The Resolution of the Twin Paradox in a One-Way Trip

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Abstract

The twin paradox is considered in a one-way trip. Usually it is considered in a two-way trip. In the usual formulation, the problem, the relative aging of the twins during the one-way trip, is hiden by the total aging of the twins in the round trip. It is shown that if we know the relative aging of the twins during the one-way trip there is no paradox and therefore the allegedly necessity to consider acceleration or the change of the frame argumentations does not emerge. It is also shown that the problem of simultaneity is irrelevant since in a one-way trip the twin can age slower or faster than the stay at home twin and therefore the assymetry must have an explanation different of the standard explanation based on the time dilation effect. It is shown that Special Relativity is enough to solve the twin paradox. The twin paradox is a classical case of a not well formulated problem (an ill-formulated problem).

Introduction

In our previous works [1] a broad approach of the Special Theory of Relativity has been formulated. The implications of this approach in the interpretation of the twin paradox will be discussed in the present paper. In Special Relativity and perhaps in all physics the twin paradox is one of the most persistent paradox ever. In the standard formulation of Special Relativity there is no agreement to solve the paradox. We have the "simultaneity", the "acceleration" and the "change of the frame" argumentations [2-10]. These argumentations are contradictory [3-6]. It is well known the arguments used by Einstein to solve the paradox [3, 4]. To an historical account of the evolution of the ideas about the resolution of the twin paradox it is recommended the article by Peter Pesic, "Einstein and the twin paradox" [6]. Following Feynman, a "paradox is a situation which gives one answer when analysed one way, and a different answer when analysed another way, so that we are left in somewhat of a quandary as to actually what would happen. Of course, in physics there are never any real paradoxes because there is only one correct answer; at least we believe that nature will act in only one way (and that is the right way, naturally). So in physics a paradox is only a confusion in our understanding " [11]. However "since it was launched by P. Langevin in 1911 (and was indeed explicit in the Einstein's 1905 famous article), the twin paradox has been at the origin of more than 25,000 articles in the literature'' [12]. So one can only suspect that "perhaps the last word on the twin paradox has yet to be said [13]."

A clock can emulate well a twin. The clocks can have the same rhythm (aging) and can have the same time reading (age) [1]. If two clocks have the same rhythms and have the same time readings the clocks emulate twins. The effect of change of aging that we are going to define occurs in the same manner, it is assumed, for clocks or real twins. Of course if one of the twins change aging they are not more twins. We are going to maintain the designation twin (because of the physical meaning of the word twin, the same aging and the same age) but we must be aware of that, and if we are aware of that we immediately acknowledge the problem of simultaneity. Contrary to the age of a real twin the time reading of a clock can be changed arbitrarily.

In section I we consider the aging of the twins in a one-way trip in a rectilinear movement with speed constant. We don't have acceleration or change of the frame and we can have the clocks in the frames where the twins are moving synchronized or not. The aging of the twins depend only on the rhythms and does not depend on the synchronization of the clocks in that frames [1, 14, 15]. In a simple way, considering a third frame to calculate the aging of the twins, it is shown that this aging does not depend of what twin is consider at rest. In a one-way trip the paradox emerge in the standard formulation because the aging of the twins allegedly depend of what twin is considered. Therefore we have a clear answer to the allegedly paradox. We consider two coordinates time, Lorentz time, t_L' and synchronized time, t' (at every point of space we have two clocks, a desynchronized clock and a synchronized clock). This permit evince the problem through an expression [1, 15] that easily show that the problem is not well formulated.

In section II we consider the paradox in the usual two-way trip. Since standard Special Relativity consider that all the frames are equivalent the allegedly standard relation between rhythms, the standard relative aging of the twins is reciprocal and therefore the paradox emerge because each twin seems aging less than the other and this is obviously impossible. The time dilation equation that standard Special Relativity consider as the relation of rhythms is mathematically correct and can be used but it is not the relation of rhythms. It is shown that in a two-way trip this relation seems to be the relation between rhythms because for the two way trip the total aging of the twins that can be calculated by that expression have the same form of the allegedly relation between rhythms, the standard time dilation in each trip when the trips to and fro have the same Einstein's speed. Since the returning twin is younger it seems odd that during all the trip both twins are being younger. It is shown that during the trips, the to and fro trips, since the correct relation between the rhythms does not depend of the frame consider the total aging of the twins calculated by the correct equation that relates rhythms is the same calculated by the allegedly expression of rhythms for the frame of the twin at rest. This explain why the standard interpretation seems correct but also show the complete solution of the problem. And also show how important is to consider the problem in a one-way trip. The aging of twins does not depend on the frame consider and also does not depend on the coordinates. It is a relation between proper times. Only depend of the speeds of the twins in relation to the frame where the one-way speed of light is isotropic.

I. One-Way Twin Paradox

Twin 1 is at rest at the origin $x'=0$ of the frame S'. Twin 2 pass by that origin with speed v' and Einstein's speed v_E' at time $t'=0$ and will pass by twin 3 located at rest at x' at time t' (the ages of twins 1 and 3, t' is the synchronized time).

Einstein's definition of speed [1] is not the usual definition of speed. This can originate misunderstandings since we are defining a new concept using the same word speed. The concept of speed is the quotient of the distance by the transit time, the time necessary to describe that distance, the time elapsed between the events departure and arrival. If the clocks at several points of the movement are synchronized between each other, the transit time is the difference between the times at arrival and at departure. This is not the case if the clocks are desynchronized. Of course if we know the desynchronization we can calculate the transit time being aware of the desynchronization. To avoid misunderstandings we call to Einstein's concept of "speed" Einstein's speed [1, 15], $v_E{}'$, maintaining the usual word speed, v', to the usual concept. Using this clear nomenclature it is easy to show that twin's paradox, or Dingle's paradox [1], is a result of not considering the desynchronization between the clocks conjugated with the misunderstanding of the two concepts of "speed" [1], Einstein's speed and speed.

