Gravity Wave Basics

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This is a gentle, explanatory overview of the fundamentals of gravity waves intended for undergraduate physics students, curious high schoolers, and brilliant 4th graders, utilizing the traditional linearized form of Einstein's field equations.

1. Preliminaries

Following Adler et al., we will denote partial differentiation using a subscripted vertical bar. For example,

$$\frac{\partial f}{\partial x^\mu} = \partial_\mu f = f_{|\mu}$$

This notation avoids the repeated use of the partial differential operator ∂ while keeping the indices of differentiated quantities conveniently close to one another (more importantly, it also allows the writer to avoid having to write the `\partial` command all over the place).

The Einstein field equations for gravity in the presence of matter (represented by the energy-momentum tensor $T_{\mu\nu}$) are given by the set of simultaneous non-linear equations

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = -\frac{8\pi G}{c^4}\, T_{\mu\nu} \tag{1.1}$$

where all Greek indices run from 0 to 3 (we will also have occasion to use Latin indices, which run from 1 to 3). Since $R_{\mu\nu}$ is symmetric there are ten equations in all, but in most cases we'll have the diagonal condition $\mu = \nu$, leaving just four equations to deal with. $R_{\mu\nu}$ is the Ricci tensor

$$R_{\mu\nu} = \Gamma^\lambda_{\mu\lambda|\nu} - \Gamma^\lambda_{\mu\nu|\lambda} + \Gamma^\lambda_{\beta\mu}\,\Gamma^\beta_{\lambda\nu} - \Gamma^\lambda_{\beta\lambda}\,\Gamma^\beta_{\mu\nu} \tag{1.2}$$

and the Ricci scalar is its contracted variant given by $R = g^{\mu\nu} R_{\mu\nu} = R^\lambda_{\ \lambda}$. The Γ quantities are the Levi-Civita connection coefficients, also symmetric in their lower indices, given by

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}\, g^{\alpha\beta}\left(g_{\mu\beta|\nu} + g_{\beta\nu|\mu} - g_{\mu\nu|\beta}\right) \tag{1.3}$$

Its contracted form is

$$\Gamma^\alpha_{\mu\alpha} = \frac{1}{2}\, g^{\alpha\beta}\, g_{\alpha\beta|\mu} \tag{1.4}$$

Lastly, when $T_{\mu\nu}$ is zero we have the free-space field equations

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = 0$$

Contraction of this with the metric tensor $g^{\mu\nu}$ shows that $R = 0$, so the field equations reduce to

$$R_{\mu\nu} = 0 \tag{1.5}$$

2. The Linearized Field Equations

When gravity is weak (say, far from a field of matter), the metric tensor can be expressed as a slight change from the flat-space Minkowski tensor $\eta_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \tag{2.1}$$

Here, the Minkowski tensor is represented by the constant matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

while $h_{\mu\nu}(x)$ is a symmetric tensor that accounts for the gravitational field. Since we are dealing with weak fields, all terms having orders of $h_{\mu\nu}$ higher than one ($|h_{\mu\nu}| \ll 1$) can be neglected. With this definition of the weak field, the connection coefficients become

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\,\eta^{\alpha\beta}\left(h_{\mu\beta|\nu} + h_{\beta\nu|\mu} - h_{\mu\nu|\beta}\right), \quad \Gamma^{\alpha}_{\mu\alpha} = \frac{1}{2}\,\eta^{\alpha\beta} h_{\alpha\beta|\mu} = \frac{1}{2}\,h_{|\mu} \tag{2.2}$$

Inspection of the definition for the Ricci tensor (1.2) shows that the last two quantities have terms in $h_{\mu\nu}$ higher than first order. Dropping them, we then have the linearized form

$$R_{\mu\nu} = \Gamma^{\lambda}_{\mu\lambda|\nu} - \Gamma^{\lambda}_{\mu\nu|\lambda} \tag{2.3}$$

