Separation, Transposition and Docking of Apollo 11 in low-Earth Orbit

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A selection of visual media from the Apollo 11 Mission, namely, the 70mm photographs AS11-36-5301 through AS11-36-5313 and the 16mm film magazine A, are shown to strongly suggest that at the time of their filming Apollo 11 craft were in low-Earth orbit.

The visual media comprise the film footage and the photographs of the craft and the Earth filmed before and during the maneuvers separation, transposition and docking (STD). The STD reportedly occured during the translunar coast some 30 minutes after the translunar injection (TLI). In the STD, the Command and Service Module (CSM) would dismount the rocket Saturn-IVB (S-IVB) carrying the Lunar Module (LM) and the CSM up to that point, then dock with and extract the LM. The S-IVB would then split from the group to fly behind the Moon and in an orbit around the Sun.

In determining the location of the Apollo 11 craft, the sizes of Earth and the S-IVB rocket and their distances from the camera are extracted from the media assuming a selection of camera lenses. The extracted CSM flight data include the turning rate and the turning angle, the maximum separation distance, and the docking velocity. From their comparison to the Flight Plan, the Mission Report and to the oral transcripts from the Apollo 11 Flight Journal, it is found that the 16mm Mauer Data Acquisition Camera (DAC) was filming with the 10mm lens, and not with the 18mm lens as NASA reported. Consequently, the photography must have been done with a Hasselblad manual camera with the 38mm lens and not with the Hasselblad electric camera with the 80mm lens as NASA reported. The visual media being recorded with these new lenses puts the craft at the time of the STD in low-Earth orbit, rather than Moon-bound after successful TLI.

CONTENTS

1. INTRODUCTION

The Apollo 11 mission plan for July 1969 featured the first lunar landing of the Apollo program. As documented in the Flight Plan [1] before the mission, and in the Mission Report [2] after the mission, the trans-lunar injection (TLI) would be the defining moment of the mission. First, the stages I and II of the Saturn V rocket would "park" the, so called, Apollo stack comprising of the Saturn-IVB stage (S-IVB) carrying the Lunar Module (LM) and the Command and Service Module (CSM) with the crew, in the low-Earth orbit (LEO). Then, for the TLI the S-IVB would fire its rocket engine for approximately five minutes and this would put the spacecraft on a trajectory towards the Moon.

The purpose of this report is to examine if the visual documentation recorded by the crew from some time before the TLI, and up to some 30 minutes after the TLI, provides an evidence that by that time the spacecraft was coasting toward the Moon following successful TLI. Previously, the public could have seen footage from the film cameras on the Saturn II showing the spent Saturn I falling to the visibly retreating Earth, then how the S-IVB separated carrying the Apollo stack. Significantly, footage from the S-IVB or from the LM showing the execution of the TLI, then of the CSM separating, turning around and docking to extract the LM, have never been admitted to exist. It seems that for lunar missions there were no cameras aboard the S-IVB, even though admittedly the S-IVB carried them in the past. [3]

To ascertain how the TLI was executed the following approach was adopted. The positions of the craft were determined from the photographs and the film footage of Earth the crew took in the period starting from LEO and ending some 30 minutes after the reported TLI when the craft completed a sequence of maneuvers termed *separation, transposition* and *docking* (STD). In particular, the lenses that were used with the cameras for filming and photography were identified not from the NASA catalogue [4, 5] but from matching the STD descriptions in technical documentation [1, 2, 6] to their appearances on film and in photographs.

The report is organized as follows. Sec. 2 describes the method used in the report to find the position of the craft. Sec. 3 lists the relevant photographs and film footage, and all the cameras and lenses carried by the mission and where they were reportedly stowed. Sec. 4 gives distances between the CSM and the Earth, and the CSM and the S-IVB for all camera/lens combinations. In the discussion in Sec. 5 the lens/camera combinations are identified that are consistent with some of the visual documentation. Interestingly, it is shown that some scientific institutions were aware of the inconsistencies in NASA documentation and proposed identical lens/camera combinations. Then, it is shown that the appearance of the Earth is inconsistent with the reported position of the spacecraft, suggesting that the image of the Earth was inserted in the footage during post-processing. Sec. 6 follows with a conclusion that the STD could not have taken place on the way to the Moon. Rather, what remains is a realization that the CSM with the crew dismounted the S-IVB prior to the TLI and stayed in LEO. Appx. A contains supporting mathematical details of the telemetry relevant to this report.

2. METHOD, MANEUVERS AND TECHNICAL DESCRIPTION

Following a successful launch, the Saturn IV-B (S-IVB) stage carrying the Command and Service Module (CSM) and the Lunar Module (LM) was brought to its parking orbit. [1, 2, 5] This was a circular low-Earth orbit (LEO) in the ecliptic plane at an approximate height of $h = 185$ km (=100 nmi) above the surface. At 2:45 Ground Elapsed Time (GET) the S-IVB performed trans-lunar injection (TLI) by firing its engine for 5 minutes and 20 seconds. This provided a kick of $\Delta V = 3,186$ m·s^{-1} (=10,451 ft/sec), which was supposed to put the craft on a trajectory towards the Moon. At 3:15 GET, when the craft was supposed to be some 7,900 km from the surface (14,200 km from Earth center), four protective side panels would be jettisoned and the CSM was supposed to separate from the S-IVB, and fly away. In a maneuver called transposition the CSM would pitch-up for 180^o(turn around its Y-axis) while continuing to fly away. The docking would follow in which the CSM would fly towards the S-IVB, and dock with the LM that the S-IVB was carrying. Approximately one hour later the CMS would extract the LM from the S-IVB, and the craft would move out of the way of the S-IVB. The S-IVB would then briefly fire its rocket engine, in what is known as “sling shot.” This would put the S-IVB on a trajectory that would have passed the Moon on the far side and into a solar orbit.

The importance of separation, docking and transposition for the telemetry is in that they were partially filmed and the film footage did catch a glimpse of the Earth in the first few seconds, but then goes on to show the S-IVB for the remainder of it. Importantly, the S-IVB was reportedly photographed using the same equipment with which Earth was photographed. The method for determining what lens was used for filming the Earth is as follows. On the one hand, the Flight Plan and the Mission Report both precisely specify the flight parameters of the maneuvers such as the CSM separation velocity, or the turning rate during transposition, the CSM docking velocity, or maximal distance from the S-IVB. And on the other hand, the same flight details can be extracted from the film footage or the photographs as a function of the focal length of the camera lens that filmed or photographed the maneuver. Thus, the cameras could have been used only with those lenses, which reproduce the flight details consistent with the technical documentation. Once the correct camera lenses were identified, the distance to Earth can be estimated from the film footage and from the photographs.

Each of the documents, the Flight Plan,[1] the Mission Report,[2], and the Apollo Flight Journal, [6] has its own description of the maneuvers. In the following, the values stated in the documents are listed as is, while the estimated values have hat above their names.

1. The Flight Plan :

   - The maneuvers would last 10 minutes.

   - The DAC would be started prior to separation, and stopped after the docking was completed. It would film at $\rho = 6$ frames-per-second (FPS) and use an 18mm lens.

   - Separation: $v_{SEP} = 0.8$ ft/sec, $t_{SEP} = 15$ s.

   - Transposition: $v_{TRANSP} = 0.5$ ft/sec (-0.3 ft/sec from v_{SEP}), turning rate $\omega = 2^o$/s. Estimated are $\hat{t}_{TRANSP} = 180/2 = 90$ s, and the distance between the CSM and the S-IVB at the end of transposition, $\hat{d}_{TRANSP} = t_{SEP} \cdot v_{SEP} + \hat{t}_{TRANSP} \cdot v_{TRANSP} = 57$ ft (=17.5 m).

   - Docking: initial closing velocity $v_{DOCK,1}$ would be in the range 0.25 to 0.5 ft/sec. For $\hat{v}_{DOCK,1} = 0.38$ ft/s (centroid of the velocity range) the estimate for the docking time is $\hat{t}_{DOCK,1} = d_{TRANSP}/\hat{v}_{DOCK,1} \simeq 152$ s.

   - Estimated duration of the first two maneuvers is $\hat{t}_{ST} \simeq t_{SEP} + t_{TRANSP} = 105$ s, and of all three maneuvers, $\hat{t}_{STD} \simeq t_{SEP} + t_{TRANSP} + \hat{t}_{DOCK} = 257$ s (=4min17sec). However, extra time may have been needed because after the craft got within few feet from each other, the docking velocity would further decrease for smooth docking.

2. The Mission Report (Secs. 4.4 and 8.6) :

   - Separation: $v_{SEP} = 0.6$ ft/sec, $t_{SEP} = 20$ s.

   - Transposition: $v_{TRANSP} = 0.5$ ft/sec (-0.1 ft/sec from v_{SEP}), the pitch-up turning rate $\omega = 1.75^o$/s was smaller than in the Flight Plan because of the problem with autopilot. Estimated are $\hat{t}_{TRANSP} = 180/1.75 \approx 105$ s, and the distance between the CSM and the S-IVB at the end of transposition, $\hat{d}_{TRANSP} = t_{SEP} \cdot v_{SEP} + \hat{t}_{TRANSP} \cdot v_{TRANSP} = 65$ ft (=20.0 m). However, in Sec. 4.4 in the Mission Report it was stated that the maximal distance may have been greater then 100 ft (=30 m).

   - Docking: the two craft docked with the velocity $v_{DOCK,2} = 0.1$ ft/sec. This velocity was set once the center-of-mass of the CSM was 12 ft from the LM. - Estimated duration of the first two maneuvers is $\hat{t}_{ST} \simeq t_{SEP} + t_{TRANSP} = 125$ s.

3. Apollo Flight Journal :

   - Separation: $v_{SEP} = 0.8$ ft/sec, $t_{SEP} = 60$ s (3:16:58 GET to 3:17:57).