The transit time is well defined and unique for the frame where the movement is considered. What are not unique are the several differences between several clocks at two points with several desynchronizations. Therefore since at a point x' we can have two clocks with "Lorentz time" $t_L{}'$ and "synchronized time" t', the difference between the instant of arrival and departure is different for those pairs of clocks. Synchronization is not a convention [1]. Synchronization is unique and only can be achieved by Einstein method of synchronization if we know the one-way speed of light in that frame. We cannot by definition impose that the speed of light is c in all frames [1]. What is c in all frames, by definition, is "Einstein's speed" of light [1]. It is not a postulate [1]. It is a definition, conjugated with a definition of time, "Lorentz time". In any frame we have only a clock rhythm and only a synchronization [1]. Therefore for each movement we have only one transit time. If we don't know the one-way speed of light we cannot synchronize the clocks in that frame with Einstein's method of "synchronization" although we can desynchronize the clocks with light in a unique manner, the Einstein method of "synchronization". The Einstein method of synchronization is a unique method of desynchronization because it is the same in all frames and leads to Lorentz Transformation. This impossibility to synchronize clocks in all frames by Einstein method of "synchronization" is the origin of the indeterminacy of Special Theory of Relativity because we don't know the "common" time of the clocks and therefore we don't know the transit time of the movement [1, 15]. However the usual language of standard Special Relativity induce to think that the transit time is the difference between the "Lorentz time" at arrival and departure. Only for one frame that can be so.

Speeds $v_E{}'$ (Einstein's speed) and v' (speed) are defined by the equations (1) and (2) [1, 15]

$$v'_E = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} \qquad (1)$$

$$v' = \frac{v_2 - v_1}{1 - \frac{v_1^2}{c^2}} \qquad (2)$$

The usual definition of speed originates equation (2) that differ from (1), Einstein's speed.
Twin 2 have a movement through the x' axis of S' with speed v_2, and v_1 is the speed of twin 1 (located at $x'=0$) and twin 3 (located at x'), the speed of S'. These speeds are defined in relation to a frame S, the Einstein's frame (*EF*), defined by the isotropy of the one-way speed of light with value c, the value of the two-way speed of light ($v'=c$ only for $v_1=0$) [1, 16]. In this frame the clocks exhibit the synchronized time that can be conceived by Einstein's method of synchronization.

For Einstein's frame since $v_1=0$ we have $v'= v_E'$. This frame is unique ($v'=c$ only for $v_1=0$). The time at twin 3 when twin 2 arrive is the time of the trip from the origin $x'=0$, $t'=0$ to x', it is t'. This time only depends of the rhythms of clocks of S'

$$t' = \frac{x'}{v'} \qquad (3)$$

This time is also the time marked by the synchronized clock at x'. It is also the age of twin 1 located at the origin since twin 3 is a real twin at x', has the same age t'.

We can calculate the aging of twin 2 and compare it with the aging of twin 1. Twin 2 is dislocating through the clocks at S and is aging τ'' (proper time) and the clocks at S display t when twin 2 pass over. We have

$$\tau'' = t\sqrt{1-\frac{v_2^2}{c^2}} \qquad (4)$$

It is the Larmor's time dilation expression that can be easily obtained with physical meaning with Feynman's clocks [17]. The meaning of (4) is that the clocks (twins) moving with speed v_2 in relation to S are displaying a lower time (aging less) then the clocks at S (twins). We have a similar expression for the clocks at S' (twins 1 and 3) since these clocks are moving with speed v_1.

$$\tau' = t\sqrt{1-\frac{v_1^2}{c^2}} \qquad (5)$$

For time t elapsed at S when twin 2 arrives at x' (twin 3) we have

$$x = x'\sqrt{1-\frac{v_1^2}{c^2}} + v_1 t \qquad (6)$$

$$x = v_2 t \qquad (7)$$

Equation (6) has the condition $t=0$ for the coordinate x correspondent to x' (position of twin 3) [17] (see this problem of the location of twin 3 in the article of Gron [5] and also the two examples at section III and the appendix). It is given by the Fitzgerald-Lorentz contraction. The Fitzgerald-Lorentz contraction is a result of the Michelson-Morley-Miller experiments supposed in vacuum and the Larmor's time dilation expression [17]. From (6) and (7)

$$t = \frac{x'}{v_2 - v_1}\sqrt{1 - \frac{v_1^2}{c^2}} \qquad (8)$$

From (5) and consistently with (3)

$$\tau' = t' = \frac{x'}{v_2 - v_1}\sqrt{1 - \frac{v_1^2}{c^2}}\sqrt{1 - \frac{v_1^2}{c^2}} \qquad (9)$$

For twin 2 we have from (4)

$$\tau'' = \frac{x'}{v_2 - v_1}\sqrt{1 - \frac{v_1^2}{c^2}}\sqrt{1 - \frac{v_2^2}{c^2}} \qquad (10)$$

Therefore the relation [1, 14, 15] between the aging of the twins 2 and 1 is, from (9) and (10)

$$\frac{\tau''}{\tau'} = \frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} \qquad (11)$$

This relation only depends from v_2 and v_1 and therefore must be true whatever the frames chosen S'' (the frame of twin 2) and S' (the frame of twin 1), independently of the coordinates, particularly the coordinate time at spatial different positions. It is a relation between proper times, a relation between rhythms that does not depend of the "synchronization" of the clocks at different spatial locations.

It is also considered evident that this relation does not depend of the "point of view" (the dialectic of relativity [18]) since from the point of view of twin 2 it is twin 1 that is moving but the relation subsist and does not change when we interchange 1 and 2.

$$\frac{\tau'}{\tau''} = \frac{\sqrt{1 - \frac{v_1^2}{c^2}}}{\sqrt{1 - \frac{v_2^2}{c^2}}} \qquad (12)$$

The false twin paradox is a result of a pretence "equivalence" of all the inertial frames that does not subsist (the equivalence) if we consider a third frame [1] as equations (11) and (12) expose. This "equivalence" seems to exist when we consider Lorentz coordinates that reveal a formal symmetry that is confounded with an ontological symmetry. Potatoes continue to be different from apples even if we designate potatoes by apples. It is only a terminological confusion. From the point of view of twin 2, allegedly, twin 1 is also aging less. This is a result of attributing the meaning of relation of rhythms to the expressions similar to eq. 4 [1] (see section II)

$$\Delta\tau^{\prime\prime} = \Delta t_L^{\prime}\sqrt{1-\frac{v_E^{\prime}}{c^2}} \qquad (13)$$

$$\Delta\tau^{\prime} = \Delta t_L^{\prime\prime}\sqrt{1-\frac{v_E^{\prime\prime}}{c^2}} \qquad (14)$$

Although (13) and (14) are true mathematical relations, (13) and (14) are not the relation of rhythms given by eq. (11) except for one frame, Einstein's frame. In (13) $\tau^{\prime\prime}$ is the time indicated by a clock (proper time) moving through clocks indicating $t_L^{\prime}$, Lorentz time displayed by desynchronized clocks. Therefore the differential of Lorentz times does not have the meaning of proper time elapsed of the clocks at $S^{\prime}$ and is a result of the time elapsed superimposed with the desynchronization [1]. Only in relation to EF this correspond to a time dilation because there is no desynchronization [1]. Eq. (14) has a similar meaning. With this relations alone we don't know who is being younger or with the same aging [1, 19] as eq. (11) reveals.

