123633	5
6	0.005085	1.025691	0.769268	0.062378	0	0.399217	0.010059	0.006036	0.162961	0.02328	0.002738	0.006046	0.079275	0.017855	0.000364	0.005085	1.794958	0.461595	0.202337	0.106278	2.570253	1.800043	6
7	0.000937	0.802482	0.524288	0.000538	0.052739	0.516845	0.004167	0.051289	0.173084	0.048463	0.004795	0.032751	0.075658	0.035419	0.001141	0.000937	1.326771	0.570122	0.277003	0.149765	2.324597	1.327707	7
8	0.000166	0.555248	0.329036	0.043666	0.18477	0.591265	0.046783	0.11228	0.149706	0.085546	0.030584	0.061533	0.055253	0.058875	0.002907	0.000166	0.884285	0.819702	0.394316	0.209152	2.30762	0.88445	8
9	2.84E-05	0.351878	0.193892	0.170948	0.341897	0.615414	0.108027	0.153827	0.103545	0.13313	0.059919	0.073735	0.028206	0.085085	0.006329	2.84E-05	0.545769	1.12826	0.498529	0.253274	2.425861	0.545798	9
10	4.74E-06	0.208735	0.108716	0.328336	0.475634	0.594542	0.151685	0.157568	0.052523	0.187581	0.073356	0.063875	0.006844	0.1095	0.012167	4.74E-06	0.317452	1.398513	0.549356	0.265741	2.531066	0.317456	10
12	1.25E-07	0.063561	0.030509	0.552964	0.584953	0.467963	0.128211	0.076927	0	0.296718	0.040614	0.014406	0.01071	0.135396	0.033849	1.25E-07	0.09407	1.605879	0.501856	0.234975	2.43678	0.09407	12
14	3.12E-09	0.01686	0.007649	0.564538	0.507839	0.311052	0.032211	0.003277	0.053521	0.374648	0.000988	0.004578	0.066068	0.117454	0.071052	3.12E-09	0.024509	1.383429	0.463656	0.26014	2.131735	0.024509	14
16	7.45E-11	0.004056	0.001766	0.439069	0.356825	0.182694	0.003427	0.032904	0.175486	0.401111	0.023634	0.054321	0.11481	0.068263	0.121356	7.45E-11	0.005823	0.978588	0.612928	0.382384	1.979722	0.005823	16
18	1.73E-12	0.000907	0.000383	0.286447	0.216954	0.097629	0.081013	0.146631	0.294473	0.378608	0.083877	0.108456	0.114906	0.019675	0.177073	1.73E-12	0.00129	0.601031	0.900724	0.503987	2.007033	0.00129	18
20	3.91E-14	0.000192	7.9E-05	0.165014	0.11864	0.048425	0.20946	0.271791	0.362388	0.32356	0.11944	0.116991	0.0722	0	0.228187	3.91E-14	0.000271	0.332079	1.167199	0.536818	2.036367	0.000271	20
22	8.66E-16	3.88E-05	1.56E-05	0.08659	0.059804	0.022613	0.317586	0.351702	0.369024	0.255154	0.102506	0.078697	0.02257	0.019781	0.26594	8.66E-16	5.44E-05	0.169007	1.293466	0.489493	1.952021	5.44E-05	22
24	1.89E-17	7.56E-06	3E-06	0.042245	0.028257	0.010047	0.367382	0.369161	0.329499	0.188285	0.052968	0.027815	6.96E-05	0.071314	0.285255	1.89E-17	1.06E-05	0.080549	1.254327	0.437422	1.772308	1.06E-05	24