   - Transposition: $v_{TRANSP} = 0.5$ ft/sec (-0.3 ft/sec from v_{SEP}), $t_{TRANSP} = 116$ s (3:17:58 GET to 3:19:54), so the estimated average turning rate was $\hat{\omega} \approx 1.55^o$/s. The distance between the CSM and the S-IVB at the end of transposition was, $d_{TRANSP} = t_{SEP} \cdot v_{SEP} + t_{TRANSP} \cdot v_{TRANSP} \approx 106$ ft

(=32.2 m), as was also stated in the Mission Report Sec. 4.4.

- Docking: $v_{DOCK,1} = 0.4$ ft/sec (3:19:52), $\hat{t}_{DOCK} = d_{TRANSP}/v_{DOCK,1} \approx 265$ s.

- Reported duration of the first two maneuvers is $t_{ST} \simeq t_{SEP} + t_{TRANSP} = 176$ s, and of all three maneuvers, $t_{STD} = t_{SEP} + t_{TRANSP} + \hat{t}_{DOCK} \approx 441$ s (=7min21sec).

- The DAC was turned on at 3:18:54, that is, for the last 60 s of transposition. This on one hand means that the DAC filming was controlled through remote control cable, rather than through the frame rate switch on the body of the DAC. Alternatively, this suggests that from the 180^o turn, only $\Delta\hat{\theta} = 60$ s $\times$ 2^o/s $\approx 120^o$ may have been recorded. Here, nominal turning rate was used for the estimate, because all the problems with the autopilot that caused average turning rate to be lower happened at the beginning of the turn. The filming rate $\rho = 6$ FPS was mentioned only once (3:21:19). However, because the DAC was operated through remote control cable a 'time' setting on the camera existed that allowed it to be triggered through the timing cable. For that reason, other filming frame rates were possible besides ones available through the switch on the body of the camera, namely 1, 6, 12 and 24 FPS.

4. Separation, Transposition and Docking in other Apollo Missions that stayed in LEO:

Apollo 7 The mission did not carry the LM, so the docking was not truly performed. In the Flight Plan [7], p.2-8, it was reported:

- Separation: $v_{SEP} = 1.05$ ft/sec, $t_{SEP} = 70$ s.

- Transposition: $v_{TRANSP} = 0.55$ ft/sec (-0.5 ft/sec from v_{SEP}), pitch up for 180^o with the turning rate of $\omega = 5^o$/s ($\hat{t}_{TRANSP} = 36$ s).

- Expected duration of the first two maneuvers is $\hat{t}_{ST} = t_{SEP} + \hat{t}_{TRANSP} \simeq 105$ s. Anticipated distance between the CSM and the S-IVB at the end of transposition would be, $\hat{d}_{TRANSP} \approx 95$ ft. In this mission the CSM would approach the S-IVB and then fly around it, as there was no LM to extract.

Apollo 9 - Separation: $v_{SEP} = 0.6$ ft/sec, $t_{SEP} = 15$ s, cf. Mission Report [8], p.10-4.

- Transposition: $d_{TRANSP} \approx 60$ ft, cf. [8], p.10-4. The duration of the first two maneuvers was, $\hat{t}_{ST} = d_{TRANSP}/v_{SEP} \simeq 100$ s. The turning rate during pitch-up was $\omega = 180^o/(\hat{t}_{ST} - t_{SEP}) \simeq 2^o$/s.

- Docking: $v_{DOCK,1} = 0.3$ ft/sec until 15 ft distance, cf. [8], p.5-17, then $v_{DOCK,2} = 0.07$ ft/sec, cf. [8], p.10-5. This yields, $\hat{t}_{DOCK} = 45\text{ ft}/v_{DOCK,1} + 15\text{ ft}/v_{DOCK,2} \simeq 290$ s.

- Reported duration of separation, transposition and docking was $t_{STD} \simeq 1243$ s (=20min43sec, from 2:41:15GET to 3:01:59GET, cf. [8], p.7-1). This time includes the crew resolving problems with the RCS: it would not fire in Y-direction. However, the total time for all three maneuvers in the absence of problems would have been $\hat{t}_{STD} \simeq 400$ s.

3. HASSELBLAD PHOTOGRAPHS AND DAC FILM FOOTAGE

Camera Film Format	Film Frame Size (mm) H	V	Aspect	Diagonal	Lens f (mm)	FOV (deg) Diagonal	H	V	Location	Comment
70-mm Hasselblad	55	55	1	77.8	38	91.3	71.8		LM	MC for lunar surface photography
					60	65.9	49.2			EC for photographing EVA
					80	51.9	37.9		CSM	EC for flight photography
					250	17.7	12.6			
16-mm Mauer DAC	10.3	7.7	1 1/3	12.8	5	104.1	91.5	75.2	CSM	Flight footage
					18	39.2	31.8	24.1		
					75	9.8	7.8	5.9		
					10	65.3	54.3	42.1	LM	EVA footage from LM

TABLE I. Still photographic and film equipment on Apollo 11 and their location from NASA sources. Hasselblad Electric Camera (HEC) featured demountable 80mm and 250mm lenses, while the Manual Camera (HMC) featured a 38mm lens and a viewfinder [9], and was manually operated. Another HEC with a 60mm lens was designed to be mounted on one of the astronauts' suit during the extra-vehicular excursion (EVA). All HECs featured an electric motor that advanced the film and cocked the shutter whenever the camera was activated.

For visual documentation, the crew carried photographic equipment including film cameras. The equipment comprised a 16mm Maurer Data Acquisition Cameras (DAC) for filming, and 70mm Hasselblad cameras for photography. Here, 16mm and 70mm refer to film format for each type of camera used. In Tbl. I information about the film and still cameras, available lenses, and their location is compiled from [5, 10, 11]. The catalog of mission photography [4] provides brief description of photographs and film footage, with the lens that was used with the camera.

The photographs are sourced from the Lunar Planetary Institute (LPI) website [11], which scanned them as JPEG files. The LPI provides JPEGs in two pixel (px) resolutions, "standard" (450px by 450px) and "print" (3,900px by 3,900px). For this report the photographs in print resolution were used.

Magazine A [4] of the film footage is available as a MPEG-2 formatted video clip from NASA's Apollo Flight Journal website. [6] The film footage comprises 3,386 frames, and is described as "short sequence of Earth with panoramic to SIVB and LM. Docking of LM to CSM." The video clip has the nominal 30 FPS, as it is targeted for the USA audience, which is independent of the frame rate of the underlying 16mm film footage. The video clip is 4 min 46 sec (=286 s) long, contains frames sized 640px by 480px. For this report the JPEG formatted images were extracted from the video clip at the rates 6, 10 and 30 FPS using the software package `ffmpeg`. [12] Individual images in a sequence extracted at rate f are referred to as $N(f)$, where N is the image ordinal number. From here, the time at which the frame was taken was $t_N = N/f$.

In the period of time from LEO to the completion of docking, some 40 minutes after the TLI, Earth was photographed in AS11-36-5301 [13, 14], AS11-36-5302 [13, 15], AS11-36-5303 [13, 16], AS11-36-5304

[13, 17], AS11-36-5305 [13, 18], AS11-36-5307 [13, 19] and AS11-36-5309 [13, 20]. Their thumbnails are provided in Fig. 1. Chronologically, the photographs of Earth are followed by the 16mm film footage of transposition and docking on the Magazine A. For reference, Fig. 2 provides two frames extracted at 10 FPS from the video clip. The frame 1(10) shows Earth midway through transposition, while the frame 80(10) shows the S-IVB at the end of the transposition. The docking to the S-IVB was also photographed by the crew in AS11-36-5310 [13, 21], AS11-36-5311 [13, 22], AS11-36-5312 [13, 23], and AS11-36-5313 [13, 24]. Thumbnails of those photographs are given in Fig. 3. On this leg of the trip the Hasselblad Electric Camera (HEC) with the 80mm lens was reportedly used for photography, [4] while the DAC was reportedly filming at 6 FPS with the 18mm lens. [1]

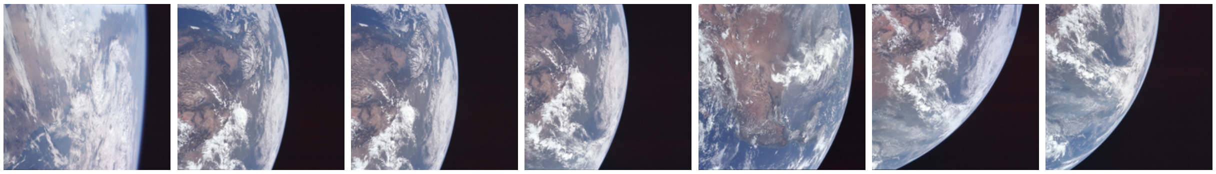

FIG. 1. Photographs AS11-36-5301 [13, 14], AS11-36-5302 [13, 15], AS11-36-5303 [13, 16], AS11-36-5304 [13, 17], AS11-36-5305 [13, 18], AS11-36-5307 [13, 19] and AS11-36-5309 [13, 20] (left to right, in that order) show Earth. NASA claims that the photographs were taken with the HEC/80mm. Tbl. III contains the Earth size for that camera and the other camera/lens combinations.

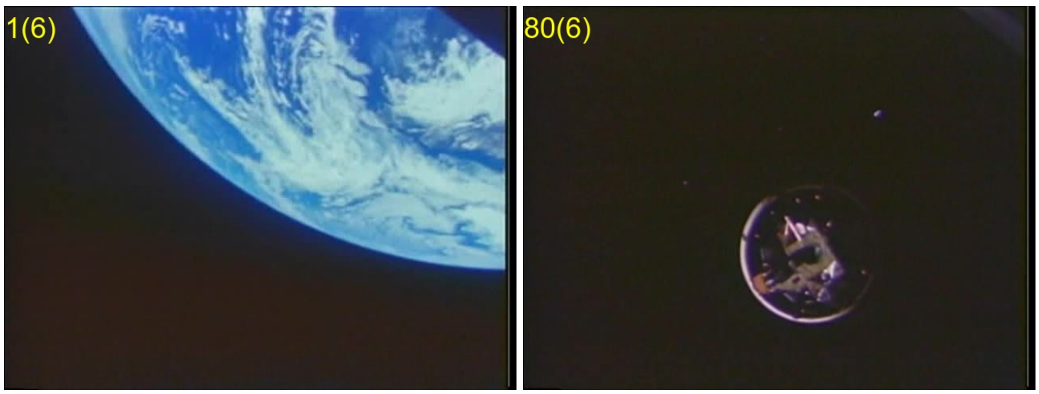

FIG. 2. View of Earth in frame 1(6) of the Magazine A video clip midway through the transposition, and of the S-IVB in the frame 80(6) toward the end of the transposition. [6] NASA claims that the footage was taken with the DAC/18mm. Tbl. III contains the Earth and S-IVB size for that and the other lenses they may have been used with the DAC.