This is why Einstein consider acceleration necessary to solve the problem of the asymmetry of the aging of the twins (and in this context, accepting that all the Lorentz clocks allegedly are truly synchronized and the frames are all equivalents, he was correct contrary to what Peter Pesic affirm [6]). Following Einstein's interpretation when twin 2 accelerate to return, twin 1 suffer a sudden aging that explains the precocious aging for the round trip [4]. Obviously eq. (11) show that there is no "*spooky effect*" at twin 1 motivated by the acceleration of twin 2 and this is also evident by the reasoning of Lord Halsbury [20].

We don´t know the relation between the rhythms if we don´t know v_2 and v_1. But this indeterminacy [19] does not permit to consider (13) and (14) as the relation between rhythms as intended by all the defenders of the standard interpretation of special relativity [1, 15, 19]. If not we have a real paradox and this cannot be consider possible as some defenders of the standard interpretation also stated [5, 6]. This is why the analysis of a paradox is so important since it gives physical meaning to a theory [3-6, 21, 22].

To maintain the "equivalence" of all inertial frames ((13) and (14) are considered by Einstein the relation between rhythms) Einstein give up, in this context of Relativity, the independence of reality from the observer, since he maintain through all his life that reality is independent of to be observed or not. Therefore observations can suggest a hidden reality, eventually not observed yet, and cannot be dismissed as no existent. This is the origin of the Theory of Relativity and also the origin of the standard interpretation. And, ironically, also the origin of Einstein's discontentment with the interpretation of Quantum Mechanics. Theory of Relativity is a result of assuming the existence of a unique EF [1] exactly the contrary that was usually said by the standard interpretation [23].

II. The Two-Way Twin Paradox

Consider now the return of twin 2 after arriving at twin 3. Twin 2 must accelerate to stop and must accelerate to return (see the example of section III where twin 2 only need to accelerate to return). But for a long trip, *x´* is far away from twin 1, this return operation can be consider only a little anomaly for the long inertial trips, to and fro. And if this is so it can be consider that for all the trips the movement is inertial. This is exactly what is consider by the standard interpretation when calculate the total aging of the twins by expression (13) with constant $v_E{}'$ during each trip in the frame of twin 1. The necessity to accelerate is not consider relevant. And this is correct. Because the same result is obtained when we consider the two different inertial frames where twin 2 is located in the two-way trip as must be [21, 22]. Standard interpretation affirms based on (14) that during the trip of twin 1 (the frame consider at rest is *S´´* the frame of twin 2) twin 1 is being younger. But this is complete nonsense (appendix). For every point of the journey the answer must be the same independent of the "point of view". It is independent of the frame.

We have the following relations from (11) for the to and fro trips

$$\tau''_{\rightarrow} = \tau'_{\rightarrow} \frac{\sqrt{1-\frac{v_{2\rightarrow}^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} \qquad (15)$$

$$\tau''_{\leftarrow} = \tau'_{\leftarrow} \frac{\sqrt{1-\frac{v_{2\leftarrow}^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} \qquad (16)$$

Twin 2 when return to twin1 has a total aging from (15) and (16)

$$\tau''_{\Leftrightarrow} = \tau''_{\rightarrow} + \tau''_{\leftarrow} = \tau'_{\rightarrow} \frac{\sqrt{1-\frac{v_{2\rightarrow}^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} + \tau'_{\leftarrow} \frac{\sqrt{1-\frac{v_{2\leftarrow}^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} \qquad (17)$$

For the particular case when twin 2 return with the same Einstein's speed v_E we have from (13)

$$\tau''_{\Leftrightarrow} = \Delta t'_{L\rightarrow}\sqrt{1-\frac{v_E^2}{c^2}} + \Delta t'_{L\leftarrow}\sqrt{1-\frac{v_E^2}{c^2}}$$

$$= (\Delta t'_{L\rightarrow} + \Delta t'_{L\leftarrow})\sqrt{1-\frac{v_E^2}{c^2}}$$

$$\tau''_{\Leftrightarrow} = \tau'_{\Leftrightarrow}\sqrt{1-\frac{v_E^2}{c^2}} \qquad (18)$$

Equation (18) seems to confirm that the relation between rhythms is given by (13) and (14), constant at every point of the trip since the "speed" is constant during the all trip, but this is not true from (11). Although (11) gives also (18), it gives more. It gives the relation between the rhythms for every point of the trip. Equation (11) can be rewritten in the form

$$\frac{d\tau''}{d\tau'} = \frac{\sqrt{1-\frac{v_2^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{v'}{v'_E}\sqrt{1-\frac{v_E'^2}{c^2}} \qquad (19)$$

Indeed since [1]

$$t'_L = t' - \frac{v_1}{c^2}x' \qquad (20)$$

Differentiating and from (1) and (2)

$$\begin{aligned} dt'_L &= dt' - \frac{v_1}{c^2}dx' = dt' - \frac{v_1 v'}{c^2}dt' \\ &= dt'\left(1-\frac{v_1 v'}{c^2}\right) = dt'\frac{v'}{v'_E} \end{aligned} \qquad (21)$$

Therefore, from (13), we obtain (19).