27	5.92E-20	6.15E-07	2.39E-07	0.012941	0.008338	0.002757	0.337037	0.308222	0.232888	0.107794	0.000791	0.002195	0.040866	0.168778	0.279	5.92E-20	8.54E-07	0.024035	0.985942	0.491631	1.501609	8.54E-07	27
30	1.81E-22	4.74E-08	1.82E-08	0.003589	0.002246	0.000702	0.243533	0.209177	0.140804	0.055874	0.034211	0.062483	0.135565	0.241005	0.241005	1.81E-22	6.56E-08	0.006537	0.649388	0.71427	1.370194	6.56E-08	30
33	5.43E-25	3.5E-09	1.32E-09	0.000921	0.000563	0.000168	0.149371	0.122606	0.075757	0.026725	0.126965	0.160433	0.220684	0.262967	0.188278	5.43E-25	4.83E-09	0.001652	0.374459	0.959327	1.335438	4.83E-09	33
36	1.6E-27	2.5E-10	9.33E-11	0.000222	0.000133	3.84E-05	0.081171	0.064357	0.037213	0.011961	0.214843	0.235272	0.258531	0.240663	0.135373	1.6E-27	3.43E-10	0.000394	0.194703	1.084683	1.279779	3.43E-10	36
40	6.64E-31	7.05E-12	2.6E-12	3.08E-05	1.81E-05	5.02E-06	0.031216	0.023889	0.012899	0.003761	0.25916	0.256223	0.235788	0.176367	0.078386	6.64E-31	9.66E-12	5.4E-05	0.071764	1.005924	1.077742	9.66E-12	36.60
45	3.81E-35	7.7E-14	2.81E-14	2.36E-06	1.36E-06	3.63E-07	0.007954	0.005889	0.002982	0.000792	0.208652	0.191425	0.15353	0.095691	0.034449	3.81E-35	1.05E-13	4.09E-06	0.017617	0.683747	0.701368	1.05E-13	36.95
50	2.14E-39	7.98E-16	2.89E-16	1.66E-07	9.44E-08	2.44E-08	0.001754	0.001267	0.000611	0.000151	0.1235	0.107935	0.078953	0.043321	0.013371	2.14E-39	1.09E-15	2.85E-07	0.003783	0.36708	0.370863	1.09E-15	37.15
55	1.17E-43	7.93E-18	2.85E-18	1.09E-08	6.11E-09	1.54E-09	0.000346	0.000245	0.000114	2.66E-05	0.059358	0.050106	0.034278	0.017106	0.004693	1.17E-43	1.08E-17	1.85E-08	0.000732	0.165542	0.166274	1.08E-17	37.28
60	6.35E-48	7.62E-20	2.72E-20	6.75E-10	3.75E-10	9.26E-11	6.26E-05	4.36E-05	1.96E-05	4.38E-06	0.024487	0.020133	0.013084	0.006065	0.001516	6.35E-48	1.03E-19	1.14E-09	0.00013	0.065285	0.065415	1.03E-19	37.38
65	3.38E-52	7.11E-22	2.52E-22	3.99E-11	2.2E-11	5.34E-12	1.05E-05	7.24E-06	3.17E-06	6.81E-07	0.008977	0.007229	0.004511	0.001971	0.000457	3.38E-52	9.63E-22	6.73E-11	2.16E-05	0.023145	0.023167	9.63E-22	37.46
70	1.78E-56	6.47E-24	2.29E-24	2.27E-12	1.24E-12	2.97E-13	1.67E-06	1.13E-06	4.86E-07	1.01E-07	0.002995	0.002371	0.001431	0.000596	0.00013	1.78E-56	8.76E-24	3.81E-12	3.39E-06	0.007523	0.007527	8.76E-24	37.53