FIG. 3. In AS11-36-5310 [13, 21], AS11-36-5311 [13, 22], AS11-36-5312 [13, 23], and AS11-36-5313 [13, 24] (left to right, in that order), the S-IVB was photographed by the crew during docking. NASA states that the photographs were taken with the HEC/80mm. Tbl. III contains the S-IVB size for that and the other camera/lens combinations.

4. RESULTS

4.1. The Earth Aperture as a Function of Distance

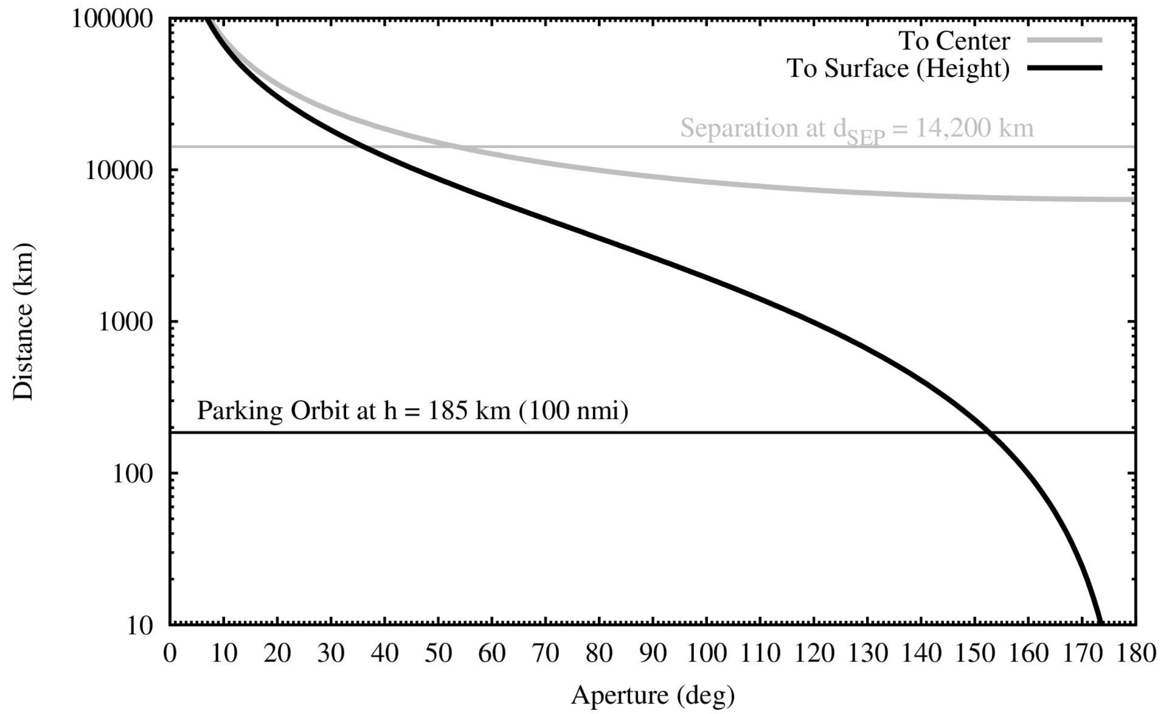

FIG. 4. Camera distances vs. the Earth aperture after Eq. (A3).

aperture σ (deg)	height (km)	distance (km)	Comment
153	185	6556	Apollo 11 low-Earth Parking Orbit
150	225	6596	
140	409	6780	International Space Station Permanent Orbit
132	620	6991	Space Shuttle Discovery Hubble Orbit(*)
130	659	7030	
120	986	7357	
90	2639	9010	
63	5822	12193	Unmanned deep space capsule ORION(a)
60	6371	12742	
53	7907	14278	Apollo 11 reported distance for SEP, TRANSP and DOCK

(*) STS-82, 2-11-1997.
(a) Exploration Flight Test 1, 12-5-2014. The first human crew is planned for Exploration Mission 2 to make a large orbit around the Moon in early 2020s. Progeny of canceled CONSTELLATION program.

TABLE II. Some special values for camera distance vs. the Earth aperture relevant to later sections of this report.

4.2. Earth and the S-IVB on Film and Photographs

The positions of Earth and the S-IVB in the photographs and the video clip with respect to the camera were determined in two steps: In the first, the edge was determined either programmatically (Earth) or by human inspection (S-IVB). In the second, depending on the underlying object the edge was fitted using implicit least squares either to a special conic section (Earth) or a circle (S-IVB), cf. Appx. A.

In Tbl. III size or aperture σ and the direction in terms of the spherical-angles θ and ϕ is reported for Earth in the photographs and in the frames extracted at 6 FPS from the film footage. Also reported is the position and distance to the camera of the S-IVB at the end of transposition, for which the frames extracted at 10 FPS from the film footage were used, and during docking, for which the photographs were used.

Please note, the dynamics of docking of the CSM to the S-IVB in the film footage is reported separately.

	Photo or Frame	16mm DAC												70mm Hasselblad								Location
		f=5mm				f=10mm				f=18mm				f=38mm				f=80mm				
		center θ (deg)	center φ (deg)	size σ (deg)	distance (ft)	center θ (deg)	center φ (deg)	size σ (deg)	distance (ft)	center θ (deg)	center φ (deg)	size σ (deg)	distance (ft)	center θ (deg)	center φ (deg)	size σ (deg)	distance (ft)	center θ (deg)	center φ (deg)	size σ (deg)	distance (ft)	
Earth	5301																	62.1	-93.0	151		Parking Orbit
	5302													48.1	-91.2	123		36.4	-91.2	86		Not Specified - HOWEVER - comparison with 16mm DAC values (on the left) suggests the same location as below.
	5303													49.5	-91.6	122		36.8	-91.6	85		
	5304													51.3	-98.2	121		37.4	-98.2	84		
	5305													29.6	-104.5	119		24.3	-104.6	79		
	5307													44.9	-128.0	119		33.9	-128.0	82		
	5309													51.4	-116.2	114		35.8	-116.1	77		
	1(6)	58.7				43.3				28.6												Separation, Transposition and Docking at approximately 30 minutes after trans-lunar injection. This puts Apollo 11 at ~14,200 km from Earth (center) where its aperture should be σ=52 deg. Mission report states that the CSM may have gotten as far as 30 m from the S-IVB during separation. The 6.6m wide S-IVB at that distance has aperture of around 13 deg.
	2(6)	58.7				43.3				28.6												
	3(6)	60.3				44.2				29.0												
	4(6)	63.1				45.6				29.8												
	5(6)	66.1				47.1				30.7												
	6(6)	68.9				48.4				31.4												
	7(6)	71.7				49.9				32.2												
	8(6)	74.6				51.4				33.0												
	9(6)	77.3				52.7				33.7												
	10(6)	80.0	-58+/- 1	121+/-2		54.1	-58+/- 1	88 +/- 1		34.5	-58+/- 1	58 +/- 1										
	11(6)	82.8				55.7				35.5												
	12(6)	85.0				56.8				36.0												
	13(6)	88.1				58.8				37.3												
	14(6)	89.6				59.4				37.5												
	15(6)	92.5				61.3				38.7												
	16(6)	94.3				62.4				39.3												
	17(6)	96.2				63.5				40.0												
	18(6)	98.4				65.1				41.0												
	19(6)	99.7				65.8				41.3												
S-IVB	68(10)	45.7				27.1				15.9												
	69(10)	45.1				26.7				15.6												
	71(10)	43.5	102+/-2	22+/-5	46+/-7	25.4	102+/-2	14 +/- 2	82+/-7	14.8	102+/-2	9 +/- 1	141+/-13									
	73(10)	41.1				23.6				13.6												
	75(10)	38.7				21.8				12.5												
	77(10)	36.6				20.4				11.7												
	5310													31.3	-71.0	12	89	16.1	-71.0	7	167	
	5311													35.6	-115.2	11	89	18.8	-115.2	7	161	
	5312													21.2	-135.4	17	66	10.4	-135.4	9	131	
	5313													4.5	-136.5	36	33	2.1	-136.5	18	69	

TABLE III. Position and size (aperture) of Earth and the S-IVB (latter with distance to the camera) in the camera coordinates in all relevant 70mm photographs and in 16mm film footage.