Indeed another way to obtain (19) is from (13), in a differential form

$$d\tau'' = dt'_L \sqrt{1 - \frac{v_E'^2}{c^2}} = \frac{dx'}{v'_E} \sqrt{1 - \frac{v_E'^2}{c^2}}$$
$$= \frac{v' dt'}{v'_E} \sqrt{1 - \frac{v_E'^2}{c^2}} = d\tau' \frac{v'}{v'_E} \sqrt{1 - \frac{v_E'^2}{c^2}} \qquad (22)$$

Therefore from (19) we have

$$\tau''_{\Leftrightarrow} = \tau''_{\rightarrow} + \tau''_{\leftarrow} = \tau'_{\rightarrow} \frac{v'_{\rightarrow}}{v'_{E\rightarrow}} \sqrt{1 - \frac{v_{E\rightarrow}'^2}{c^2}} + \tau'_{\leftarrow} \frac{v'_{\leftarrow}}{v'_{E\leftarrow}} \sqrt{1 - \frac{v_{E\leftarrow}'^2}{c^2}} \qquad (23)$$

From (1), (2) and (23), imposing the same Einstein's speed for the returning trip (26) is obtained

$$v'_{E\rightarrow} = -v'_{E\leftarrow} = v_E \qquad (24)$$

$$\tau''_{\Leftrightarrow} = \tau'_{\rightarrow} \left(1 - \frac{v_1 v'_{\rightarrow}}{c^2}\right) \sqrt{1 - \frac{v_E^2}{c^2}} + \tau'_{\leftarrow} \left(1 - \frac{v_1 v'_{\leftarrow}}{c^2}\right) \sqrt{1 - \frac{v_E^2}{c^2}} \qquad (25)$$

$$\tau''_{\Leftrightarrow} = \left(\tau'_{\rightarrow} + \tau'_{\leftarrow} - \left(\tau'_{\rightarrow} \frac{v_1 v'_{\rightarrow}}{c^2} + \tau'_{\leftarrow} \frac{v_1 v'_{\leftarrow}}{c^2}\right)\right) \sqrt{1 - \frac{v_E^2}{c^2}} \qquad (26)$$

Indeed from (1) and (2) (eliminating v_2) and from (24) we obtain (26) that gives (18)

$$v'_E = \frac{v'}{1 - \frac{v_1 v'}{c^2}} \qquad (27)$$

$$v'_{E\to} = \frac{v'_{\to}}{1 - \frac{v_1 v'_{\to}}{c^2}} \qquad (28)$$

$$v'_{E\leftarrow} = \frac{v'_{\leftarrow}}{1 - \frac{v_1 v'_{\leftarrow}}{c^2}} \qquad (29)$$

$$\tau'_{\to} v'_{\to} + \tau'_{\leftarrow} v'_{\leftarrow} = \frac{|x'|}{|v'_{\to}|} v'_{\to} + \frac{|x'|}{|v'_{\leftarrow}|} v'_{\leftarrow} = 0 \qquad (30)$$

Eq. (18) is therefore obtained from (23).

Equations (25) and (30) reveals that during the trips to and fro although twin 2 has the same aging twin 1 is aging differently. This explain why it is possible that during the trips twin 1 (the older for the two-way trip) can be younger for a one-way trip (see one example at section III and appendix). The condition is that in module v_2<v_1. Only for $v_1=0$ this condition is not satisfied (twin 2 is aging less for the two trips) and this is what standard interpretation affirm for every frame. Standard interpretation is always in EF (appendix) and therefore it is inevitable that only one answer emerge and every frame seems exhibit complete similitude with no apparent conflict. The conflict however emerge in the paradox (appendix).

III. The distance between the Twins, the relativity principle and the acceleration

Let us calculate explicitly from the point of view of twin 2 the distance between twin 2 and twin 1 when twin 2 arrive at twin 3 (this is the same distance from twin 2 to twin 3 when twin 2 is at the origin of *S´*).

From (14)

$$\frac{|x'|}{|v'_{\to}|} = \tau'_{\to} = \Delta t''_{L\to} \sqrt{1 - \frac{v''_{E\to}}{c^2}} \qquad (31)$$

and from (28)

$$\Delta t''_{L\to} = \frac{|x''_{\to}|}{|v_E|} = \frac{|x'|}{|v'_{\to}|} \frac{1}{\sqrt{1 - \frac{v_E^2}{c^2}}} = \frac{|x'|}{|v_E|} (1 + \frac{v_1 v_E}{c^2}) \frac{1}{\sqrt{1 - \frac{v_E^2}{c^2}}} \qquad (32)$$

$$\left|x''_{\rightarrow}\right| = \left|x'\right|\left(1+\frac{v_1 v_E}{c^2}\right)\frac{1}{\sqrt{1-\frac{v_E^2}{c^2}}} \qquad (33)$$

Note that equation (33) can be rewritten, as must be

$$\frac{\left|x''_{\rightarrow}\right|}{\left|x'\right|} = \frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_{2\rightarrow}^2}{c^2}}} \qquad (34)$$

For the fro trip similarly

$$\left|x''_{\leftarrow}\right| = \left|x'\right|\left(1-\frac{v_1 v_E}{c^2}\right)\frac{1}{\sqrt{1-\frac{v_E^2}{c^2}}} \qquad (35)$$

$$\frac{\left|x''_{\leftarrow}\right|}{\left|x'\right|} = \frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_{2\leftarrow}^2}{c^2}}} \qquad (36)$$

This problem of correctly calculate the distance between twins is crucial (appendix). When twin 2 arrive at twin 3 the distance between 2 and 1 exist but only can be known if v_1 and v_2 is known. It is a distance between two events well defined. In the frame of twin 2 (one to the trip to and other for the fro trip) that distance is given by (33) and (35). And if this is done correctly (O. Grϕn refer this problem of the determination of distance by stipulation [5]) no paradox emerge and of course it is not necessary, in a returning trip, to consider the acceleration or change of frame explanation [4, 5, 12, 21, 22].
Let us check it in two particular cases (see also the appendix for a numerical exercise). The first, twins 1 and 3 are at rest in EF (*S´*, $v_1=0$) (in the second case we are going to consider it is twin 2 that is at rest in EF ($v_1=0$) for the first trip, the twins that are

moving in relation to EF are twins 2 ad 3 [5]). The problem is the correct evaluation of the distances, between twin 2 and twin 3 when twin 2 pass over twin 1 for the trip to and between twin 3 and twin 1for the returning trip. The distance between twin 1 and 3 is x' in the frame where 1 and 3 are located. And the distance between twin 2 and twin 3 is x'' in the frame of twin 2, correspond to a coordinate in the frame of twin 2 that passes over x' where twin 3 is located, when twin 2 is over twin 1, simultaneously. This is the most primitive notion of simultaneity that standard interpretation does not ruled out. When coordinate $x''=0$ is over $x'=0$ there is a coordinate x'' that is over x' where is twin 3. It is like an instantaneous transmission of a signal, but it is not “spooky” because the events are connected by rods [23]. Therefore from (33)

$$\left|x''_{\rightarrow}\right| = \left|x'\right| \frac{1}{\sqrt{1-\frac{v_E^2}{c^2}}} \qquad (37)$$

