The two-dimensional Vavilov-Čerenkov effect with radiative corrections

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May 6, 2018

Abstract

We derive the photon power spectrum, including the radiative corrections, generated by charged particle moving within 2D graphene sheet with implanted ions forming dielectric medium. It enables the experimental realization of the Vavilov-Čerenkov radiation. The relation of the Vavilov-Čerenkov radiation to light emission diode (LED) is discussed. LED dielectric sheets can be the crucial components of detectors in experimental particle physics. So, the article represents the starting point of the unification of graphene physics with the physics of elementary particles.

1 Introduction

The fast moving charged particle in a medium when its speed is faster than the speed of light in this medium produces electromagnetic radiation which is called the Vavilov-Čerenkov radiation.

The prediction of Cerenkov radiation came long ago. Heaviside (1889) investigated the possibility of a charged object moving in a medium faster than electromagnetic waves in the same medium becomes a source of directed electromagnetic radiation. Kelvin (1901) presented an idea that the emission of particles is possible at a speed greater than that of light. Somewhat later, Sommerfeld (1904) proposed the hypothetical radiation with a sharp angular distribution. However, in fact, from experimental point of view, the electromagnetic Čerenkov radiation was first observed in the early 1900's by experiments developed by Marie and Pierre Curie when studying radioactivity emission. In essence they observed the emission of a bluish-white light from transparent substances in the neighborhood of strong radioactive source. But the first attempt to understand the origin of this was made by Mallet (1926; 1929a; 1929b), who observed that the light emitted by a variety of transparent bodies placed close to a radioactive source always had the same bluish-white quality, and that the spectrum was continuous, with no line or band structure characteristic of fluorescence.

Unfortunately, these investigations were forgotten for many years. Čerenkov experiments (Čerenkov, 1934) was performed at the suggestion of Vavilov who opened a door to the true physical nature of this effect[1] (Bolotovskii, 2009).

This radiation was first theoretically interpreted by Tamm and Frank (Tamm et al., 1937) in the framework of the classical electrodynamics. The source theoretical description of this effect was given by Schwinger et al. (Schwinger et al., 1976) at the zero temperature regime and the classical spectral formula was generalized to the finite temperature situation and for the massive photons by author (Pardy, 1989; 2002). The Vavilov-Čerenkov effect was also used by author (Pardy, 1997;) to possible measurement of the Lorentz contraction.

2 Source theory of the Vavilov-Čerenkov effect

Let us start with the source theory formulation of the problem (Schwinger et al., 1976). The basic formula in the source theory is the vacuum to vacuum amplitude:

$$< 0_+|0_- >= e^{\frac{i}{\hbar}W(S)}, \tag{1}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after space-time region where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add.

The electromagnetic field is described by the amplitude (1) with the action

$$W(J) = \frac{1}{2c^2}\int (dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{2}$$

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is the charge and current densities, where quantity J_μ is conserved. The symbol

[1] So, the adequate name of this effect is the Vavilov-Čerenkov effect. In the English literature, however, it is usually called the Čerenkov effect.

$D_{+\mu\nu}(x-x')$, is the photon propagator and its explicit form will be determined later.

It may be easy to show that the probability of the persistence of vacuum is given by the following formula (Schwinger et al., 1976):

$$|<0_+|0_->|^2 = \exp\{-\frac{2}{\hbar}\mathrm{Im}\,W\} \stackrel{d}{=} \exp\{-\int dtd\omega\frac{P(\omega,t)}{\hbar\omega}\}, \quad (3)$$

where we have introduced the so called power spectral function $P(\omega,t)$ (Schwinger et al., 1976). In order to extract this spectral function from $\mathrm{Im}\,W$, it is necessary to know the explicit form of the photon propagator $D_{+\mu\nu}(x-x')$.