4.3. The Playback Frame Rate and Filming of Transposition

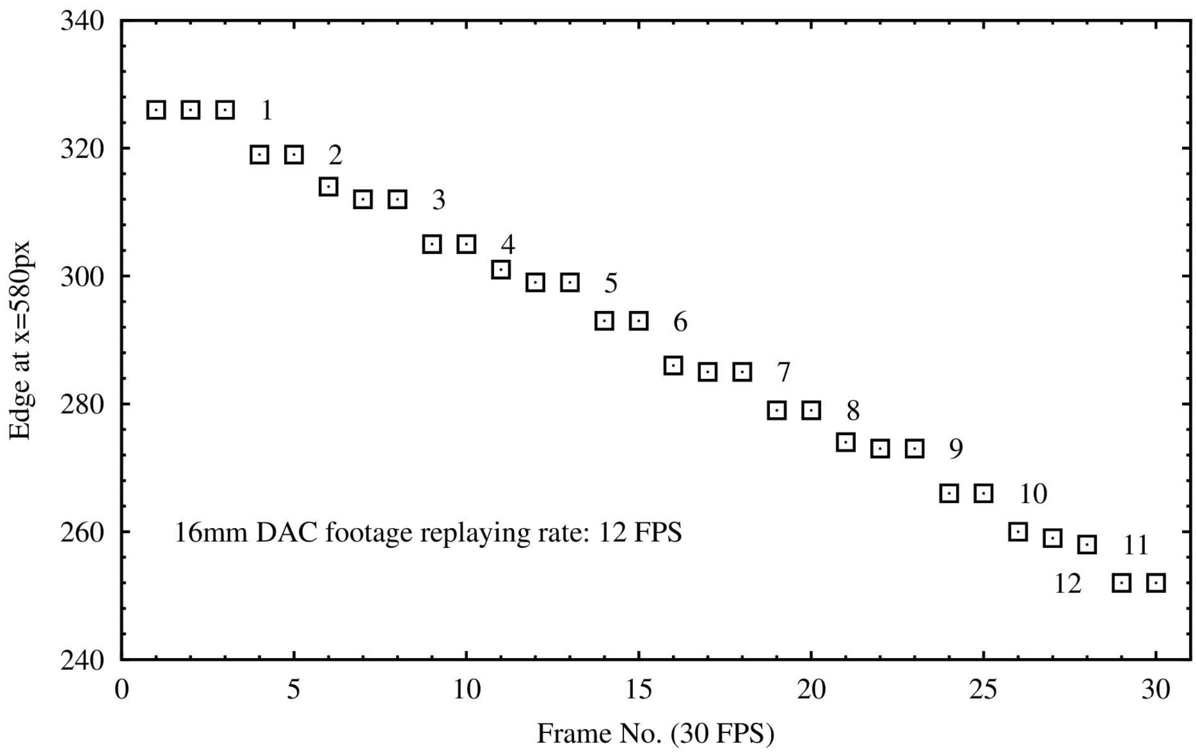

FIG. 5. The way the edge of Earth moves in the video clip reveals that the 16mm DAC film footage was played back at 12 FPS for conversion to video.

The flight documentation [1, 2] states that the DAC would start filming before the separation so it would film all three maneuvers in their entirety. The filming frame rate would be $\rho = 6$ FPS, and that the CSM turning rate was $\omega \simeq 2^o$/s.

On the other hand, the video clip starts with a 15 s long segment of the end of the transposition. From the conversation between the crew and the ground control [6], it follows that the last 60 s of the transposition were filmed, cf. Sec. 2. All these suggest (i), that $\Delta\theta \simeq 120^o$ of the transposition's 180^o turn was filmed; and (ii), in order to compress 60 s of recording into 15 s of playing time requires that for the conversion to the video clip the original film footage had to be played back at four times higher rate, $\hat{\Pi} = 4 \cdot \rho = 24$ FPS.

For the analysis of the flight dynamics captured in the video clip it is thus important to establish or to verify the filming and playback rates, ρ and Π.

The playback rate Π can be determined as follows. The video clip is 286 s long and comprise compressed version of the 3386 frames from the original 16mm footage. This suggests the playback rate to be $\Pi = 3386/286 = 12$ FPS. One then looks at the granularity of the motion of Earth in the frames extracted at the highest possible rate of 30 FPS. As Fig. 5 shows, during the first second of the video clip the edge of

i	Time t_i (s)	Earth Center from Tab.II θ_i (deg)			Comment
		5mm	10mm	18mm	
1	0.2	58.7	43.3	28.6	Earth in view
2	3.2	99.7	65.8	41.3	
		13.7	7.5	4.3	ω' (deg/s)
3	9.2	182	111	67	$\theta_{E,S}$ Earth to S-IVB (deg)
TRANSP (180 deg)		123	67	38	Filmed Δθ (deg)
		57	113	142	non-filmed Δθ (deg)
		5	16	34	Duration non-filmed (s)

TABLE IV. From the motion of Earth in the video clip, the average turning rate $\omega' = \langle d\theta/dt' \rangle$ of the CSM in the time scale of the video clip, and the turning angle $\Delta\theta$ are estimated, see discussion in the text. The actual turning rate ω depends on the ratio of the recording rate ρ of the 16mm film footage to its playback rate Π, cf. Eq. (4).

Earth moves in 12 steps, confirming that the playback rate was indeed 12 FPS. This analysis can also be performed using the S-IVB, and the result is the same.

While ρ cannot be directly estimated, the average turning rate ω' in the reference frame of the video clip (thus prime designation, as compared to the recording, or the true, time frame), and the filmed turning angle $\Delta\theta$ can be measured with respect to Earth. From Tbl. III, during transposition the Earth center moves in such a way that $\phi_i \simeq -60^o \simeq$ const., for all considered frames i. The change in the angular position of Earth between frames i and j is,

$$\cos\angle(\hat{n}_i, \hat{n}_j) = \sin\theta_i \sin\theta_j \cos(\phi_i - \phi_j) + \cos\theta_i \cos\theta_j \approx \cos(\theta_i - \theta_j), \tag{1}$$

where $\hat{n}_i$ is the direction of the Earth center in the camera coordinate system of the frame i. The CSM turning rate ω' can be estimated from the video clip assuming it was filmed with 5mm, 10mm, or 18mm lens.

Tbl. IV summarizes for each DAC lens the average turning rate ω', the angle $\theta_{E,S}$ between the Earth center and the S-IVB, and how much of the 180^o-turn was filmed and how much was not. Fig. 7 illustrates $\theta_{E,S} \sim 180^o$ obtained for the DAC and the 5mm lens. The calculation is done as follows:

- Turning rate ω': The pitch θ_1 is found from frame 1(6) at time $t_1 = 1/6$ s, and θ_2 from frame 19(6) at time $t_2 = 19/6$ s, where the average rate ω' is

$$\omega' \approx \frac{\theta_2 - \theta_1}{t_2 - t_1}. \tag{2}$$

- The angle $\theta_{E,S}$ between the centers of Earth and the S-IVB from the perspective of the CSM: It is assumed that the CSM continues to turn with the same turning rate until the time $t_3 = 9\ 1/6$ s at which the S-IVB is approximately in the center of the frame. The CSM motion for the next six seconds can be neglected as the CSM slows down and re-centers onto the S-IVB. At t_3 the angle between the Earth center and the S-IVB, $\theta_{E,S} \equiv \theta_3$, is given by

$$\theta_3 = \theta_2 + \omega' \cdot (t_3 - t_2). \tag{3}$$

- The filmed angle $\Delta\theta$ is thus $\Delta\theta = \theta_3 - \theta_1$, while $180^o - \Delta\theta$ was not filmed. From the non-filmed turning angle and the estimated turning rate ω', the duration of the non-filmed part of transposition $t'_{nf/T}$ can also be found.

If the turning rate ω is known fairly certainly, then ρ is given by,

$$\rho = \frac{\omega}{\omega'} \times \Pi, \tag{4}$$

while the real (recording) time scale t relates to the playback time scale t' as

$$t = \frac{\Pi}{\rho} \times t'. \tag{5}$$

As for the velocities, v' from the video clip relate to their real-time values v as

$$v = \frac{\rho}{\Pi} \times v'. \tag{6}$$

Note that the turning angle $\Delta\theta = \omega \cdot t = \omega' \cdot t'$, is time scale invariant - it only depends on the DAC lens that was used in filming.

4.4. "Constellation of Debris" and the Question of When the Separation Started

Prior to separation of the CSM from the LM and the S-IVB, four panels that were protecting the LM were jettisoned using explosive charges. The crew claimed this created "constellation" of debris some of which have been captured on photographs AS11-36-5310 through AS11-36-5312, and on the 16mm footage.

In the following, the motion of the debris is used to calculate the time at which the separation started. Firstly, notice that the center of the DAC frame follows the ecliptic plane (yellow line in Fig. 6 that connects centers of Sun, Earth, and the S-IVB and the CSM). As found in Tbl. III, the ecliptic is at an angle $\varphi \simeq -58 \pm 1^o$ with respect to the DAC frame X-axis. From the first 3 seconds of the video clip it is found that

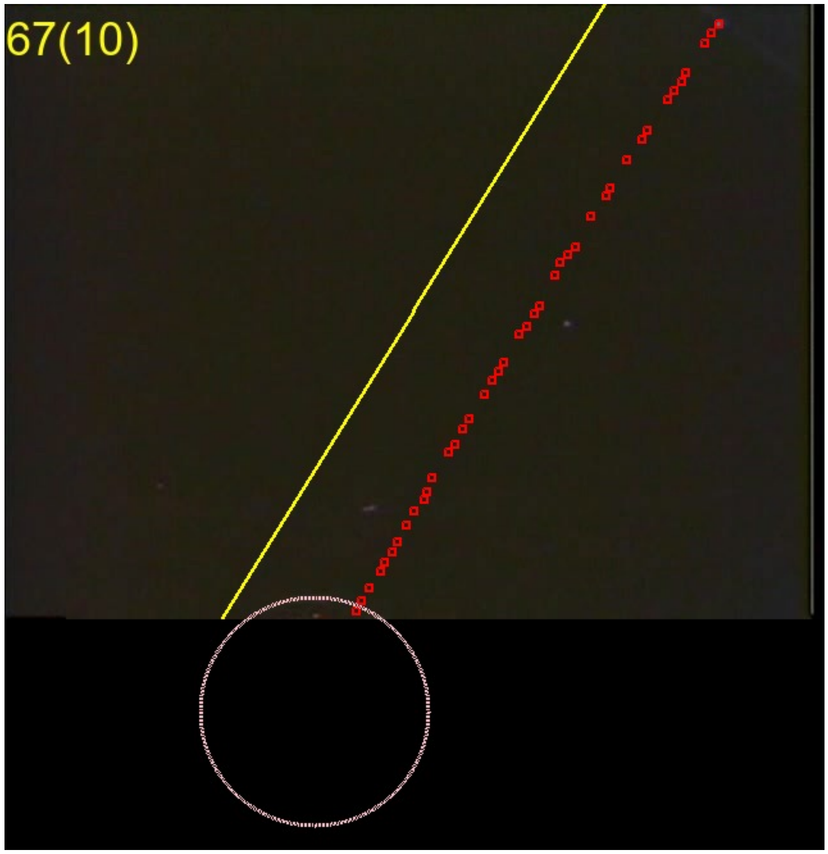

FIG. 6. Expanded frame 67(10) from the video clip with superimposed motion of the fastest fragment (red dots, collected from the frames 17(10) through 67(10)), with the ecliptic plane (in yellow, calculated from the motion of the CSM with respect to Earth). While the CSM moves downwards along the ecliptic with the screen speed v', the fastest fragment moves upwards with an apparent screen speed v''. The actual screen speed of the fragment is thus $\Delta v'' = v'' - v'$ upwards, allowing one to calculate the time when the fragment was ejected from the S-IVB (pink circle, extrapolated from the edge captured on the frame 67(10)).

the DAC moves away from Earth with the screen speed of

$$v' = 93 \text{ px/s}, \tag{7}$$

along the ecliptic. This procedure is illustrated also in Fig. 10, later in the text.