For this particular case $v_E=v_2=v'$ and since S' is the EF ($v_1=0$)

$$\Delta t' = \Delta t'_L = \tau' = \left|x'\right| \frac{1}{v_E} \qquad (38)$$

For the first trip twin 1 is dislocating through the x'' axis with speed

$$v'' = \frac{-v_2}{1-\frac{v_2^2}{c^2}} \qquad (39)$$

Therefore the time elapsed in the trip between twin 1 and twin 3 is given by

$$\tau'' = \frac{\left|x''\right|}{\left|v''\right|} = \frac{\left|x'\right|}{\left|v_E\right|} \frac{1}{\sqrt{1-\frac{v_E^2}{c^2}}} \left(1-\frac{v_E^2}{c^2}\right) = \tau' \sqrt{1-\frac{v_E^2}{c^2}} \qquad (40)$$

A similar expression (for this particular case since the twins that does not return are located at the EF) is obtained for one eventual returning trip and the two-way trip calculation gives as expected the same value obtained with the calculation for the frame of twin 1. Of course it essential the determination of the correct distance between the twins if not we don´t have a complicated paradox but simply a wrong calculation [5] (appendix). Indeed, in the second case, we consider that twin 2 is located initially at the EF (S) and the twins 2 and 3 are moving through the x axis of the frame S. At $x=0$ and $t=0$ where twin 2 is located twin 1 (located at the origin of S', $x'=0$) pass with speed v_1 and after t' pass with speed v_1 twin 3 (located at x'). The distance between twin 1 and twin 3 is the absolute value of x'. The distance between twin 2 and 3 is for the instant $t=0$

$$|x| = |x'|\sqrt{1-\frac{v_1^2}{c^2}} \qquad (41)$$

And the time elapsed in the frame of twin 2 is

$$\tau = \frac{|x|}{v_1} = \frac{|x'|}{v_1}\sqrt{1-\frac{v_1^2}{c^2}} \qquad (42)$$

Since S' is moving with speed v_1 in relation to EF the time dilation applies

$$\tau' = \tau\sqrt{1-\frac{v_1^2}{c^2}} = \frac{|x'|}{v_1}\sqrt{1-\frac{v_1^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}} = \frac{|x'|}{v_1}\left(1-\frac{v_1^2}{c^2}\right) \qquad (43)$$

For the returning trip we must consider that twin 2 acquire “instantaneously” speed v_2 in relation to EF moving in direction to twin1. The condition that must be imposed is that v_2 is consistent with the same Einstein speed v_1 in the frame S'.

From

$$v'_E = v_1 = \frac{v_2 - v_1}{1-\frac{v_2 v_1}{c^2}} \qquad (44)$$

we obtain

$$v_2 = \frac{2v_1}{1+\frac{v_1^2}{c^2}} \qquad (45)$$

and

$$v' = \frac{v_2 - v_1}{1-\frac{v_1^2}{c^2}} = \frac{v_1}{1+\frac{v_1^2}{c^2}} \qquad (46)$$

Therefore the time elapsed in the frame *S´* for the returning trip is

$$\tau_r^{´} = \frac{|x´|}{|v´|} = \frac{|x´|}{v_1}(1+\frac{v_1^2}{c^2}) \qquad (47)$$

Adding (43) and (47) we confirm that the total time is the same obtained with the first analysis but the times elapsed for the two trips are different.

We can also verify that we obtain consistency with the new frame *S´´* of twin 2. The distance between twin 2 and twin 1 when twin 2 instantaneously accelerate to return is given by eq. (33). And the aging for twin 2 is obtained calculating the time for the trip of twin 1 in this frame *S´´*. The speed of twin 1 in this frame is

$$v´´ = \frac{-v_1}{1-\frac{v_2 v_1}{c^2}} \qquad (48)$$

Since v_2 is given by (45) we obtain

$$v´´ = \frac{-v_1}{1-\frac{v_1^2}{c^2}}(1+\frac{v_1^2}{c^2}) \qquad (49)$$

Therefore the time for the returning trip for twin 2 is

$$\tau_r^{´´} = \frac{|x´´|}{|v´´|} = \frac{|x´|}{|v´|}(1-\frac{v_1^2}{c^2})\frac{1}{(1+\frac{v_1^2}{c^2})}(1+\frac{v_1^2}{c^2})\frac{1}{\sqrt{1-\frac{v_1^2}{c^2}}} \qquad (50)$$

$$\tau_r^{´´} = \frac{|x´´|}{|v´´|} = \frac{|x´|}{|v´|}\sqrt{1-\frac{v_1^2}{c^2}} \qquad (51)$$

With a new frame and without any paradox [5, 21, 22].

Until now we avoid the use of Lorentz Transformation. But now it is easy to reveal what is the problem with the interpretation of the Lorentz transformation. If *S* is the EF ($v_1=0$) and S' is a frame moving with speed v_1 in relation to EF we have the following equations that relates the spatial coordinates and time

$$x' = \frac{x - v_1 t}{\sqrt{1 - \frac{v_1^2}{c^2}}} \tag{52}$$

$$t'_L = \frac{t - \frac{v_1}{c^2} x}{\sqrt{1 - \frac{v_1^2}{c^2}}} \tag{53}$$

If S'' is a frame moving with speed v_2 we have similar relations with the new speed v_2. The relations between the coordinates of S'' and S' has also a similar form with the Einstein speed v_E

$$x'' = \frac{x' - v_E t'_L}{\sqrt{1 - \frac{v_E^2}{c^2}}} \tag{54}$$

$$t''_L = \frac{t'_L - \frac{v_E}{c^2} x'}{\sqrt{1 - \frac{v_E^2}{c^2}}} \tag{55}$$

Since *S* is the EF the clocks of *S* are synchronized. Therefore for ($x=x$, $t=0$) we have from (52) and (53)

$$x' = \frac{x}{\sqrt{1 - \frac{v_1^2}{c^2}}} \tag{56}$$

$$t'_L = \frac{-\frac{v_1}{c^2}x}{\sqrt{1-\frac{v_1^2}{c^2}}} = -\frac{v_1}{c^2}x' \qquad (57)$$

And similarly

$$x'' = \frac{x}{\sqrt{1-\frac{v_2^2}{c^2}}} \qquad (58)$$

$$t''_L = \frac{-\frac{v_2}{c^2}x}{\sqrt{1-\frac{v_2^2}{c^2}}} = -\frac{v_2}{c^2}x'' \qquad (59)$$

Therefore the coordinates x' and x'' depend from v_1 and v_2 and are not knowable with only the knowledge of v_E. There is an indeterminacy [19]. We don't know the localization of x'' in relation to x' when the origin of the frames coincide eq. (56-59). This localization clearly show that the knowledge of v_1 and v_2 is essential and cannot be avoided. The EF is not superfluous [23]. This is the astonishing conflict referred by Zbigniew Oziewicz [24].