Note-1: the a (=5.29E-11 meters for Hydrogen atom) in above R(n,l) formulas is replaced by r_1=1.74E+8 meters (for Solar {N,n} QM structure).
Note-2: due to the calculated $r^2|R(n,l)|^2$ values were in the range of 1E-9, it is not easy to mark this small value in plot, so I use r_1=0.174 (with unit of E+9 meters) in Table 2 calculation, it brings the calculated $r^2|R(n,l)|^2$ values down to the range of single digit, it only changes the relative scale of y-axis, but not the shape of curve.
Note-3: Here a single object's QM probability density in the orbit space is treated as equivalent to the whole collection of all objects' mass density in the same orbit space.



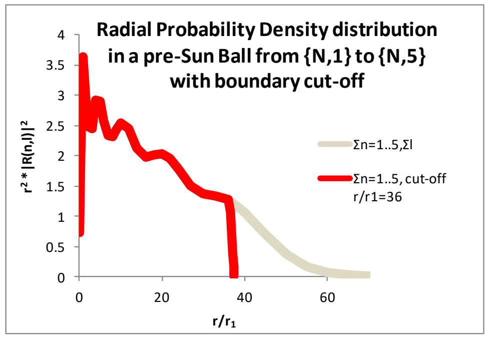


Figure 2a. Probability Density (of Mass) vs. r/r_1 in a pre-Sun Ball from {N,1} to {N,6}, presented as $r^2|R(n,l)|^2$ vs. r/r_1 for n=1 to 5.

Figure 2b. A pre-Sun ball's mass density distribution curve, with a logarithmical compression at $r/r_1 \geq 36$ (=36 + log(r/r_1-36)) is added to the curve of Σn=1…5.

With the known R(n,l), I plotted $r^2|R(n,l)|^2$ vs. r/r_1 for n=1 to 5 as in Table 2 and Figure 2. From Figure 2a we can see that the space between {N,1} and {N,6} can be simplified as 5 orbit space shells, each one has quantum number n=1,…5 from inner shell to outer shell. For each n shell, it can be further divided into n-numbered sub-shells (*l*=0, 1, … n-1), with *l*=0 sub-shell at out-most, and *l*= n-1 sub-shell at the inner-most. For example, as shown in Figure 2a, shell of n=3 has three sub-shells (*l*=0, 1, 2), and the sub-shell (n=3, *l*=0) is at outside of sub-shell (n=3, *l*=2). The total probability curve for this super-shell (named as Σn=1..5 total probability curve), is shown as the thickest line in Figure 2a. This super-shell Σn=1..5 total probability curve will be used in many figures for the r-dimension probability (or mass) density analysis in this paper and in other papers.

In the classical mechanics, the orbit energy of E(r) continuously decreases as r decreasing. In pre-Sun ball QM, E(r) is averaged within each n, so it becomes E_n, which is step-wise decrease as r (or n) decreasing. For shell n=3, all three sub-shells (*l*=0, 1, 2) have the same E(n=3), so ΔE = 0 between E(3,0) and E(3,2), or between E(3,1) and E(3,2), and so on. So E(n,l) is degenerate for *l*. An object has the same probability to stay in any one of these three sub-shells. Or, for all of countless objects (including Hydrogen atoms, solid fragments, etc.) in n=3 shell of pre-Sun ball, they also have the same probability to stay in any one of these three *l* sub-shells, this cause them evenly distributed in these three sub-shells, or the same mass density throughout the n=3 shell.

Figure 2a also shows that n=3 shell has a higher mass density than that of n=4 shell, and has a lower mass density than that of n=2 shell, as determined by theirs $r^2|R(n,l)|^2$ vs. r/r_1 . Remember this is based on the non-spin pre-Sun model. As that will be explained later, this degeneracy will be removed in a spinning pre-Sun model.

As shown in Figure 2a, *l* sub-shells are overlapping with each other, so do the n shells. In Figure 2a, we can see that the sum of sub-shells of |n,l> =|3,0>, |3,1>, and |3,2> forms a shell of n=3. Then the sum of shells n=1, 2, 3, 4, 5 forms a continuously decreasing (and bumpy) intensity curve (curve Σn=1..5) and it is the single object probability density distribution curve along r. From physics sense, I believe that the mass density distribution of all countless objects along r (in the real pre-Sun ball) should directly correlate to the probability distribution of a single object along r.