Secondly, the fastest fragment is identified as the one that appears at the bottom near the center of the frame 17(10), and exits the frame 67(10) at the top. Its position in the frames in-between are given in Fig. 6 as red squares, which are superimposed on top of the last frame 67(10). There it can be seen that its apparent trajectory is parallel to the ecliptic. It takes the fragment $\Delta t' = 5.0$ s to traverse the distance $\Delta l = 538.4$ px across the screen from the rim of the S-IVB (pink circle in Fig. 6, which radius is $r_S \simeq 89$ px) to its last filmed position on the frame 67(10). Apparent screen speed of the fastest fragment is thus,

$$v'' = \frac{\Delta l}{\Delta t'} = 107.7 \text{ px/s.} \tag{8}$$

Were the DAC stationary, the actual screen speed of the fastest fragment would thus be,

$$\Delta v'' = v'' - v' = 14.7 \text{ px/s.} \tag{9}$$

From there, the actual time $\Delta t''$ it took the fragment to cross the distance Δl is

$$\Delta t'' = \frac{\Delta l}{\Delta v''} \simeq 36.6 \text{ s.} \tag{10}$$

This time is a sum of three times, namely, the duration of the (non-filmed) separation t'_{SEP}, the duration of the non-filmed part of the transposition $t'_{nf/T}$, and the time instance of the frame 67(10) $t_{67} = 6.7$ s. It thereby follows that in the video clip time the separation started

$$t'_{SEP} + t'_{nf/T} \lesssim 30 \text{ s,} \tag{11}$$

sometimes before the beginning of the video clip.

Lastly, the filmed transposition is some $t'_{f/T} \simeq 15$ s long, so the total duration of the separation and the transposition is

$$t'_{ST} = t'_{SEP} + t'_{nf/T} + t'_{f/T} \simeq \text{between 40 and 45 s.} \tag{12}$$

Note, using Tbl. IV the minimal duration of the transposition can be estimated as $t'_{TRANSP} = t'_{f/T} + t'_{nf/T} \simeq$ 20 s, 31 s and 50 s, for the 5mm, 10mm and 18mm DAC lens, respectively.

These findings are summarized as follows:

- The DAC could not have filmed with the 18mm lens because $t'_{TRANSP} > t'_{ST}$.
- If the DAC was filming with the 10mm lens then the separation lasted $t'_{SEP} \approx 15$ s, while the transposition lasted $t'_{TRANSP} \approx 30$ s, all in the video clip time.
- If the DAC was filming with the 5mm lens then the separation and the transposition both lasted the same, $t'_{TRANSP} \approx t'_{SEP} \approx 20$ s.

4.5. Pitch-up

In the documentation, cf. Sec. 2, the 180^o turn was described as "pitch-up."

The DAC was filming the latter part of the turn supposedly from underneath left rendezvous window, where it should have been mounted using the Right Angle Mirror accessory. The accessory itself was described as the device that would facilitate photography through the spacecraft rendezvous window along a line of sight parallel to the CSM X-axis with a minimum interference to the crew, and which adapted to the 18mm and 75mm lenses by means of bayonet fittings. As the commander of the craft sat on the left side, this would allow the DAC to film what the commander was seeing while docking the craft.

There would be two indications that this was followed by the crew. Firstly, because the DAC frame would now be oriented like the crew, the S-IVB would be entering the view from the top of the screen. Secondly, at the end of docking the DAC frame would be occupied by the *docking target* on the LM, which the commander of the craft would use to align the craft for docking.

However, in the video clip the DAC captured the S-IVB coming from the bottom-left of the screen; and the DAC frames end with the *docking window* instead. The two docking features can be easily identified on the LM, e.g., see [25], Fig. 1-2, on p.17.

Considering the geometry of the CSM rendezvous windows illustrated in the Panel (c) of Fig. 7, it transpires that the DAC was mounted in the top left corner of the right window (upside-down from the perspective of the crew). For that purpose another accessory, the Data Acquisition Camera Mount had to be used (described in documentation as a device used to facilitate in-flight mounting at the spacecrafts left or right randezvous wihdows, comprises a quick disconnect hand-grip that may be attached to a dovetail adapter at either randezvous window).

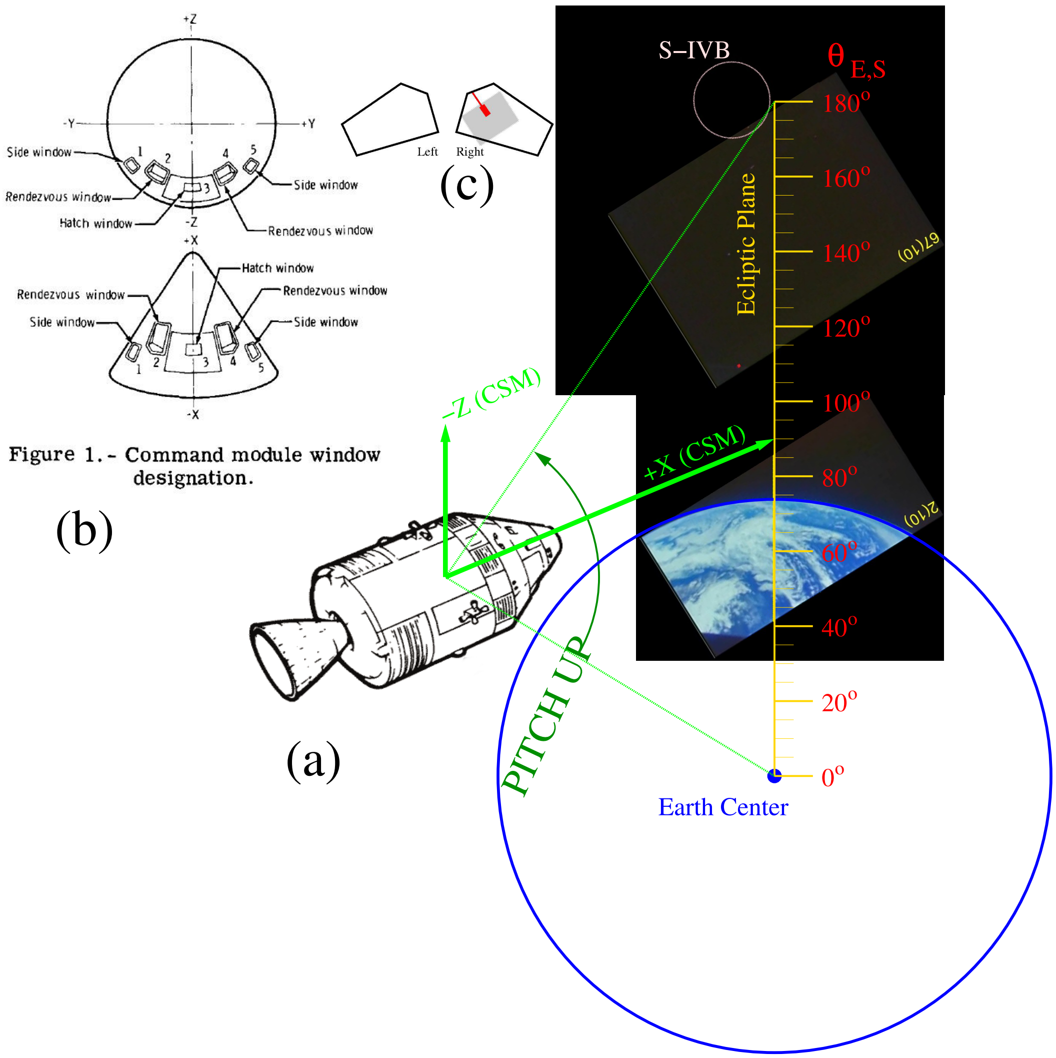

FIG. 7. An illustration of the 180^o turn performed during the transposition assuming the DAC was filming with the 5mm lens. All angles are approximations given the uncertainties of the optical distortions. Panel (a): The angular distance from the perspective of the CSM between the S-IVB and the Earth center is $\theta_{E,S} \sim 180^o$, where the Earth size is $\sigma_E \sim 150^o$ (when fitted to circle), or $\sim 120^o$ (when fitted to conic section, cf. Tbl. III). From the way the windows were positioned on the CSM, Panel (b), reprinted from [26], Fig.1, it follows that the DAC had to be mounted from the upper left corner of the right randezvous window (shown in red in Panel (c), with the outline of the camera frame in grey).