Let us check it directly from (54) and (55)

$$t''_L = \frac{-\frac{v_1}{c^2}x' - \frac{v_E}{c^2}x'}{\sqrt{1-\frac{v_E^2}{c^2}}} = \left(-\frac{v_1}{c^2} - \frac{(v_2 - v_1)\ {}^1\!/_{c^2}}{1-\frac{v_1 v_2}{c^2}}\right)x'\frac{1}{\sqrt{1-\frac{v_E^2}{c^2}}}$$

$$= \left(\frac{-\frac{v_2}{c^2}\left(1-\frac{v_1^2}{c^2}\right)}{1-\frac{v_1 v_2}{c^2}}\right)x'\frac{1}{\sqrt{1-\frac{v_E^2}{c^2}}} = -\frac{v_2}{c^2}x'' \qquad (60)$$

From (60) we obtain

$$x'' = \left(\frac{(1-\frac{v_1^2}{c^2})}{1-\frac{v_1 v_2}{c^2}}\right)\frac{1}{\sqrt{1-\frac{v_E^2}{c^2}}}\,x' = \frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}}\,x' \qquad (61)$$

From (54) and (55) formal similar relations is obtained for $t'_L=0$

$$x'' = \frac{x'}{\sqrt{1-\frac{v_E^2}{c^2}}} \qquad (62)$$

$$t''_L = \frac{-\frac{v_E}{c^2}x'}{\sqrt{1-\frac{v_E^2}{c^2}}} \qquad (63)$$

But this relations are a mathematical result that does not can be interpreted as an equivalence. For example eq. (62) does not mean that the distance of the origin of $S''$for $t_L''=0$ to coordinate x' is x'' except if $v_1=0$ [5] since that distance is given by (62). This new event with coordinates x'' and t_L''given by (63) is another event and cannot be misinterpret as the same event [1, 15, 16, 19]. The two-way trip is the same for every frame, and does not depend in the one-way trips that are different and cannot be descripted only with Lorentz coordinates. This is the meaning of the relativity principle [19].

Now we can understand more deeply why Einstein consider necessary to introduce acceleration [4]. From the point of view of twin 2, eq. (14) gives for the two-way trip

$$\tau'_{\Leftrightarrow} = \tau'_{\rightarrow} + \tau'_{\leftarrow} = (\Delta t''_{L\rightarrow} + \Delta t''_{L\leftarrow})\sqrt{1-\frac{v_E^2}{c^2}} \qquad (64)$$

This equation means, it is interpreted, that twin 1 is aging less than twin2. But the analysis from the point of view of twin 1 (the first analysis) show that it is twin 2 that is aging less. Therefore something must be wrong. And, not consistently with the first analysis, Einstein introduce another effect, if not the standard interpretation collapse. Since in this case the addition of the Lorentz times does not give the total proper time of the trip for twin 2, this difference has been interpreted as an effect of acceleration [4-6, 12].

However, the explanation is very simple. From (12) and applying the formalism to the two frames of twin 2 (65) is obtained with generality, for any two frames (eventually one is the EF).

$$\tau'_{\Leftrightarrow} = \tau'_{\rightarrow} + \tau'_{\leftarrow} = (\Delta t''_{L\rightarrow} + \Delta t''_{L\leftarrow})\sqrt{1-\frac{v_E^2}{c^2}} =$$

$$= (\tau''_{\rightarrow} + \tau''_{\leftarrow})\frac{1}{\sqrt{1-\frac{v_E^2}{c^2}}} \tag{65}$$

And it is not necessary to consider any other effect. The acceleration allegedly cause a sudden jerk in the clock miles away from the accelerated twin that do not have any own effect and, from (65), we confirm that this interpretation of Einstein is based on the necessity to maintain the equivalence of all the frames.

However eq. (38) can be rewritten

$$\tau'_{\Leftrightarrow}\sqrt{1-\frac{v_E^2}{c^2}} = (\tau'_{\rightarrow} + \tau'_{\leftarrow})\sqrt{1-\frac{v_E^2}{c^2}} = (\Delta t''_{L\rightarrow} + \Delta t''_{L\leftarrow})(1-\frac{v_E^2}{c^2}) = \tag{66}$$

$$= (\tau''_{\rightarrow} + \tau''_{\leftarrow})$$

$$(\tau'_{\rightarrow} + \tau'_{\leftarrow}) = (\Delta t''_{L\rightarrow} + \Delta t''_{L\leftarrow})\sqrt{1-\frac{v_E^2}{c^2}} =$$

$$(\tau''_{\rightarrow} + \tau''_{\leftarrow})\sqrt{1-\frac{v_E^2}{c^2}} + (\Delta t''_{L\rightarrow} + \Delta t''_{L\leftarrow})\frac{v_E^2}{c^2}\sqrt{1-\frac{v_E^2}{c^2}} = \quad (67$$

$$= (\tau'_{\rightarrow} + \tau'_{\leftarrow})_i + (\tau'_{\rightarrow} + \tau'_{\leftarrow})_a = 2\frac{x'}{v_E}$$

Therefore it is possible to identify formally two terms

$$(\tau'_{\rightarrow} + \tau'_{\leftarrow})_i = (\tau''_{\rightarrow} + \tau''_{\leftarrow})\sqrt{1 - \frac{v_E^2}{c^2}} \qquad (68)$$

$$(\tau'_{\rightarrow} + \tau'_{\leftarrow})_a = 2\frac{x' v_E}{c^2} \qquad (69)$$

When we have acceleration it is the second term that affect twin 1 at distance *x´*, it has been interpreted and have not any physical meaning.

IV. Conclusions

In our previous works [1] a broad approach of the Theory of Relativity has been formulated. The standard interpretation of the theory can be explained. It is a result of considering that the existence of a frame where the one way speed of light is isotropic independent of the source, named Einstein's frame (EF), is superfluous. Only the relative movement between two frames must be considered. This cannot be so. There is an indeterminacy of the theory [17]. This can be demonstrated considering a third frame where the speed of light is isotropic and analysing the relative movement [1, 14-17]. The twin paradox is analysed as an example well known of one the difficulties of the standard interpretation. The twin paradox has never been solved and cannot be solved with that interpretation. The paradox is not specifically with the twins in relative movement but with the standard interpretation [21, 22].