The electromagnetic field is described by the four-potentials $A^\mu(\varphi,\mathbf{A})$ and it is generated, including a particular choice of gauge, by the four-current $J^\mu(c\varrho,\mathbf{J})$ according to the differential equation, (Schwinger eta l., 1976):

$$\left(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)J_\nu \quad (4)$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_+^{\mu\nu} = \frac{\mu}{c}(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu)D_+(x-x'), \quad (5)$$

where $\eta^\mu \equiv (1,\mathbf{0})$, μ (in the fraction μ/c)is the magnetic permeability of the dielectric medium with the dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_+(x-x')$ was derived by (Schwinger et al., 1976) in the following form:

$$D_+(x-x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \quad (6)$$

Using formulas (2), (3), (5) and (6), we get for the power spectral formula the following expression (Schwinger eta l., 1976):

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos[\omega(t-t')]\times$$

$$\times\left\{\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')\right\}. \quad (7)$$

3 Spectral formula for the two-dimensional Vavilov-Čerenkov effect

Now, we apply the last formula to the situations of the two-dimensional dielectric medium. We derive here the power spectrum of photons generated by charged particle moving within the plane of the graphene-like structure with index of refraction n. However, we cannot immediately apply the formula (7) to the graphene-like 2D structures because the index of refraction n is $n(x,y,z) = 1, z > 0$, $n(x,y,z) = const > 1, z = 0$ and

$n(x,y,z) = 1, z < 0$. It means that the situation is not the Vavilov-Čerenkov problem but the problem with the transition radiation which was solved by Ginzburg and Tsytovič (Ginzburg et al., 1984) for thin dielectric film. The problem of the transition radiation when electron is moving with the arbitrary angle with respect to the boundary is discussed by Bass et al. (Bass et al., 1965). Our goal is to solve only the Vavilov-Čerenkov radiation of charge when moving within the plane of dielectric sheet. So, it needs some modified approach.

While the graphene sheet is conductive, some graphene-like structures, for instance graphene with implanted ions, or, also 2D-glasses, are dielectric media, and it means that it enables the experimental realization of the Vavilov-Čerenkov radiation. Some graphene-like structure can be represented by graphene-based polaritonic crystal sheet (Bludov et al., 2012) which can be used to study the Vavilov-Čerenkov effect. We calculate it from the viewpoint of the Schwinger theory of sources.

The charge and current density of electron moving with the velocity $\mathbf{v}$ and charge e is as it is well known:

$$\varrho = e\delta(\mathbf{x} - \mathbf{v}t); \quad \mathbf{J} = e\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t). \tag{8}$$

In case of the the two-dimensional Vavilov-Čerenkov radiation by source theory formulation, the form of equations (2) and (3) is the same with the difference that $\eta^\mu \equiv (1, \mathbf{0})$ has two space components, or $\eta^\mu \equiv (1,0,0)$, and the Green function D_+ as the propagator must be determined by the two-dimensional procedure. In other words, the Fourier form of this propagator is with $(dk) = dk^0 d\mathbf{k} = dk^0 dk^1 dk^2 = dk^0 k dk d\theta$

$$D_+(x - x') = \int \frac{(dk)}{(2\pi)^3} \frac{1}{\mathbf{k}^2 - n^2(k^0)^2} e^{ik(x-x')}, \tag{9}$$

or, with $R = |\mathbf{x} - \mathbf{x}'|$

$$D_+(x - x') = \frac{1}{(2\pi)^3} \int_0^{2\pi} d\theta \int_0^\infty k dk \int_{-\infty}^\infty \frac{d\omega}{c} \frac{e^{ikR\cos\theta - i\omega(t-t')}}{k^2 - \frac{n^2\omega^2}{c^2} - i\varepsilon}. \tag{10}$$