4.6. The LM Docking Window

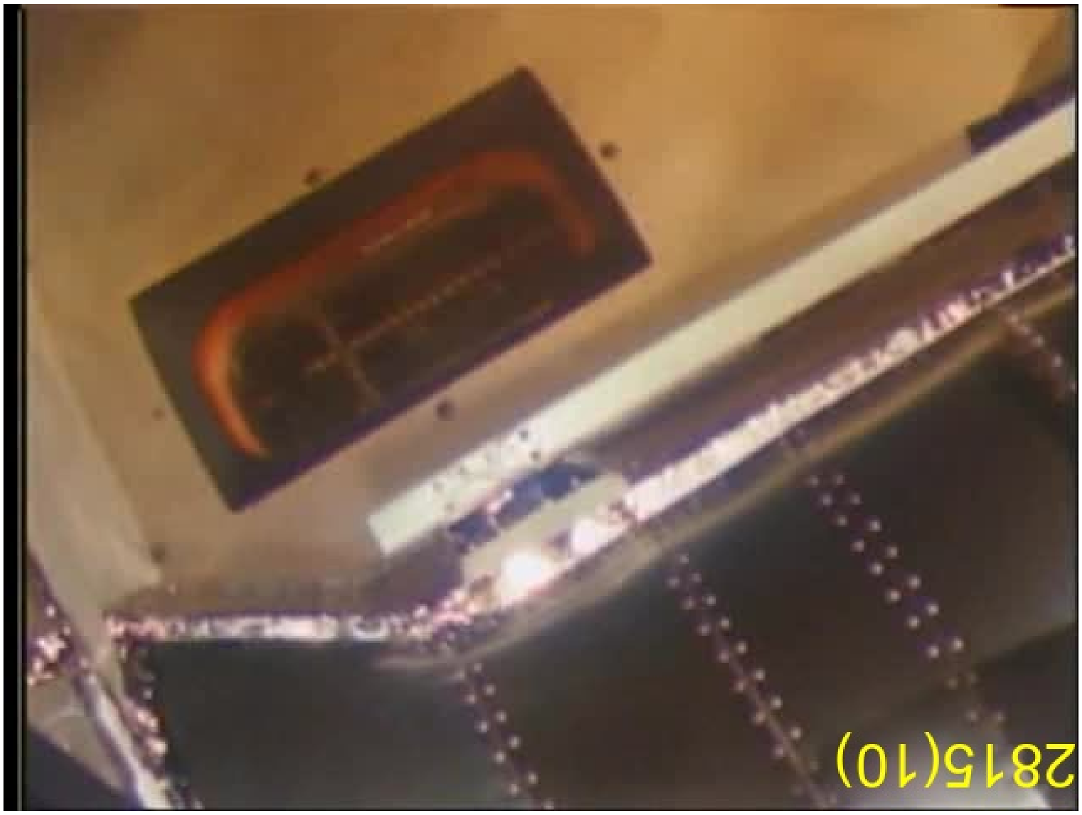

FIG. 8. The 16mm DAC film footage ends with the view of the 16-by-5 in *docking window* on the LM, rather than with the view of the *docking target* that was used by the CSM pilot to align the craft for docking. This puts the DAC behind the right randezvous window upside-down as illustrated in Panel (c) in Fig. 7.

16mm DAC		docking window in frame 2815(10)				
Lens focal length	**scale (deg/px)**	**length (px)**	**length (deg)**	**length (ft)**	**distance to camera (ft)**	**length (deg)**
5mm	0.157		34			
10mm	0.088	218	19	1.33	4	18.9
18mm	0.050		11			

TABLE V. From the final size of the *docking window* in the 16mm film footage, it follows that the DAC was filming with the 10mm lens.

4.7. The Docking Velocity, Maximal Distance and Duration, vs. the DAC lens

Irrespective of the finding that the DAC was most likely filming with the 10mm lens, the following analysis is performed for all available DAC lenses.

For this analysis the frames were extracted from the video clip at 10 FPS, and the docking segment was isolated from times 00:08 (frame 80(10)) to 02:03 (frame 1230(10)). In few tens of selected frames the edge of the S-IVB was determined by human inspection. The aperture of the S-IVB was then converted to distance from the camera as a function of time, and this is reported in units of feet for each lens in Fig. 9. The docking velocity $v'_{DOCK,1}$ is approximated by the slope of the distance vs. time over the linear part of the curve. For all lenses the maximal distance d_{DOCK} is evaluated at 16 s. The duration of docking $\Delta t'_{DOCK,1}$ was estimates as when the craft were 5 ft apart based on the docking velocity.

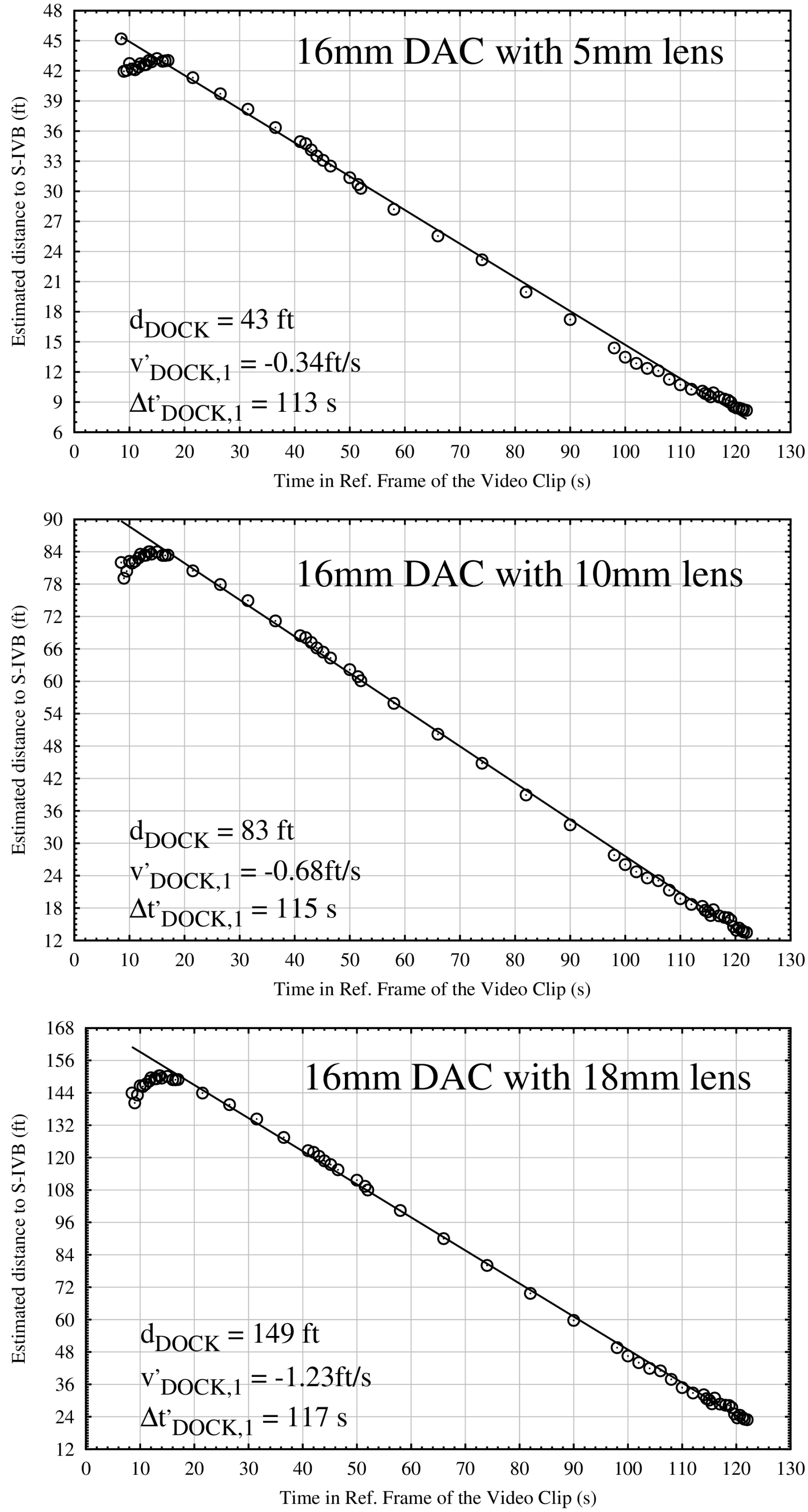

FIG. 9. The distances between the DAC and the S-IVB as the functions of video clip time for DAC lenses 5mm (top), 10mm (middle) and 18mm (bottom), allow the docking distances d_{DOCK}, velocities $v'_{DOCK,1}$ and times $t'_{DOCK,1}$ to be estimated.

5. DISCUSSION

5.1. Flight Dynamics

Quantity	Filming/Photography *DAC/18mm and HEC/80mm*	Technical Documentation	Comment
σ_{EARTH}	58 ± 1 ° (DAC)	53 ° down to 45 °	Given the long length of the lens σ should be very close to documented value - cf. AS11-36-5301 in Tbl. III - so this discrepancy is important.
ρ		6 FPS (DAC)	$\Pi/\rho = 2$
Π	12 FPS (DAC)		
ω		2 °/s	$\Pi/\rho = 2$
ω'	4 °/s		
$t_{f/T}$		60 s	$\Pi/\rho \approx 4$
$t'_{f/T}$	15s		
t_{ST}		176 s	$\Pi/\rho \approx 4$
t'_{ST}	45 s (lens independent)		
$v_{DOCK,1}$		0.1 to 0.4 ft/s	$\Pi/\rho \geq 3$ and/or the DAC lens is shorter then 18mm
$v'_{DOCK,1}$	1.2 ft/s		
d_{DOCK}		100 ft at most	the DAC lens must be shorter then 18mm
d'_{DOCK}	140 ft (DAC), 157 ft (HEC)		
length of the LM docking window captured at the end of the DAC footage strongly suggests that the 10mm lens was used			

TABLE VI. Comparison of technical documentation and visual media all suggest that the DAC could not have been filming with the 18mm lens. On the contrary, some basic evidence, e.g., the size of the LM *docking window* at the end of the transposition in the 16mm footage strongly points to a 10mm lens being used, see next Tbl. VII.