In section I it is considered the Twin Paradox in a one-way trip. Considered in a one-way trip the paradox allegedly emerge because the standard interpretation affirm that both twin are being younger. This affirmation is the result of the time dilation expression that standard Special Relativity consider the relation between the rhythms. It is shown that the relations of rhythms is given by another expression that relates the proper times. This expression is obtained calculating the rhythms of every twin in relation to EF and relating them. In relation to the EF the time dilation expression is the relation between the rhythms of every twin in relation of the rhythm of the EF. This clearly show that the problem is not well formulated by the standard interpretation. The relation of rhythms is only dependent of the speed of the twins in relation to the EF and cannot be expressed in function of the relative speed defined by the standard interpretation. Therefore this relation of rhythms does not depend of the frame considered. It is the same for both frames. The results must be the same and it is. Does not depend of the point of view.

In section II we formulate the two-way formulation of the paradox. This correspond to consider that one of the twins return. The standard interpretation consider legitimate to calculate, using the standard equation, the total aging of both twins without considering acceleration in a first calculation in the frame of the twin that stays at home. Therefore in that calculation there is no acceleration associated to the change of the frame. However the returning twin must accelerate. This show that this calculation must be an approximation and this is what has been accepted. For a long trip the effect of changing speed does not have a significant effect in the calculation and can be ruled out. Or we can consider that the "returning twin" is another twin with the same age of the twin that must return, moving in returning opposing direction. This is a way to avoid the acceleration and consider only inertial frames. Therefore in a second calculation for the frames of the returning twin it must give the same result of the first calculation. It is shown that it does, as expected. Therefore the affirmation that Special Relativity cannot be applied because the acceleration, or, because the frames are different [21, 22], is ruled out. If we apply the relation between the rhythms we obtain the same result. It does not depend of the frame contrary to the standard explanation [21, 22]. In section III the distance between the twins for the several frames are equated and shows an answer that is dependent of the two frames consider. It is not only a problem of relative movement between two frames. This problem of the determination of the distance is intimately related with the problem of simultaneity since we are considering the ageing of the twins at a given distance at the same time. If we have a rod moving we know for sure that the extremities of the rod are simultaneous at two points of another frame. The difficulty to solve the paradox with the standard interpretation is only a result to attribute a wrong physical interpretation to a mathematical equation that does not have the meaning of relation between rhythms. And Special Theory of Relativity is a result of assuming the existence of a frame where the one-way speed of light is isotropic independently of the speed of the source and can solve the twin paradox.

If we consider the frame of the returning twin, twin 2, we obtain the same relations for the to and fro frames. This is what equations (11) and (12) expose. And also expose why it is not possible to obtain consistently from twin 2 frame the relation between rhythms with expression (13), the standard dilation equation, allegedly the relation of rhythms. It is because (13) is not the relation between rhythms. It is not because we cannot apply the expression (appendix). The expression can be applied and give the relation between the proper times to the variation of Lorentz times. And we can convert this expressions to the relation between rhythms. Without any paradox. Of course the knowledge of Einstein's speed is not enough to solve the problem and this is why standard Special Relativity does not explicitly can solve the paradox. We know the distance to twin 3 in the frame of twin 1 but we don't know the distance in the frame of twin 2 only with the knowledge of Einstein speed. We need to know v_1. This is why never standard special relativity can solve the paradox [1, 2-10, 12, 18, 19, 20-24]. Einstein was right: "*We cannot solve our problems with the same thinking we used to create them*" [25].

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Appendix

Following the important and interesting paper of O. Grϕn [5] consider that the twin Alpha Proxima (AP) is L_0=4 ly (light years) from the twin Eartha (E). Twin Stella (S) is moving in relation to AP and E with v_1=0.8 c.

Consider now that Twin Stella (S) for the first trip is at rest in EF (frame S) and AP and E (for the two trips) are moving with speed v_1=0.8 c in relation to EF. The travel time in the frame of E is the aging of twins S and E (it is possible to consider that Stella is moving in relation to EF with speed v_1=0.8 c and E and AP are at rest in relation to EF but this is the first case of our previous analysis not consistent with the distance between S and AP stipulated by Grϕn (but only valid for the first trip as we are going to show) and can be considered as another case with the same relative movement).

In Section III from our previous analysis, the second case, we consider that twin 2 (S) is located initially at the EF (S) and the twins 1 (E) and 3 (AP) are moving through the x axis of the frame S. At $x=0$ and $t=0$ where twin 2 (S) is located, twin 1 (E) (located at the origin of S', $x'=0$) pass with speed v_1 and after t' pass with speed v_1 twin 3 (AP) (located at x'). The distance between twin 1 (E) and twin 3 (AP) is the absolute value of x'. The distance between twin 2 (S) and 3 is for the instant $t=0$ given by (41) (this is the distance stipulated by Grϕn [5])

$$|x| = L = |x'|\sqrt{1-\frac{v_1^2}{c^2}} = L_0\sqrt{1-\frac{v_1^2}{c^2}} = 4ly\sqrt{1-\frac{(0.8c)^2}{c^2}} = 2.4ly \quad (A1)$$

And the time elapsed in the frame of twin 2 (S), the aging of S is for the to trip

$$\begin{aligned}\tau &= \frac{|x|}{v_1} = \frac{|x'|}{v_1}\sqrt{1-\frac{v_1^2}{c^2}} = \frac{L_0}{v_1}\sqrt{1-\frac{v_1^2}{c^2}} = \\ &= \frac{4ly}{0.8c}\sqrt{1-\frac{(0.8c)^2}{c^2}} = 3years\end{aligned} \quad (A2)$$

$$\tau' = \tau\sqrt{1-\frac{v_1^2}{c^2}} = \frac{|x'|}{v_1}\sqrt{1-\frac{v_1^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}} = \frac{|x'|}{v_1}(1-\frac{v_1^2}{c^2}) = \qquad (A3)$$
$$= 3years \times 0.6 = 1.8years$$

Since S' is moving with speed v_1 in relation to EF the Larmor's time dilation applies

The time elapsed in the frame S' for the returning trip is from (47)

$$\tau'_r = \frac{|x'|}{|v'_r|} = \frac{|x'|}{v_1}(1+\frac{v_1^2}{c^2}) = \frac{4ly}{0.8c}(1+\frac{0.8^2c^2}{c^2}) = 8.2years \qquad (A4)$$

The aging of twins E and AP are different for the to and from trip.