Therefore in Figure 2b, a simplified mass density (according to the probability density) was plotted (in red thick line) for {N,n=1..5}o orbits of pre-Sun ball. Within each n, the mass density is constant. Between n, the mass density increases as n decreases. This is for N super-shell. For the inner N-1 period, the mass density distribution is exactly the same, except now density of {N-1,6} should equivalent (or at least close to) the mass density of {N,1}.So we can extent the same pattern to all inner N-1, N-2, N-3, ... Super-shells. The final mass density distribution will show that the more inner in the pre-Sun ball, the more higher the mass density it will be (like in Figure 3b).

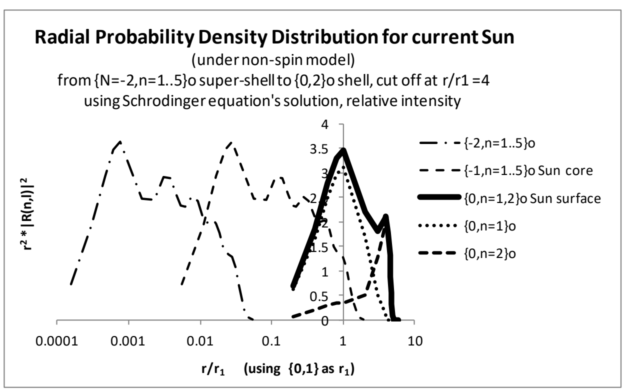
For the surface of pre-Sun ball, it ends at {N,n=6} (not the current Sun which ends at n=2), Figure 2a shows its mass density has a long tail extend outward. This is due to that the original R(n,l) is a function of r from 0 to infinity. For pre-Sun, it need to add a boundary condition so that at the Sun surface the $|R(n,l)|^2$ quickly decay to zero. So in Figure 2b, I simply make a logarithmical compression at $r/r_1 \geq 36$, therefore it becomes 36 + log(r/r_1 - 36).

Similar as *l* degeneracy, in each *l* sub-shell, E(n,l) is independent to m, so E(n,l) is degenerated to m. For example, for |nlm> = |3,2,m>, ΔE = 0 between |3,2,0> and |3,2,2>, or between |3,2,1> and |3,2,-1>, and so on (remember this is in the 2D spherical θφ-dimension). Or, for all of countless objects in *l*=2 sub-shell of n=3 shell, they have the same probability to stay in any one of m=-2, -1, 0, 1 ,2 orbits, this cause them evenly distributed in these orbits, or the same mass density throughout m layers in the *l*=2 sub-shell of n=3 shell. Again, remember this is based on the non-spin pre-Sun model. As that will be explained later, this degeneracy will be removed in a spinning pre-Sun model.

I-f. Construct a current Sun ball by adding the mass probability density distribution for super-shell spaces of N= -1, -2, etc.

Now, by using the radial wave function of the pre-Sun ball's Schrodinger equation, combining with Solar QM {N,n} model, we are able to construct a true {N,n//6} QM structure for the current Sun ball , with multiple N super-shells, a base-5*6^ of n, a increasing mass density from surface to center, and a clear drop of mass density at the surface of the ball.

In Figure 3a, I constructed the radial probability (or mass) density distribution for current Sun ball from {-2,n=2..6} super-shell to {0,2} shell under non-spin model. The N= -1 super shell (which is sun core)'s Σn=1..5 total probability curve is directly copied from Figure 2a. The N= -2 super shell (the inner core of Sun) 's Σn=1..5 total probability curve is directly copied from that of N= -1, but shift the r axis by a factor of 1/36. In the N=0 super shell, only n=1 and n=2 of $r^2*|R(n,l)|^2$ make contribution. We know that the mass distribution stopped at r of {0,2}, so I made a boundary cut off at $r/r_1 \geq 4$, or at >4, $r/r_1 = 4 + \log(r/r_1 - 4)$. According to the rule of "all mass between r_n and r_{n+1} belong to orbit n (see paper SunQM-3s2)", the space between {0,1} and {0,2} is belong to orbit {0,1}o. This rule is a little bit over simplified. A more accurate estimation (shown in Figure 3a) is that in space between {0,1} and {0,2}, ~80% mass is at n=1 orbit, and ~20% mass is at n=2 orbit. For a complete solar system (including all eight planets)'s radial mass density distribution, and the detailed explanation of how it is plotted, refer to Figure 4 in paper SunQM-3s1.