Tbl. VI summarizes major innconsistencies in the description of the maneuvers on film and in the technical documentation. These all strongly suggests that the filming could not have been done with the DAC/18mm. Consequently, neither the HEC/80mm could have been used when photographing the maneuvers. Rather, as shown in Tbl. VII, the DAC must have been filming with the 10mm lens. The immediate consequence is that it must have been the HMC/38mm that was used to photograph the S-IVB in AS11-36-5310 through AS11-36-5313.

It is intriguing that the Lunar Planetary Institute (LPI) might have guessed that the 10mm lens was used with the DAC for filming the maneuvers. On the institute's web pages describing the Apollo 11 film equipment it is stated that at the time of the maneuvers the 10mm lens was in the CSM, while the 18mm was in the LM. [11] The probability that the HMC/38mm was used for the in-flight photography (and not only for the lunar EVA as NASA states) was also claimed by a professional photographer on his web site `www.kenrockwell.com`.

While the DAC/10mm and HMC/38mm being used for filming and photography resolves many inconsistencies between the visual media and technical documentation, it also raises some important questions:

1. The Earth size that was captured on the 16mm film is $\sigma_E = 88 \pm 1^o$. This size puts the craft at an altitude (distance) 2,800 km or 1,500 nmi above Earth (or 9,000 km from its center) at the time the

Quantity	Filming/Photography *DAC/10mm and HMC/38mm*	Technical Documentation	Comment
σ_{EARTH}	88 ± 1 ° (DAC)	53 ° down to 45 °	Impossible location
ρ		6 FPS (DAC)	Recording rate is either $\rho \approx$ 3 FPS ($\Pi/\rho \approx 4$), or $\rho \approx$ 6 FPS ($\Pi/\rho \approx 2$)
Π	12 FPS (DAC)		
ω		2 °/s	*$\Pi/\rho \approx$ 4*
ω'	7.5 °/s		
$t_{f/T}$		60 s	
$t'_{f/T}$	15s		
t_{ST}		176 s	
t'_{ST}	45 s (lens independent)		
$v_{DOCK,1}$		0.34 ft/s	*$\Pi/\rho \approx$ 2, otherwise total time for all three maneuvers exceeds 10 min*
$v'_{DOCK,1}$	0.68 ft/s		
d_{DOCK}		100 ft at most	*$\Pi/\rho \approx$ 2, so that the craft after 90sec can reach some 90ft separation flying at 1ft/sec separation velocity.*
d'_{DOCK}	83 ft (DAC), 89 ft (HMC)		

TABLE VII. The DAC being used with 10mm lens implies that the HMC/38mm was used for the photography and that $\Pi/\rho = 2$ and not 4. This in turn strongly suggests that (i),the timing of the maneuvers, as fimed on the 16mm footage, was different from what was planned or reported in the technical documentation; and (ii), that the mission was at an impossible location, see discussion in the text.

maneuvers were performed. However, that point was allegedly passed at around 2:55 GET, [6] (20 minutes earlier, or 10 minutes after the supposed TLI). At the time when the CSM separated from the S-IVB, the mission could have been on one of the two locations: Either it commenced its translunar coast (TLC) and at the moment it reached some 14 thousand kilometers from (the center of) Earth; or, it was in low-Earth orbit (LEO) preparing to stage the TLC for the audiences on Earth. As has been demonstrated, the mission was not at the location consistent with the TLC, and so by elimination one can reasonably conclude that the mission was only in LEO preparing to stage the trans-lunar coast.

2. It has to be $\Pi/\rho = 2$, that is, the filming rate was in fact the reported $\rho = 6$ FPS, for at least two reasons:
   By the first, if this was not the case then the docking speed would have been too low, and the docking time would have made all three maneuvers last more then 10 minutes.
   By the second, in video clip time frame the duration of separation and transposition is $t'_{ST} \simeq 45$ s, while the maximal separation of the craft recorded in the film footage and on the photographs is $d_{DOCK} \simeq 90$ ft. Considering that the documented separation velocity in all Apollo missions was $v_{SEP} \sim 1$ ft/s, this implies that the actual duration of the two maneuvers had to be $t_{ST} = d_{DOCK}/v_{SEP} \simeq 90$ s, so $\Pi/\rho = t_{ST}/t'_{ST} = 2$. If $\Pi/\rho = 2$, however, then the maneuvers were performed differently then what was orally communicated by the crew to the ground control. As it would not be possible for the crew to talk about one way of performing the maneuvers while performing them the other way, it thus follows that at the time the crew communicated with the ground control the maneuvers have already been performed. In other words, the crew staged its communication with the ground

control describing the progress of maneuvers. For the craft to be in a similar position along the low-Earth parking orbit, the maneuvers thus had to be performed some 90 minutes earlier.

5.2. The Scene

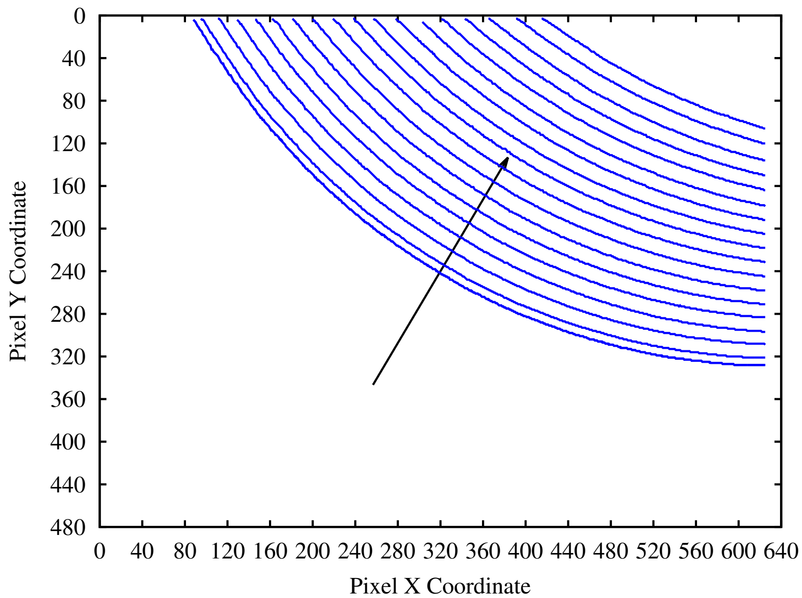

FIG. 10. The way the Earth leading edge (blue lines) moves in video clip frames 1(6) through 17(6) is consistent with a circle being dragged along the ecliptic (black arrow in the direction of apparent Earth motion) with a constant screen velocity.

What remains to be discussed in the context of staging is how the Earth was captured on the 16mm film footage, and where in the orbit the staging occured.

It is very likely that the image of Earth was superimposed onto 16mm film footage, because the Earth eccentricity in the footage is fixed. Consider the way the Earth edge appears on the footage from frames 1(6) through 17(6), given in Fig. 10. Their appearances are consistent with a disk of fixed size moving away along ecliptic with a constant screen velocity. Were these the edges of "real" Earth, they would comprise a sequence of conic sections with increasing eccentricity as the camera was turning away from Earth. As is known, the eccentricity is zero for a circle, between 0 and 1 for an ellipse, 1 for a parabola and greater than 1 for hyperbole. Most easily available source of the Earth image used in this manipulation would have been footage of Earth filmed with the 5mm lens while the craft were still orbiting in "standard" orientation, that is, +X axis of the craft pointed in the direction of orbital motion, with -Z axis (crew heads) pointed toward Earth. Once the lens was swapped for the 10mm lens and the right position in low-Earth orbit reached, the craft would have made an undocumented change of pitch and yaw in preparation for (filming of) the

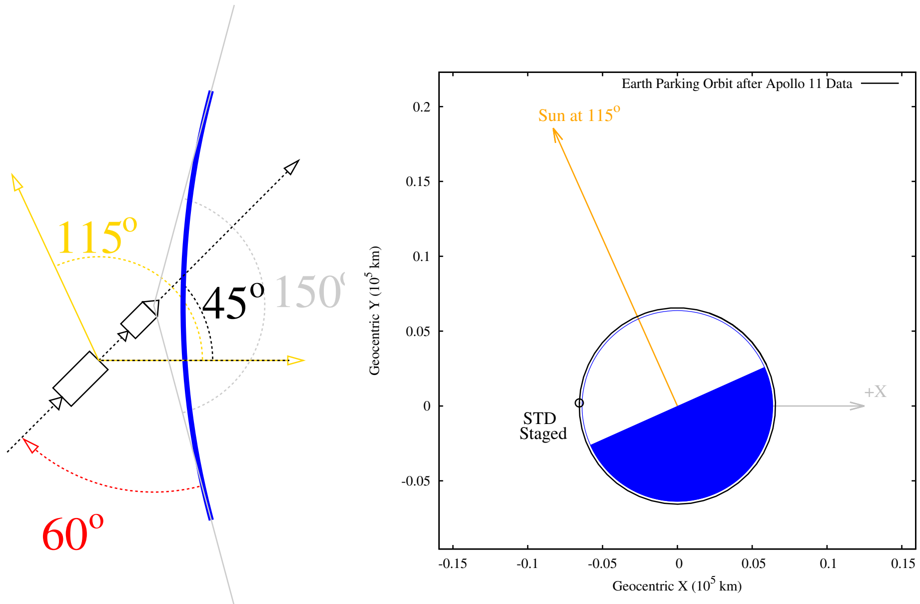

FIG. 11. The astronomical position of the craft with respect to the Sun (the Sun at $\sim 115^o$, in orange; the S-IVB at $\sim 45^o$, in black, Earth size of $\sim 150^o$ in grey) in combination with the empty space between the S-IVB and the leading Earth edge ($\sim 60^o$ from footage, in red) strongly suggests that Separation, Transposition and Docking (STD) were staged at a point in orbit for which the angle was close to 180^o.

separation.