Using $\exp(ikR\cos\theta) = \cos(kR\cos\theta) + i\sin(kR\cos\theta)$ and $(z = kR)$

$$\cos(z\cos\theta) = J_0(z) + 2\sum_{n=1}^\infty (-1)^n J_{2n}(z)\cos 2n\theta \tag{11}$$

and

$$\sin(z\cos\theta) = \sum_{n=1}^\infty (-1)^n J_{2n-1}(z)\cos(2n-1)\theta, \tag{12}$$

where $J_n(z)$ are the Bessel functions (Kuznetsov. 1962), we get after integration over θ:

$$D_+(x - x') = \frac{1}{(2\pi)^2} \int_0^\infty k dk \int_{-\infty}^\infty \frac{d\omega}{c} \frac{J_0(kR)}{k^2 - \frac{n^2\omega^2}{c^2} - i\varepsilon} e^{-i\omega(t-t')}. \tag{13}$$

The ω-integral in (13) can be performed using the residuum theorem after integration in the complex half ω-plane.

The result of such integration is the propagator D_+ in the following form:

$$D_+(x - x') = \frac{i}{2\pi c}\int_0^\infty d\omega J_0\left(\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|\right) e^{-i\omega|t-t'|}. \tag{14}$$

The spectral formula for the two-dimensional Vavilov-Čerenkov radiation is the analogue of the formula (7), where the charge density and current involves only two-dimensional velocities and integration is also only two-dimensional.

The difference is in the replacing mathematical formulas as follows:

$$\frac{\sin\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \quad \longrightarrow \quad J_0\left(\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|\right). \tag{15}$$

So, After insertion the quantities (8) and (9) into (7), we get:

$$P(\omega, t) = \frac{e^2}{2\pi}\frac{\mu\omega v}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right)\int dt' J_0\left(\frac{nv\omega}{c}|t - t'|\right)\cos[\omega(t - t')], \quad \beta = v/c, \tag{16}$$

where the t'-integration must be performed. Putting $\tau = t' - t$, we get the final formula:

$$P(\omega, t) = \frac{e^2}{2\pi}\frac{\mu\omega v}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right)\int_{-\infty}^\infty d\tau J_0(n\beta\omega\tau)\cos(\omega\tau), \quad \beta = v/c. \tag{17}$$

The integral in formula (17) is involved in the tables of integrals (Gradshteyn et al., 1963). Or,

$$J = \int_0^\infty dx J_0(ax)\cos(bx) = \frac{1}{\sqrt{a^2 - b^2}}, \quad 0 < b < a,$$

$$J = \infty,\ a = b;\ J = 0, \quad 0 < a < b, \tag{18}$$

In our case we have $a = n\beta\omega$ and $b = \omega$. So, the power spectrum in eq. (16) is as follows with $J_0(-z) = J_0(z)$:

$$P = \frac{e^2}{\pi}\frac{\mu v}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right)\frac{2}{\sqrt{n^2\beta^2 - 1}}, \quad n\beta > 1,\ \beta = v/c. \tag{19}$$

and

$$P = 0;\ n\beta < 1, \tag{20}$$

which means that the physical meaning of the quantity P is really the Vavilov-Čerenkov radiation. And it is in our case the two-dimensional form of this radiation.

The fundamental features of the 3D and 2D Vavilov-Čerenkov radiation are as follows: 1) The radiation arises only for particle velocity greater than the velocity of light in the dielectric medium.

2) It depends only on the charge and not on mass of the moving particles
3) The radiation is produced in the visible interval of the light frequencies and partly in the ultraviolet part of the frequency spectrum. The radiation does not exists for very short waves, which follows from the dispersion theory of the index of refraction n, where $n < 1$.
4) The spectral dependency on the frequency is linear for the 3D homogeneous medium.
5) The radiation generated in the 3D medium at given point of the trajectory spreads on the surface of the Mach cone with the vertex at this point and with the axis identical with the direction of motion of the particle. The vertex angle of the cone is given by the relation $\cos\Theta = c/nv$.
6) There is no Mach cone in the 2D dielectric medium. There is only the Mach angle in the 2D sheet. It follows from the fact that Vavilov-Čerenkov effect is the result of the collective motion of the 2D dielectric medium and it also follows from the quantum definition of the Vavilov-Čerenkov effect in the 2D structures. The conservation laws of momentum and energy for the Vavilov-Čerenkov effect is as follows:

$$\mathbf{p}_i = \mathbf{p}_f + \hbar\mathbf{k}, \tag{21}$$

$$E_i = E_f + \hbar\omega, \tag{22}$$

where index i concerns the initial momentum and energy of an electron and index f concerns the final momentum and energy of an electron. Symbol $\mathbf{k}$ is the wave vector of emitted photon and $\hbar\omega$ is its energy. With regard to the situation that the motion of an electron is realized in the plane $x-y$, the 3D Mach cone cannot be realized (The existence of Mach cone in our situation is the nonphysical escape of photons from 2D plane to the extra-dimension). So, the nonexistence of the Mach cone in the 2D structures is not mysterious.

4 The 3D an 2D Vavilov-Čerenkov efect with radiative coeec- tions

According to (Dittrich, 1978; Schwinger, 1973) the photon propagator with radiative correction is in the momentum representation of the form:

$$\tilde{D}(k) = D(k) + \delta D(k), \tag{23}$$

or,

$$\tilde{D}(k) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 - i\epsilon} +$$

$$+\int_{4m^2}^{\infty} dM^2 \frac{a(M^2)}{|\mathbf{k}|^2 - n^2(k^0)^2 + \frac{M^2c^2}{\hbar^2} - i\epsilon}, \tag{24}$$

where the last term in equation (24) is derived on the virtual photon condition

$$|\mathbf{k}|^2 - n^2(k^0)^2 = -\frac{M^2c^2}{\hbar^2}, \tag{25}$$

where n is the index of refraction of the medium. The weight function $a(M^2)$ has been derived in the following form (Dittrich, 1978; Schwinger, 1973)

$$a(M^2) = \frac{\alpha}{3\pi}\frac{1}{M^2}(1+\frac{2m^2}{M^2})(1-\frac{4m^2}{M^2})^{1/2}. \tag{26}$$

The x-representation of $D(k)$ in eq. (23) is as follows:

$$D_+(x-x') = \int\frac{(dk)}{(2\pi)^4}e^{ik(x-x')}D(k). \tag{27}$$

Or,

$$D_+(x-x') = \int\frac{(dk)}{(2\pi)^4}\frac{e^{ik(x-x')}}{|\mathbf{k}^2|-n^2(k^0)^2-i\epsilon} =$$

$$= \frac{i}{c}\frac{1}{4\pi^2}\int_0^\infty d\omega\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{28}$$

Now, with regard to the definition of x-representation (27) and (28) of the $D_+(x-x')$, we get the x-representation of the δD_+ in the following form:

$$\delta D_+(x-x') = \frac{i}{c}\frac{1}{4\pi^2}\int_{4m^2}^\infty dM^2a(M^2)\times$$

$$\times\int d\omega\,\frac{\sin[\frac{n^2\omega^2}{c^2}-\frac{M^2c^2}{\hbar^2}]^{1/2}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{29}$$

The function (29) differs from the the original function D_+ especially by the factor

$$\left(\frac{\omega^2n^2}{c^2}-\frac{M^2c^2}{\hbar^2}\right)^{1/2} \tag{30}$$

and by the additional mass-integral which involves the radiative corrections to the original Čerenkov effect. In order determine the explicit analytical radiative contribution, we employ the Schwinger source theory (Dittrich, 1978; Schwinger, 1973).

So, we get after extracting $P(\omega,t)$ the following general expression for this spectral function:

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\times$$

$$\left[\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}+\int_{4m^2}^\infty dM^2a(M^2)\frac{\sin[\frac{n^2\omega^2}{c^2}-\frac{M^2c^2}{\hbar^2}]^{1/2}]|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\right]\times$$

$$\cos[\omega(t-t')][\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')] \tag{31}$$

Now, let us apply the formula (14) in order to get the Čerenkov radiation with radiative corrections. The Čerenkov radiation is produced by charged particle of charge e moving at a constant velocity $\mathbf{v}$. In such a way we can write for the charge density and for the current density (8), where $v = |\mathbf{v}|$.