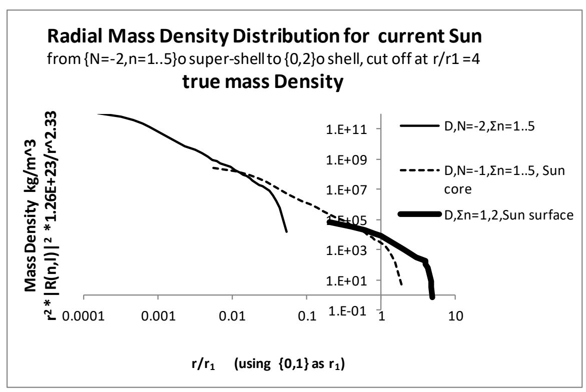


Figure 3a. Using Schrodinger equation solution to construct the radial probability density distribution for the current Sun ball from {N=-2,n=1..5} super-shell to {0,2} shell. The N=0 super-shell cut-off at $r/r_1 \geq 4$. It can be seen that in {0,n=1..2}o orbit, ~80% probability (estimated) from n=1, ~20% (estimated) from n=2. Note: The intensities of $r^2*|R(n,l)|^2$ in Figure 3a are not on scale for each N super-shell.
Figure 3b, I normalized the maximum value of probability $r^2|R(n,l)|^2$ (in Figure 3a) to one, then multiply to D=1.26E+23 / r^2.33, so it is the true mass density vs. r inside Sun ball.

Now I'd like to re-plot Figure 3a by using the true mass density inside the Sun. To do that, I need to deduce the mass density D(r) for each {N,n} super-shell inside the Sun. In paper SunQM-1s1, I have derived out the mass density formula for Solar system with its radius greater than {1,2}, that is, D = 4.37E+28 / r^3.279. Now I assume that D(r) inside Sun has the same formula form: D= A/r^B , where A and B are unknown constant values. From wiki "Solar core", "t*he core inside 0.20*

of the solar radius contains 34% of the Sun's mass, but only 0.8% of the Sun's volume." With this information, I can calculate A and B for D(r) formula. So I manually fit the following integration formula
Mass = ∫ D dV = ∭ D r^2 sin(θ) dr dθ dφ = 4π ∫ D r^2 dr = 4π *A *∫ (r^2) / (r^B) dr
to the two conditions:
1) Integration of r from 0 to 6.96E+8 m should be the total Sun mass = 1.99E+30 kg. Using online integration calculator "WolframAlpha" at" https://www.wolframalpha.com/", "integrate 4*pi*1.26E+23 / x^(0.33) dx, x=0 to 6.96E+8" should be the total Sun mass = 1.99E+30 kg,

$$\int_0^{6.96\times 10^8} \frac{4\,\pi\,1.26\times 10^{23}}{x^{0.33}}\,dx = 1\,986\,340\,954\,382\,331\,234\,968\,666\,112\,000$$

2) Integration of r from 0.2*6.96E+8=1.39E+8 m to 6.96E+8 m should be the 66% of total Sun mass (=0.66*1.99E+30 =1.3E+30 kg). Or, "integrate 4*pi*1.26E+23 / x^(0.33) dx, x=1.39E+8 to 6.96E+8" =1.31E+30

$$\int_{1.39\times 10^8}^{6.96\times 10^8} \frac{4\,\pi\,1.26\times 10^{23}}{x^{0.33}}\,dx = 1\,311\,307\,149\,053\,706\,941\,736\,788\,426\,752$$