We can also calculate the aging of E and AP for the to trip using the frame of E and AP.

The trip of Stella in this frame is given by

$$\tau' = \frac{|x'|}{|v'|} = \frac{|x'|}{v_1}(1-\frac{v_1^2}{c^2}) = \frac{4}{0.8c}(1-\frac{0.8^2c^2}{c^2}) = 1.8years \qquad (A5)$$

The aging of Stella for the to and fro trip can be calculated using the frame of E and AP. We can use the standard dilation expression (contrary to the calculation of Gron based on the Lorentz contracted distance)

$$\tau = \Delta t'_L\sqrt{(1-\frac{v_1^2}{c^2})} = \frac{|x'|}{|v_1|}\sqrt{(1-\frac{v_1^2}{c^2})} = \qquad (A6)$$
$$\frac{L_0}{v_1}\sqrt{(1-\frac{v_1^2}{c^2})} = \frac{4}{0.8c}\sqrt{(1-\frac{0.8^2c^2}{c^2})} = 3years$$

The aging of E and AP for the returning trip can be also calculated in the returning frame of twin S. The distance between E and Stella in the begining of the trip is given by (33). Therefore we have

$$L_{fro} = L_0(1+\frac{v_1^2}{c^2})\frac{1}{\sqrt{(1-\frac{v_1^2}{c^2})}} = \\ = 4ly(1+\frac{0.8^2c^2}{c^2})\frac{1}{\sqrt{(1-\frac{0.8^2c^2}{c^2})}} = 10.933ly \qquad (A7)$$

Using the standard dilation expression in this frame of Stella

$$\tau_r^{'} = \Delta t_L^{''}\sqrt{(1-\frac{v_1^2}{c^2})} = \frac{L_{fro}}{0.8c}\sqrt{(1-\frac{v_1^2}{c^2})} = \\ = \frac{10.933}{0.8c}\sqrt{(1-\frac{0.8^2c^2}{c^2})} = 8.2years \qquad (A8)$$

The speed of E in the frame of Stella is given by (49). Therefore

$$v_r^{''} = \frac{-v_1}{(1-\frac{v_1^2}{c^2})}(1+\frac{v_1^2}{c^2}) = \frac{0.8c}{0.36}\times 1.64 = -3.64c \qquad (A9)$$

Therefore the aging of Stella for the fro trip is

$$\tau_r^{''} = \frac{L_{fro}}{|v_r^{''}|} = \frac{10.933ly}{3.64c} = 3years \qquad (A10)$$

The aging of E for the two way trip is

$$\tau'_{total} = \tau' + \tau'_r = \tau\sqrt{1-\frac{v_1^2}{c^2}} + \frac{|x'|}{|v'_r|} = \frac{|x'|}{v_1}\sqrt{1-\frac{v_1^2}{c^2}}\sqrt{1-\frac{v_1^2}{c^2}} + \frac{|x'|}{v_1}(1+\frac{v_1^2}{c^2}) = \qquad (A11)$$

$$= \frac{|x'|}{v_1}(1-\frac{v_1^2}{c^2}) + \frac{|x'|}{v_1}(1+\frac{v_1^2}{c^2}) = \frac{2|x'|}{v_1} = \frac{2L_0}{v_1} = 10\,years$$

The aging of E can be calculated in another way since the frame of E and AP does not change and the total change of Lorentz time in the two way trip is the aging of E since the frame is the same and the initial anf final clock is also the same, the problem of the desynchronization of Lorentz clocks disapear

$$\tau'_{total} = \Delta t'_{L(to)} + \Delta t'_{L(fro)} = \frac{L_0}{v_1} + \frac{L_0}{v_1} = \frac{8ly}{0.8c} = 10\,years \qquad (A12)$$

Of course the aging of E is different for the to and fro trip as eq. (A 11) reveals.

Let us see what Stella predicts for the ageing of E. For the first trip (eq. A11)

$$\tau' = \tau\sqrt{1-\frac{v_1^2}{c^2}} = \frac{L_0\sqrt{1-\frac{v_1^2}{c^2}}}{v_1}\sqrt{1-\frac{v_1^2}{c^2}} = \frac{L}{v_1}\sqrt{1-\frac{v_1^2}{c^2}} = \qquad (A13)$$

$$= \frac{4\times 0.6}{0.8c}\times 0.6 = \frac{2.4ly}{0.8c}\times 0.6 = 3years\times 0.6 = 1.8years$$

And for the fro trip (eq. A11) we can calculate the aging of E by the standard dilation time eq. in the frame of S

$$\tau'_r = \frac{L_{fro}}{v_1}\sqrt{1-\frac{v_1^2}{c^2}} = \frac{10.933ly}{0.8c}\times 0.6 = 8.2years \qquad (A14)$$

Contrary to Gron affirmation (based on the following eq. A15),

$$\tau'_{total} = \frac{2L}{v_1}\sqrt{1-\frac{v_1^2}{c^2}} = \frac{2\times 2.4ly}{0.8c}\times 0.6 = 3.6years \neq 10years \quad (A15)$$

there is no disagreement when the calculation for the aging of E by S and therefore does not emerge any paradox.

Consider now that twins E and AP are at rest in the EF, the first case analysed previously in Section III. The aging of E and AP for the to and fro trip are equal

$$\tau_{to} = \tau_{fro} = \frac{L_0}{v_1} = \frac{4ly}{0.8c} = 5years \qquad (A16)$$

And the aging of S is by the dilation expression valid for the two trips

$$\tau'_{to} = \tau'_{fro} = \frac{L_0}{v_1}\sqrt{1-\frac{v_1^2}{c^2}} = 5\times 0.6 = 3years \qquad (A17)$$

Stella predicts the same results for the to and fro frames [21, 22]. The distance *L* from AP for the to trip and from *E* for the fro trip is

$$L = \frac{L_0}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{4ly}{0.6c} = 6.66ly \qquad (A18)$$

and the aging of E and AP is for the two trips, by the standard dilation eq.

$$\tau = \frac{L_0}{\sqrt{1-\frac{v_1^2}{c^2}}} \times \frac{1}{v_1} \times \sqrt{1-\frac{v_1^2}{c^2}} = 5 years \qquad (A19)$$

and the aging of S is the time trip of E and AP in the frames of S

$$\tau' = \frac{L_0}{\sqrt{1-\frac{v_1^2}{c^2}}} \times \frac{(1-\frac{v_1^2}{c^2})}{v_1} = 3 years \qquad (A20)$$

without any paradox.