Plugging in the reduced connection coefficents, along with some indexing raising with $\eta^{\alpha\beta}$ and rearrangement of terms, we have

$$R_{\mu\nu} = \frac{1}{2}\,\Box^2 h_{\mu\nu} - \frac{1}{2}\left(h^{\alpha}_{\ \nu|\alpha} - \frac{1}{2}\,h_{|\nu}\right)_{|\mu} - \frac{1}{2}\left(h^{\alpha}_{\ \mu|\alpha} - \frac{1}{2}\,h_{|\mu}\right)_{|\nu} \tag{2.4}$$

where $\Box^2$ is the four-dimensional d'Alembertian differential operator

$$\Box^2 = \frac{1}{c^2}\,\frac{\partial^2}{\partial t^2} - \nabla^2$$

The last two terms in (2.4) look remarkably like divergences which, in general relativity and quantum mechanics, have a nice tendency to vanish. Indeed, by multiplying either term by $\eta^{\mu\nu}$ we have what appears to be the divergence of a legitimate scalar density:

$$S^{\mu}_{\ |\mu}, \quad \text{where} \quad S^{\mu} = h^{\alpha\mu}_{\ \ |\alpha} - \frac{1}{2}\,\eta^{\alpha\mu}\,h_{|\alpha}$$

Consequently, we can simplify (2.4) enormously if we can set both these terms to zero; that is, we want to set

$$\left(h^{\alpha}_{\ \nu} - \frac{1}{2}\,h_{|\nu}\right) = \text{constant (even zero)} \tag{2.5}$$

This expression is not a tensor quantity, so it will not be true in every coordinate system. But there is a clever work-around that will solve this problem.

Consider an infinitesimal change or coordinates given by

$$\bar{x}^{\mu} = x^{\mu} + \xi^{\mu} \quad \rightarrow \frac{\partial \bar{x}^{\mu}}{\partial x^{\lambda}} = \delta^{\mu}_{\ \lambda} + \xi^{\mu}_{\ |\lambda} \tag{2.6}$$

where $\xi^{\mu}(x)$, like $h_{\mu\nu}$ is some small arbitrary vector field in which terms like $|\xi^{\mu}|^2$ and $|\xi^{\mu}|\,|h_{\mu\nu}|$ vanish. Working to first order, it can be shown that

$$\bar{g}_{\mu\nu} = g_{\mu\nu} - g_{\mu\alpha}\,\xi^{\alpha}_{\ |\nu} - g_{\alpha\nu}\,\xi^{\alpha}_{\ |\mu} \tag{2.7}$$

Similarly, by (2.1) we have

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \xi_{\mu|\nu} - \xi_{\nu|\mu} \tag{2.8}$$

Using these identities we find, after a laborious calculation,

$$\left(\bar{h}^{\alpha}_{\ \nu|\bar{\alpha}} - \frac{1}{2}\,\bar{h}_{|\bar{\nu}}\right) = \left(h^{\alpha}_{\ \nu|\alpha} - \frac{1}{2}\,h_{\nu}\right) - \Box^2 \xi_{\nu} \tag{2.9}$$

where the $|\bar{\alpha}$ notation in the derivative stands for $\partial/\partial\bar{x}^\alpha$. Note that we can now set the last two terms in (2.4) to zero provided we can find a coordinate change in which

$$\Box^2 \xi_\nu = 0 \tag{2.10}$$

Since ξ^μ is arbitrary, we assume that we can always find a suitable vector quantity that satisfies (2.10). The two identities

$$h^\alpha_{\ \nu|\alpha} - \frac{1}{2} h_{|\nu} = \text{constant (including zero)}\,, \quad \Box^2 \xi_\nu = 0 \tag{2.11}$$

constitute what is known as the de Donder (or harmonic) *coordinate gauge* condition for the linearized field, which effectively sets a total of eight constraints on $h_{\mu\nu}$. Since $h_{\mu\nu}$ is symmetric, in four dimensions it has a total of ten independent terms. The conditions in (2.11) remove eight of these, leaving the linearized field equations with just two independent terms in $h_{\mu\nu}$. This is a highly significant finding, and we will see that the remaining two terms define the allowed polarization states of a gravitational wave.