After insertion of eq.(8) into eq. (31), we get

$$P_{total}(\omega,t) = P(\omega,t) + \delta P(\omega,t) \tag{32}$$

with

$$P(\omega,t) = \frac{e^2}{4\pi^2}\frac{v\omega\mu}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\sin(n\beta\omega\tau)\cos\omega\tau \tag{33}$$

and

$$\delta P(\omega,t) = \frac{e^2}{4\pi^2}\frac{v\mu\omega}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\times$$

$$\int_{4m^2}^{\infty} dM^2 a(M^2)\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\sin\left([\frac{n^2\omega^2}{c^2}-\frac{M^2c^2}{\hbar^2}]^{1/2}v\tau\right) cos\omega\tau \tag{34}$$

where we have put $\tau = t'-t, \beta = v/c$. In case of eq. (33) it was derived by Schwinger et al., 1976), that

$$P(\omega,t) = \begin{cases} 0 & n\beta < 1 \\ \frac{e^2}{4\pi}\frac{\mu\omega}{c^2}v\left(1-\frac{1}{n^2\beta^2}\right) & n\beta > 1, \end{cases} \tag{35}$$

which is the well known spectral formula for the classical Čerenkov spectrum, with the threshold behavior $n\beta > 1$.

In case of the $\delta P(\omega,t)$ the situation is more complex. The first step is the necessity of evaluation of the τ-integral. For this integral we have:

$$\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\sin\left([\frac{n^2\omega^2}{c^2}-\frac{c^2M^2}{\hbar^2}]^{1/2}v\tau\right)\cos\omega\tau =$$

$$= \begin{cases} \pi, & 0 < M^2 < \frac{\hbar^2\omega^2}{c^2v^2}(n^2\beta^2-1) \\ 0, & M^2 > \frac{\hbar^2\omega^2}{c^2v^2}(n^2\beta^2-1). \end{cases} \tag{36}$$

From eq. (36) immediately follows that $M^2 > 0$ implies the Čerenkov threshold $n\beta > 1$. From eq. (34) and (36) we get that the radiative corrections to the original spectral formula of the Čerenkov radiation are given by the formula

$$\delta P = \frac{e^2}{4\pi}\frac{v\omega\mu}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\int_{M_1^2}^{M_2^2} dM^2\, a(M^2) \tag{37}$$

where

$$M_1^2 = 4m^2, \qquad M_2^2 = (n^2\beta^2 - 1)\frac{\hbar^2\omega^2}{c^2v^2}. \tag{38}$$

By the substitution

$$t = \left(1 - \frac{4m^2}{M^2}\right)^{1/2} \tag{39}$$

and after some elementary integration , we get the radiative contribution to the Čerenkov effect in the following form:

$$\delta P(\omega, t) = \alpha\frac{e^2}{4\pi^2}\frac{v\mu\omega}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right) \times$$

$$\left(\frac{s^2}{9} - \frac{2}{3}s + \frac{1}{3}\ln\left|\frac{1+s}{1-s}\right|\right), \qquad n\beta > 1 \tag{40}$$

where

$$s = \left(1 - \frac{4mc^2v^2}{(n^2\beta^2 - 1)\hbar^2\omega^2}\right)^{1/2}, \qquad s \geq 0 \tag{41}$$

The condition $s \geq 0$ in eq. (41) implies the existence of the radiative corrections to the original Frank-Tamm formula for

$$\omega^2 > \frac{4m^2c^2v^2}{\hbar^2(n^2\beta^2 - 1)} \tag{42}$$

For $n = \sqrt{2}$ and $v \approx c$, we get from eq. (42) $\hbar\omega \approx 2mc^2$, which can be interpreted as the condition for creation of the electron-positron pair by the gamma quantum.