The fitted result is A=1.26E+23, B=2.33. So, inside Sun ball, the mass density radial distribution is estimated to be

$$\mathbf{D = 1.26 * 10^{23} / r^{2.33}} \quad (\mathrm{kg/m^3}) \qquad \text{eq-16}$$

In Figure 3b, I normalized the probability (actually only the maximum value of $r^2|R(n,l)|^2$ curve in Figure 3a) to one, then multiply to D=1.26E+23 / r^2.33. So Figure 3b gives the true (or close to true) mass density of a Sun ball which is constructed by using Solar {N,n} QM structure, the Schrodinger equation solution, and Sun's D(r) formula. A more accurate prediction of Sun's mass density vs. r (calculated from {N,n} QM probability function) will be given in paper SunQM-3s6.

This example shows that we are able to construct a central-gravity formed celestial body (like Sun, Jupiter, Earth, etc.) by solely using Schrodinger equation solution and {N,n} QM structure. What a great success for QM! The time of "QM is only for micro-world" is gone forever. Long live QM !

I-g. Thermal pressure is the sustaining force to stabilize a {N,1}RF pre-Sun ball

So now we are able to construct a pre-Sun ball {N,n} QM structure by solely using Schrodinger equation and solution. But what is the sustaining force to stabilize a {N,1}RF pre-Sun ball? The answer is the thermal pressure. As mentioned in wiki "Degenerate matter": "*Most stars are supported against their own gravitation by normal thermal gas pressure*". Let us first compare the sustaining forces of Sun, white dwarf, and neutron star. We know (from wiki "degenerate matter") that the sustaining force for both white dwarf and neutron start is Pauli exclusion principle, it generates degeneracy pressure (or Fermi pressure) if all the lowest energy (or Fermi energy) quantum states are filled. Degeneracy pressure does not depend on the temperature but only on the density of the fermions. Similarly, the sustaining force for the current Sun is the thermal pressure which is generated from the heat of hydrogen fusion.

The forming of {N,1}RF (={N-1,n=2..6}RF) pre-Sun ball is due to the collapse of {N+1,1}RF (={N,n=2..6}RF) pre-Sun ball. During collapsing, > 99% of objects in space of {N, n=2..6} flew inward, and most of them stopped at space of {N-1,n=2..6} through collision with the existing objects. While flying inward, their total energy lost some potential energy (U), and transformed into kinetic energy (KE), and part of KE transformed into temperature through collision, and thus transformed into heat. Similar as that in the idea gas PV=nRT, with constant V (the shell space volume of {N-1,n=2..6} is fixed), and increasing both T and mass (=n), the P in the shell of {N-1,n=2..6} will have to increase.

So in shell of {N-1,n=2..6} of pre-Sun ball where large amount of objects just filled in (from {N,n=2..6}), the mass density related pressure is similar to the degeneracy pressure (or Fermi pressure) of Pauli exclusion principle. However, it is not the dominant factor here because the mass density here is far below the maximum value. Only the T related pressure (=Thermal pressure) is the dominant factor because collisions produced very high T in shell {N-1,n=2..6}. As mentioned in wiki "Degenerate matter": "*All matter experiences both normal thermal pressure and degeneracy pressure, but in commonly encountered gases, thermal pressure dominates so much that degeneracy pressure can be ignored*." After {N+1}RF collapsed to {N,1}RF, the greatly increased thermal pressure sustained the {N,1}RF ball structure from the immediate further collapse to {N-1,1}RF ball. Also in θφ 2D-dimention, both T and P is homogeneous, so it provided the spherical surface pressure to prevent this spherical surface further collapse by G-force.

After long time of heat losting (into space through the EM radiation), the T of super-shell {N-1,n=2..6} decreased so much that its thermal pressure can no longer sustain the {N,1}RF structure against the gravity, so the {N,1}RF pre-Sun ball will quantum collapse into a {N-1,1}RF pre-Sun ball.