With the de Donder gauge in hand, we see that both the Ricci tensor and the Ricci scalar R vanish in matter-free space, and that

$$R_{\mu\nu} = \frac{1}{2} \Box^2 h_{\mu\nu} = 0 \tag{2.12}$$

$$R = \frac{1}{2} \Box^2 \eta^{\mu\nu} h_{\mu\nu} \quad \text{or} \quad \Box^2 h = 0 \tag{2.13}$$

(We will interpret the latter equation to mean that the trace of h is identically zero.) These two identities will be of use later when we examine the matrix representation of the $h_{\mu\nu}$ field.

But before moving on, let us reflect on the nature of the de Donder gauge condition. In electrodynamics, the four-potential A_μ defines the electric and magnetic fields via the covariant expressions

$$F_{\mu\nu|\lambda} + F_{\lambda\mu|\nu} + F_{\nu\lambda|\mu} = 0\,, \quad \left(\sqrt{-g}\, F^{\mu\nu}\right)_{|\nu} = \sqrt{-g}\, J^\mu$$

where $F_{\mu\nu}$ is the antisymmetric electromagnetic tensor

$$F_{\mu\nu} = A_{\mu|\nu} - A_{\nu|\mu}$$

and J^μ is the source vector. The electric and magnetic fields are then determined via

$$\vec{E} = -\vec{\nabla} A_0 - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\,, \quad \vec{B} = \nabla \times \vec{A}$$

The well-known principle of a local *gauge transformation* allows us to make a change in the A_μ by adding an arbitrary gradient that does not affect the electric and magnetic fields. That is, the change

$$A_\mu \to A_\mu + \partial_\mu \Lambda(x) \tag{2.14}$$

has no effect on the electric and magnetic fields or on $F_{\mu\nu}$. Compare this with the coordinate change of $h_{\mu\nu}$ in (2.8), which is structurally similar.

To summarize, in electromagnetism we have the four-vector A_μ, which can be varied via a gauge transformation, while in linearized gravity we have the tensor $h_{\mu\nu}$, which by (2.13) is allowed to vary by a change in the coordinate system without affecting $R_{\mu\nu}$. Although gravity and electromagnetism are two very different phenomena, the underlying gauge invariance of both is at once unmistakable and intriguing.

3. Gravitational Radiation

By utilizing the de Donder coordinate gauge, the linearized gravitational field equations for free space are simplified to

$$\Box^2 h_{\mu\nu} = 0 \tag{3.1}$$

We recognize this as a *wave equation*, with the effects propagating at the speed of light. If gravitational radiation exists, then it propagates at the same speed as light. We can even go so far as to assume that, like photons of light, *gravitons* exist that make up the gravitational field.

Since we are still in a flat Minkowski frame, we can express the d'Alembertian of the $h_{\mu\nu}$ field in terms of waves propagating along some plane in a specified direction. We can therefore write the field in plane-wave form as

$$h_{\mu\nu} = \epsilon_{\mu\nu} e^{ik\cdot x} \tag{3.2}$$

where $\epsilon_{\mu\nu}$ is a constant symmetric tensor, with $k \cdot x = k_\mu x^\mu$, where k_μ is the wave vector. Plugging this expression into (3.1) (or taking the associated Fourier transform), we immediately have the two identities

$$k_\mu k^\mu = 0\,, \quad k^\mu \epsilon_{\mu\nu} = 0$$

Let us assume we have a gravitational wave moving along the z axis. The associated wave vector can be described by the simple row vector

$$k_\mu = \frac{\omega}{c}\,[1 \;\; 0 \;\; 0 \;\; 1]$$

where ω is the wave frequency. If we represent the tensor $\epsilon_{\mu\nu}$ as the square matrix

$$\epsilon_{\mu\nu} = \begin{bmatrix} \epsilon_{00} & \epsilon_{01} & \epsilon_{02} & \epsilon_{03} \\ \epsilon_{01} & \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{02} & \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{03} & \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{bmatrix}$$

then

$$k^\mu \epsilon_{\mu\nu} = \frac{\omega}{c}\,[\epsilon_{00}+\epsilon_{03} \quad \epsilon_{01}+\epsilon_{13} \quad \epsilon_{02}+\epsilon_{23} \quad \epsilon_{03}+\epsilon_{33}] = 0$$