The radiative corrections (40) to the original power spectral formula of the Čerenkov radiation is derived in the framework of the source theory for the first time and to our knowledge there is no conventional derivation of this effect in QED and at the same time it has no classical analogue.

The two-dimensional reduction of the formula (14) is analogical to the transformation (15), where the transformation involves also the mass term. Or,

$$\frac{\sin[\frac{n^2\omega^2}{c^2} - \frac{M^2c^2}{\hbar^2}]^{1/2}]|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \longrightarrow J_0\left([\frac{n^2\omega^2}{c^2} - \frac{M^2c^2}{\hbar^2}]^{1/2}]|\mathbf{x} - \mathbf{x}'|\right) \tag{43}$$

It means that after some operations we get instead the formula (34) the following radiative correction caused by the massive term:

$$\delta P(\omega, t) = \frac{e^2}{4\pi^2}\frac{v\mu\omega}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right) \times$$

$$\int_{4m^2}^{\infty} dM^2 a(M^2) \int_{-\infty}^{\infty} \frac{d\tau}{\tau} J_0\left([\frac{n^2\omega^2}{c^2} - \frac{M^2c^2}{\hbar^2}]^{1/2}v\tau\right) cos\omega\tau \tag{44}$$

In order to find the final contribution by mass term, we are forced to determine the τ-integral in eq. (44). The integral in formula (44) is involved in the tables of integrals (Gradshteyn, 1963) . Or,

$$J = \int_0^\infty dx J_0\left(ax\right)\cos(bx) = \frac{1}{\sqrt{a^2-b^2}}, \quad 0 < b < a,$$

$$J = \infty,\; a = b;\; J = 0, \quad 0 < a < b, \tag{45}$$

In our case we have

$$a = [\frac{n^2\omega^2}{c^2} - \frac{M^2c^2}{\hbar^2}]^{1/2}, \quad b = \omega \tag{46}$$

.

Or,

$$\int_{-\infty}^{\infty}\frac{d\tau}{\tau}J_0\left([\frac{n^2\omega^2}{c^2} - \frac{c^2M^2}{\hbar^2}]^{1/2}v\tau\right)\cos\omega\tau =$$

$$= \begin{cases} \frac{1}{\sqrt{\omega^2(n^2\beta^2-1)-\frac{v^2c^2}{\hbar^2}M^2}}, & 0 < M^2 < \frac{\hbar^2\omega^2}{c^2v^2}(n^2\beta^2-1) \\ 0, & M^2 > \frac{\hbar^2\omega^2}{c^2v^2}(n^2\beta^2-1) \end{cases} \tag{47}$$

From eq. (41) immediately follows that $M^2 > 0$ implies the Čerenkov threshold $n\beta > 1$. From eq. (44) and (47) we get that the radiative corrections to the original spectral formula of the Čerenkov radiation are given by the formula (with $J_0(-z) = J_0(z)$):

$$\delta P = \frac{e^2}{4\pi^2}\frac{v\omega\mu}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right)\int_{M_1^2}^{M_2^2} dM^2\, a(M^2)\frac{1}{\sqrt{\omega^2(n^2\beta^2-1) - \frac{v^2c^2}{\hbar^2}M^2}}, \tag{48}$$

where

$$M_1^2 = 4m^2, \qquad M_2^2 = (n^2\beta^2-1)\frac{\hbar^2\omega^2}{c^2v^2}. \tag{49}$$

By the substitution

$$t = \left(1 - \frac{4m^2}{M^2}\right)^{1/2} \tag{50}$$

we get the integral boundaries as follows:

$$t_1 = 0; \quad t_2 = s = \left(1 - \frac{4mc^2v^2}{(n^2\beta^2-1)\hbar^2\omega^2}\right)^{1/2}, \qquad s \geq 0 \tag{51}$$

Then, after some elementary integration , we get the radiative contribution to the Čerenkov effect (48) in the following form:

$$\delta P = \frac{e^2}{4\pi^2}\frac{v\omega\mu}{c^2}\left(\frac{\alpha}{3\pi}\right)\left(1 - \frac{1}{n^2\beta^2}\right)\times$$

$$\int_0^s \frac{t^2(3-t^2)}{\sqrt{1-t^2}} \frac{\hbar dt}{\sqrt{\omega^2\hbar^2(1-t^2)(n^2\beta^2-1)-4v^2m^2c^2}}, \tag{52}$$

The integral in eq. (52) is not tableted and it must by evaluated by the special integration technique.