For a non-spin pre-Sun ball, all mass from {N-1,n=2..6} super-shell collapse into {N-1,1} RF ball, so there will be no mass left over to form planets in {N-1,n=2..6} super-shell. As the result, a non-spin star will not have any (original) planetary system that is formed from the collapse of pre-Sun ball, although it may have captured planets.

For a spinning pre-Sun ball, the result in paper SunQM-1, and SunQM-3s1 tells us that during collapsing, only < 1% of mass (at the high end of Boltzmann velocity distribution) in each n shell will have high enough v, and with orbit close enough to nLL (or orbital angular momentum close enough to $+L_z$), to survive from the collapse, and transform theirs RF heat (micro random) movement velocity into macro movement orbit velocity, and re-gain the sustaining force (now it is $F = ma = m\, v_n^2/r_n$) to stay in the shell ! My next paper SunQM-3s1 will present how this will happen in QM analysis (named as nLL QM effect). Meanwhile, < 1% of mass (also at the high end of Boltzmann velocity distribution) with theirs angular momentum in x-y plan (L_{xy}) will be expelled out as the bipolar overflow driven by the nL0 QM effect.

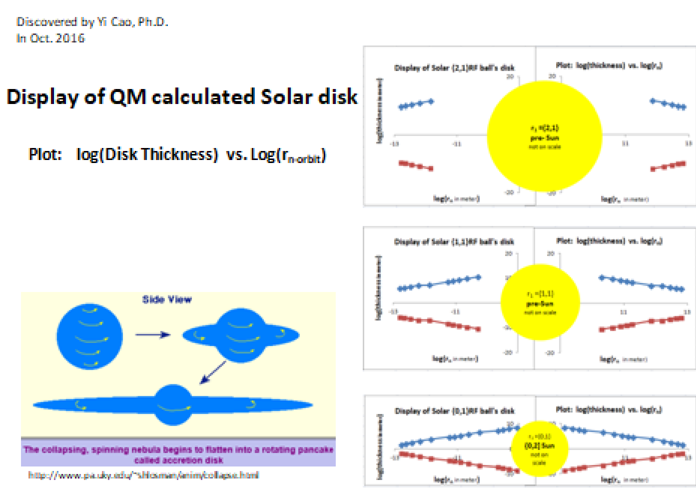
One evidence I mentioned in paper SunQM-1s1 is: In a series of quantum collapse of pre-Sun ball, at the stage of {2,1}, hydrogen fusion was ignited at the center of the pre-Sun ball. It quickly expanded its size from smaller than {-5,1} to {-2,1}. When pre-Sun ball collapsed from {2,1} to {0,1}, the Hydrogen fusion ball expanded from {-2,1} to {0,1}. Then, a {0,1} sized hydrogen fusion ball provides the exact thermal pressure for a {0,1} sized Sun ball, to prevent it from collapse under G-force. So this is a super stable {N,n} QM structure, it will last for 10 billion years.

II. Further analysis of Solar system and planets by combining {N,n} QM structure and Schrodinger equation solution

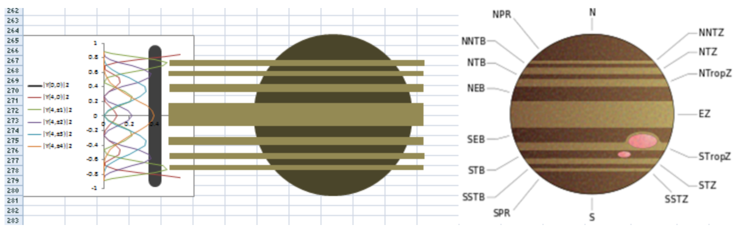
The combination of Solar {N,n} QM structure and Schrodinger equation solution has enabled me to solve a number of problems related to Solar system evolution, or planet structure.

In paper SunQM-3s1, I used the 1st order spin-perturbation to solve the pre-Sun ball disk-lyzation problem.

In paper SunQM-3s2, I used SunQM-3s1's result to study the evolutionary process of the pre-Sun ball's collapse and disk-lyzation. Below shows a (snap-shot) picture of pre-Sun ball disk-lyzation from {5,1} down to {0,1}, directly calculated from the probability density $|R_{nl}Y_{lm}|^2$.