In view of this, let us set all the terms in this vector to zero. Then

$$\epsilon_{00} = \epsilon_{01} = \epsilon_{02} = \epsilon_{03} = 0,$$

$$\epsilon_{13} = \epsilon_{23} = \epsilon_{33} = 0$$

This leaves only three non-zero components $\epsilon_{11}, \epsilon_{12}$ and ϵ_{22} in $h_{\mu\nu}$. However, from (2.13) we also know that the trace h of $h_{\mu\nu}$ vanishes, so we must also have $\epsilon_{11} = -\epsilon_{22}$. The plane-wave form of $h_{\mu\nu}$ thus reduces to

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_+ & \epsilon_\times & 0 \\ 0 & \epsilon_\times & -\epsilon_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{ik_z z} \tag{3.3}$$

where we have given special names for the two non-zero components, which in fact represent the two allowed degrees of polarization of a gravitational wave. We now ask: what possible effect can the $h_{\mu\nu}$ field have upon a physical body?

4. Gravitational Distortion of a Physical Object

We continue with the results of the previous section, in which we assumed that a gravitational effect moving along the z axis is imparted by the weak $h_{\mu\nu}$ field given by (3.3). Assume that somewhere along this axis we place a vector A^μ describing some non-point object, such as a thin rod. The length L of this vector is given by the invariant quantity

$$L^2 = g_{\mu\nu}\, A^\mu A^\nu = \left(\eta_{\mu\nu} + h_{\mu\nu}\right) A^\mu A^\nu$$

or

$$L^2 = L_0^2 + h_{\mu\nu}\, A^\mu A^\nu$$

where L_0 is the length the object would have in the absence of a gravitational field. Since the gravitational field is weak, we can approximate the object's length in the field by

$$L = \sqrt{L_0^2 + h_{\mu\nu} A^\mu A^\nu} \approx L_0 \left(1 + \frac{h_{\mu\nu} A^\mu A^\nu}{2 L_0^2} \right)$$

The difference in length that the object will experience is then $\delta L = L - L_0$, or

$$\delta L = \frac{\epsilon_{\mu\nu} A^\mu A^\nu \, e^{i k_z z}}{2 \, L_0^2}$$

Since there are only two non-zero components in $\epsilon_{\mu\nu}$, this reduces to

$$\delta L = \frac{\epsilon_+ \left[(A^1)^2 - (A^2)^2 \right] + 2 \epsilon_\times A^1 A^2}{2 \, L_0^2} \, e^{i k_z z} \tag{3.4}$$

Obviously, according to this expression the object will suffer small distortions in length. Assume that the rod is initially positioned along either the x or y axis. Because of the presence of the oscillatory exponential term, the ϵ_+ component will cause the rod to oscillate in length only along those axes, alternatively expanding and shortening the rod as the wave passes by. Conversely, if the rod is initially rotated along some arbitary x-y axis, then the $\epsilon_\times$ component will cause the rod to alternatively lengthen and shorten along that particular axis.

It should be noted that for objects very far from a gravitating source, the distortion in length will be extremely small and difficult to detect. Indeed, the recent discovery of gravitational waves emanating from distant black hole and neutron star mergers involved distortions that are almost unbelievably small, much smaller than the diameter of a proton. It is a tribute to experimental gravitational-wave physicists that detection and precise analysis of such events in now possible.

5. Weak-Field Analysis in the Presence of Gravitating Matter

The linearized field equations can be applied to the case where a gravitating source of matter is present. Given that situation, we need specific information for the energy-momentum tensor $T_{\mu\nu}$ along with some idea of how that body of matter is behaving (moving, rotating, interacting, oscillating, etc.). For simplicity, we will consider only the case of a single, static massive body having a fixed size of constant density ρ. Even then the analysis is not elementary, so we will merely sketch the approach.