The condition $s \geq 0$ in eq. (51) implies the existence of the radiative corrections to the original Frank-Tamm formula for

$$\omega^2 > \frac{4m^2c^2v^2}{\hbar^2(n^2\beta^2-1)}. \tag{53}$$

For $n = \sqrt{2}$ and $v \approx c$, we get from eq. (51) $\hbar\omega \approx 2mc^2$, which can be interpreted as the condition for creation of the electron-positron pair by the gamma quantum.

5 Discussion

While the formula for the three dimensional (3D) Vavilov-Čerenkov radiation is well known from textbooks and monographs, the two-dimensional (2D) form of the Vavilov-Čerenkov radiation is new. Zuev (2009) considers the Vavilov-Čerenkov phenomenon in nanofilms from Au, Ag, Cu, where the Vavilov-Čerenkov phenomenon is realized only as the surface plasmons which cannot escape the 2D medium.

The fundamental importance of the Vavilov-Čerenkov radiation is in its use for the modern detectors of very speed charged particles in the high energy physics. The detection of the Vavilov-Čerenkov radiation enables to detect not only the existence of the particle, however, also its direction of motion and its velocity and also its charge. The two-dimensional Vavilov-Čerenkov radiation was still not applied in physics, nevertheless, it is the promising application in LED, the light-emitting diode.

The light-emitting diode, LED, consists of several layers (sheets) of semiconducting materials. Electrical voltage drives electrons (from the n-layer) and holes (from the p-layer) to the active layer, where they recombine forming light. Anode, the p-electrode, and cathode, the n-electrode are connected to the voltage element as a source. The LED is no larger than a grain of sand. In case of the blue LED lamp, it consists of several different layers of gallium nitride (GaN). By mixing in indium (In) and aluminium (Al), the Nobel prize laureates, Isamu Akasaki (Nagoya University, Japan), Hiroshi Amano (Nagoya University, Japan), Huji Nakamura (American citizen, University of California, Santa Barbara, USA) succeeded in increasing the lamps efficiency (Royal Swedish Academy of Sciences, 2014). White LEDs currently reach more than 300 lm/W, representing more than 50% wallplug efficiency.

The Relation of Vavilov-Čerenkov effect to LED is based on the following arguments. Namely, when LED (with additional dielectric sheet) is irradiated by high-energy electrons with velocity greater than the velocity of light in the sheet, then LED produces the 2D Vavilov-Čerenkov radiation if and only if the electrons moves within the dielectric sheet inside the LED. The set of small grain-sand LED (fixed in adequate viscous gel emulsion) forms then the new detector of elementary particle physics. The two-dimensional Vavilov-Čerenkov radiation was still not applied, nevertheless, it is not excluded that it is the crucial effect in LED.

The condition (47) concerns the gamma photons rather than the optical ones. The possibility of the existence of the gamma Čerenkov radiation is discussed by Ion and Stocker (1993) in nuclear physics. The so called nuclear gamma Čerenkov radiation requires a special experimental technique in order to extract such radiation from the background produced by other mechanism. Such "coherent" techniques are well known in nuclear physics and we can expect that sooner or later the existence of the gamma Čerenkov radiation in nuclear physics and graphene physics will be confirmed together with the corresponding radiative corrections.

The present theory can be generalized to the situation with the string-like objects which play important role in the present theoretical physics of elementary particles (Manoukian, 1991; 1992). The article is based on the author articles (Pardy, 2015, 1994).

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