In paper SunQM-3s3, by using Schrodinger equation solution, I have solved the problem of how the atmosphere bands are formed on the Jupiter's surface. The figure below shows a (snap-shot) picture of Jupiter atmosphere's ring pattern, directly calculate out by using probability density $|R_{nl}Y_{lm}|^2$ (see SunQM-3s3 for details). The Earth atmosphere circulation problem has also been solved by using the same method.



In paper SunQM-3s4, Saturn's ring will be analyzed by using Solar QM {N,n} structure and multiplier n'. In paper SunQM-3s5, nL0 effect of {N,n} QM structure will be explored for stars' bipolar outflow. In paper SunQM-3s6, all planets' internal cores (size and mass density) will be analyzed by using interior {N,n} QM structure.

Conclusions

1) Schrodinger equation has been established for the non-spin Solar QM {N,n} structure model. Solving the Schrodinger equation gives the orbit energy $E_n = -2m[\pi GM/(Hn)]^2$, where $H = 2\pi GM/v_{1\text{-orbit}}$, or, $E_n = -3.81E+11 *(m/n^2)$, where m is the orbit moving object's mass (in kg), and n is the total n of the orbit using Sun core {0,1} as r_1, and E_n in unit J.
2) I have constructed the radial probability (or mass) density distribution for the current Sun ball from {N=-2,n=1...6} super-shell to {0,2} shell (shown in Figure 3a).
3) Combining $r^2|R_{nl}|^2$ to the deduced Sun's interior mass density formula D=1.26E+23 / r^2.33, the true mass density vs. r inside Sun ball (from {N,n} QM calculation) has been plotted (in Figure 3b).

References

[1] A series of my papers that to be published (together with current paper):
SunQM-1: Quantum mechanics of the Solar system in a {N,n//6} QM structure.
SunQM-1s1: The dynamics of the quantum collapse (and quantum expansion) of Solar QM {N,n} structure.
SunQM-1s2: Comparing to other star-planet systems, our Solar system has a nearly perfect {N,n//6} QM structure.
SunQM-1s3: Applying {N,n} QM structure analysis to planets using exterior and interior {N,n} QM.
SunQM-2: Expanding QM from micro-world to macro-world: general Planck constant, H-C unit, H-quasi-constant, and the meaning of QM.
SunQM-3: Solving Schrodinger equation for Solar quantum mechanics {N,n} structure.
SunQM-3s1: Using 1st order spin-perturbation to solve Schrodinger equation for nLL effect and pre-Sun ball's disk-lyzation.
SunQM-3s2: Using {N,n} QM model to calculate out the snapshot pictures of a gradually disk-lyzing pre-Sun ball.
SunQM-3s3: Using QM calculation to explain the atmosphere band pattern on Jupiter (and Earth, Saturn, Sun)'s surface.
SunQM-3s6: Predict radial mass density distribution for Earth, planets, and Sun based on {N,n} QM probability distribution.
SunQM-5: C-QM (a new version of QM based on interior {N,n}, multiplier n', |R(n,l)|^2 |Y(l,m)|^2 guided mass occupancy, and RF) and its application from string to universe.
SunQM-5s1: White dwarf, neutron star, and black hole re-analyzed by using C-QM.

[2] The citation of wiki "Solar core" means it is obtained from the Wikipedia online searching for "Solar core". Its website address is: https://en.wikipedia.org/wiki/Solar_core. This website address can be generalized for all other searching items.

[3] Major QM books, data sources, software I used for this study are:
Douglas C. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. 2009.
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Stephen T. Thornton & Andrew Rex, Modern Physics for scientists and engineers, 3rd ed. 2006.
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Free software: R
Microsoft Excel.
Public TV's space science related programs: PBS-NOVA, BBC-documentary, National Geographic-documentary, etc.
Journal: Scientific American.