We reiterate the Einstein field equations in the presence of matter:

$$R_{\mu\nu} - \frac{1}{2} \, g_{\mu\nu} R = - \frac{8\pi G}{c^4} \, T_{\mu\nu} \tag{5.1}$$

Linearization gives us

$$R_{\mu\nu} = \frac{1}{2} \Box^2 h_{\mu\nu}, \quad R = \frac{1}{2} \Box^2 h$$

so that (5.1) becomes

$$\Box^2 \left(h_{\mu\nu} - \frac{1}{2} \, \eta_{\mu\nu} h \right) = - \frac{16\pi G}{c^4} \, T_{\mu\nu} \tag{5.2}$$

Let us now define a new field $\tilde{h}_{\mu\nu}$ given by

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \, \eta_{\mu\nu} h \tag{5.3}$$

so that

$$\Box^2 \, \tilde{h}_{\mu\nu} = - \frac{16\pi G}{c^4} \, T_{\mu\nu} \tag{5.4}$$

We can also write this in raised form as

$$\Box^2 \, \tilde{h}^{\mu\nu} = - \frac{16\pi G}{c^4} \, T^{\mu\nu}$$

Taking the divergence, we see that the de Donder gauge condition guarantees that the left side vanishes, while $T^{\mu\nu}{}_{|\nu}$ also vanishes by virtue of mass-energy conservation.

The simplest possible gravitating object would be a static body composed of cold, catalyzed matter having only density $\rho(x)$ and pressure $P(x)$ components. In that case the energy-momentum tensor is

$$T_{\mu\nu} = \begin{bmatrix} c^2\rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{bmatrix}$$

In view of the magnitude of the coefficient c^2, we will assume that the T_{00} term dominates, so that the corresponding $\bar{h}_{ii}$ solutions will be small by comparison.

In the presence of matter, we can no longer assume a plane-wave solution to the linearized field equations. However, we can use a Green's function to obtain a solution. The Green's function is defined via

$$\Box^2 \, G(\vec{x}^{\mu\prime}; \vec{x}^{\mu}) = \delta(\vec{x}' - \vec{x})\, \delta(ct' - ct)$$

or, for the static case,

$$\nabla^2 G(\vec{x}'; \vec{x}) = \delta(\vec{x}' - \vec{x})$$

where the arguments are delta functions. We then have

$$G(\vec{x}'; \vec{x}) = \frac{1}{4\pi\, |\vec{x}' - \vec{x}|}$$

which is the textbook definition for a Green's function in three-dimensional space for the operator ∇^2. The solution to (5.4) is then

$$\tilde{h}_{\mu\nu}(\vec{x}) = -\frac{16\pi G}{c^4} \int T_{\mu\nu}(\vec{x}')\, G(\vec{x}'; \vec{x})\, d^3\vec{x}'$$

or

$$\tilde{h}_{\mu\nu}(\vec{x}) = -\,\frac{4G}{c^4} \int \frac{T_{\mu\nu}(\vec{x}')}{|\vec{x}' - \vec{x}|}\, d^3\vec{x}' \tag{5.5}$$

Let us now assume that the gravitating body in question is a static mass of constant density ρ. In that case, (5.5) integrates immediately to

$$\tilde{h}_{00} = -\,\frac{4\,GM}{c^2\, |\vec{x}' - \vec{x}|}$$

or

$$\tilde{h}_{00} = -\,\frac{4\,GM}{c^2\, r} \tag{5.6}$$

where r is the distance from the source. This looks remarkably like the expected Schrödinger solution $g_{00} = \eta_{00} + h_{00} = 1 - 2GM/c^2 r$, but our (5.6) has a 4 instead of a 2. This is because we're still working with $\bar{h}_{00}$ and not h_{00}. To rectify this, note that we have assumed that the lower diagonal terms T_{ii} and their corresponding solutions $\bar{h}_{ii}$ will be small compared to $\bar{h}_{00}$. With these conditions in mind, we then have approximately

$$\bar{h} = \eta^{\mu\nu}\, \bar{h}_{\mu\nu} = \bar{h}_{00} \tag{5.7}$$

From (5.3), we see that $\bar{h} = -h$, a condition logically called *trace-reversal*. But now we have

$$\bar{h}_{00} = h_{00} - \frac{1}{2} h = -h \quad \text{or} \quad h = -2\, h_{00} \tag{5.8}$$

so that $\bar{h}_{00} = 2\, h_{00}$. We then have

$$h_{00} = -\,\frac{2GM}{c^2\, r} \tag{5.9}$$

as expected. Since $\bar{h}_{ii} \approx 0$, we also have $h_{ii} = h/2$, or $h_{ii} = h_{00}$. Thus, the complete line element for the linearized field equations are

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left(\eta_{\mu\nu} + h_{\mu\nu}\right) dx^\mu dx^\nu$$

or

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 + \frac{2GM}{c^2 r}\right)\left(dx^2 + dy^2 + dz^2\right) \tag{5.10}$$

which agrees with the Schwarzschild line element in an isotropic coordinate system.

5. Comments

By assuming a strictly diagonal energy-momentum tensor $T_{\mu\nu}$ for the linearized field equations, we have intentionally omitted any contribution from cross terms such as T_{03}, which in fact account for the gravitational effects of a rotating mass. The interested student is encouraged to pursue this situation further, where she will discover the approximate metric for a spinning body, first derived by Lense and Thirring in 1918 and subsequently solved for the exact non-linear case by Kerr in 1963.

We have also omitted any discussion on non-static cases, such as non-radially oscillating masses or the gravitational effects resulting from colliding or co-rotating binary masses. These problems are the focus of much current research, given the recent exciting discovery of gravitational waves resulting from merging black holes and neutron stars, but their treatment goes far beyond the scope of what is presented here. The student should note, however (and admire) the analytical complexity of such problems, given that the associated field equations require high-speed numerical computations to obtain even approximate solutions for the linearized case.

Although gravity waves are generally weak, over time they are responsible for the gradual loss of mass-energy from sources such as co-rotating binary star systems due to the emission of gravitational radiation. As mass is radiated away, the stars' orbital distance shrinks with the concurrent decrease in orbital period. The Hulse-Taylor binary neutron system PSR B1913+16, consisting of a neutron star and a pulsar, has been observed since 1974, its orbital period being accurately determined thanks to the regular periodicity of the pulsar's radio emission. After forty-five years of observation, the calculated change in the stars' orbital period exactly matches the observations. The discovery of this star system and its calculated agreement with Einstein's general relativity theory earned Hulse and Taylor the 1993 Nobel Prize in physics.

Lastly, the student should note that the linearized field equations were derived many years ago, first by Einstein himself, who recognized the likely possibility of gravitational radiation. But until the advent of large, highly sensitive instruments such as LIGO (the Laser Interferometer Gravitational-Wave Observatory) the technology simply did not exist for the detection of gravity waves.

References

1. R. Adler, M. Bazin and M. Schiffer, *Introduction to General Relativity*, McGraw-Hill, 2nd Edition, 1975. A dated but very readable text for the undergraduate physics student who wants to learn the basics of general relativity. It includes a detailed discussion on gravity waves, but the treatment is archaic by modern standards and omits the de Donder gauge approach, making the discussion difficult to follow.

2. M. Gasperini, *Theory of Gravitational Interactions*, Undergraduate Lecture Notes in Physics, Springer-Verlag Italia, 2013. Another good text on general relativity, the section on gravitational radiation details more advanced topics such as multipole field expansion, radiative power generation and the gravitational field of a co-rotating binary star system.

3. A. Zee, *Einstein Gravity in a Nutshell*, Princeton University Press, 2013. Arguably the best book available on gravitation for the student, the section on linearized gravity is rather terse but includes a great discussion on the similarities between gravitational and electromagnetic radiation.