As for the position along orbit where the staging took place, this is most easily done in the geocentric ecliptic coordinate system, in which all angles are measured with respect to the +X axis shown in Fig. 11 (right panel) as follows. The orbital period of the craft along the parking orbit is some 90 min, so the orbital angular velocity is some 4^o/min. On the day of the mission launch, the Sun was at 115^o and the terminator (line between day and night) was at $25^o/205^o$. In photographs, the angle between the direction of the S-IVB and the Sun is determined from the length of the shadow of the Lunar Module (LM) randezvous antena to be 70^o (or equivalently, the Sun is 20^o above the LM). This means that the S-IVB is pointed at 45^o. Earlier analysis of the 16mm film footage of the end of transposition suggests that with the 10mm lens there is an empty space of some $\lambda \simeq 60^o$ between the Earth leading edge and the S-IVB. All this consistently points to the staging taking place at a location at around $\varphi \simeq 180^o$, as shown in Fig. 11. Interestingly, if $\lambda > 60^o$, this means thay $\varphi > 180$, that is, closer to the terminator. Conversely, if $\lambda < 60^o$, then $\varphi < 180$, and the staging took place further away from the terminator. At $\varphi = 180^o$ time allowed for staging is $(205-180)/4 \simeq 6:15$ min, which is some 3 mins shorter then the length of the 16mm film footage ($\sim 2 \times 4:46 = 9:30$).

Obviously, the uncertainty in position of the craft at the time of maneuvers comes from the lack of knowledge of what was λ exactly.

6. CONCLUSION

In this report various inconsistencies in the description of the separation, transposition and docking (STD), between the visual media and the technical documentation have been exposed. Consequently, it has been shown how these inconsistencies could be resolved if one assumes that the STD were staged in the low-Earth parking orbit, but filmed using camera and photographic lenses different from the reported. The staging implies that what the visual media shows is the Command and Service Module (CSM) dismounting the Saturn IV-B (S-IVB) rocket prior to the S-IVB attempting the moonshot. In other words, even if the S-IVB reached the Moon the CSM did not, but stayed in low-Earth orbit.

7. RETRO-OUTLOOK

The S-IVB in all probability attempted to perform translunar injection, because the firing of its rocket engiens would have been visible in some parts of the globe. The planned extraction of the LM by the CSM may have not been attempted, and the S-IVB could have continued to carry the LM to the Moon orbit. Once in the lunar orbit, the ground control may have tried to land the LM. This would have been an achievement on par with the Soviets lunar landings that were part of the Luna program: The Apollo 5 mission demonstrated that the LM could dismount the S-IVB on its own and could go through pre-programmed flight steps using a sequencer. Once it landed on the Moon, the ground control just had to initiate an ascent which would leave the spent Descent Stage on lunar surface as a testament that the mission reached the Moon.

Conversely, the Apollo missions may have put space junk in the low-Earth orbit, which by all accounts should not be there. Consider four large Lunar Module protective panels which were jettisoned at the beginning of separation. They were photographed only during the Apollo 7 mission, because on that mission the panels were moved by the actuators but otherwise remained with the S-IVB. Failure of one panel to fully deploy motivated NASA to jettison them in subsequent Apollo missions. Now, in Apollo 11 mission the panels were not captured on the film, even though images of much smaller and faster pieces of debris were. This strongly suggests that the panels were either removed from the footage when the Earth image was pasted in, or that they were ejected as the craft were repositioning to stage the STD. One such intact panel resembles the, so called, “Black Knight” satellite that would be much later photographed by the Space Shuttle crew.

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Appendix A: Camera Pin-Hole Model

1. Image of Sphere

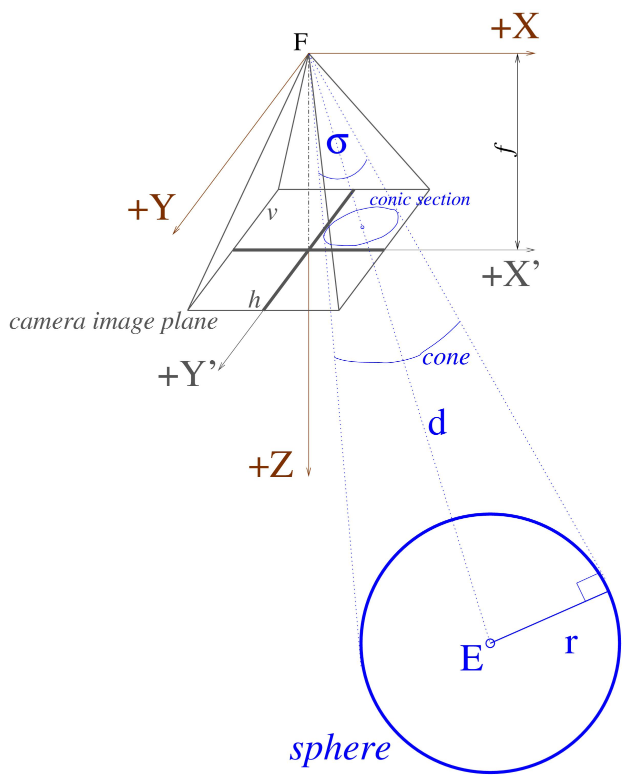

FIG. 12. Geometric representation of a pin-hole camera with a lens with focal length f, the film frame size h-by-v (horizontal by vertical), and how it captures an image of a sphere of radius r at a distance d from the camera.

Consider how an idealized camera represented via a pin-hole model registers a sphere shown in Fig. 12. The coordinate system of the camera comprise its center F and the axes X,Y,Z, while the image is created in a plane at a focal distance f from the center in the Z+ direction. The image plane thus has its own coordinate axes X' and Y' parallel with X and Y, while the Z axis is common for the camera and for the image plane.

To find an image of the sphere, one first constructs the tangents to the sphere passing through the camera center F. Let the center of the sphere in the coordinate system of the camera be given by the vector $\vec{FE} = \hat{n} \cdot d$, where to the unit-vector $\hat{n}$ we refer to as the direction of the sphere,

$$\hat{n} = [\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta], \tag{A1}$$

while d is the distance between the camera center F and the center of the sphere E. Then the tangents comprise points A such that the vector $\vec{FA}$ satisfies,

$$\hat{n} \cdot \vec{FA} = |\vec{FA}| \cos \frac{\sigma}{2}, \tag{A2}$$

where σ is the aperture of the cone. The cone aperture σ, on the other hand, satisfies,

$$\sin \frac{\sigma}{2} = \frac{r}{d}, \tag{A3}$$

where r is the radius of the sphere

Now, it is not difficult to see that the image of the sphere in the image plane of the camera will be a conic section, which is subset of points A above, call it R, given by $\vec{FR} = [x', y', f]$. This can be written as,

$$x' \sin\theta \cos\phi + y' \sin\theta \sin\phi + f \cos\theta = \sqrt{x'^2 + y'^2 + f^2} \cos \frac{\sigma}{2}. \tag{A4}$$

Here, it is important to remark that the center of the conic section $\vec{c}' = [x'_c, y'_c, f]$, given by

$$\begin{aligned} x'_c &= -f \tfrac{\cos\phi \sin 2\theta}{\cos 2\theta + \cos\sigma}, \\ y'_c &= -f \tfrac{\sin\phi \sin 2\theta}{\cos 2\theta + \cos\sigma}, \end{aligned} \tag{A5}$$

is different from the projection of the center of the sphere that is at $\vec{e}' = [x'_e, y'_e, f]$, given by

$$\begin{aligned} x'_e &= -f \cos\phi \tan\theta, \\ y'_e &= -f \sin\phi \tan\theta. \end{aligned} \tag{A6}$$

The two are close to each other in the limit of $\theta \to 0$, that is, when the center of the sphere is close to the camera Z-axis, and $\sigma \to 0$, that is, when the sphere itself is small.

Please note, in Fig. 12 the two centers are not distinguished. Also, from Eq. (A6) it follows that when the center of the sphere approaches the camera plane, $\theta \to 90^o$ ($\tan\theta \to \infty$), it is no longer possible to determine accurately the sphere center position in the camera plane (unlike its spatial position that is fully determined by angles θ and ϕ).

2. Image of Circle

As shown in Fig. 13, a circle in space in the camera coordinate system is specified by its center C, its radius r, and with the unit vector $\hat{q}$ which is perpendicular to the plane of the circle. As before, the position of the center C in the coordinate system of the camera is given by $\vec{FC} = d \cdot \hat{n}$, where $\hat{n}$ is the direction of the circle

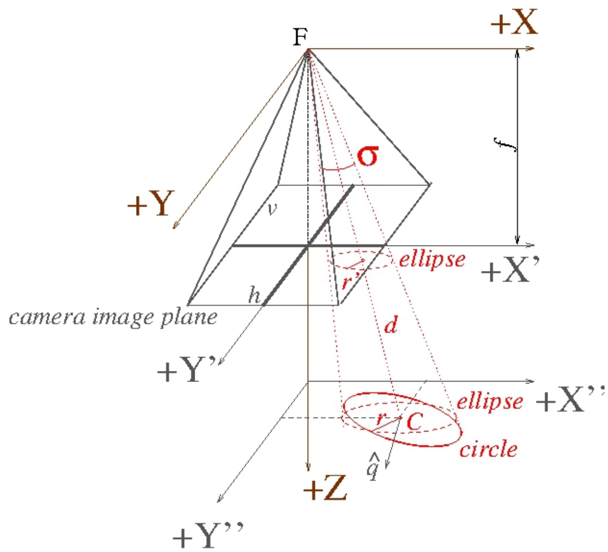

FIG. 13. Geometric representation of a pin-hole camera and how it captures an image of a distant circle of radius r fairly close to the optical axis +Z. See discussion in the text.

with the same parameters θ and ϕ as in Eq. (A1).

The orthogonal base of the plane in which circle lies is constructed as follows. First, the unit-vector $\vec{q}$ can be written as

$$\hat{q} = [\sin\delta\cos\eta, \sin\delta\sin\eta, \cos\delta], \tag{A7}$$

First orthogonal vector to $\hat{q}$, call it $\hat{e}_2$ reads,

$$\hat{e}_2 = [-\cos\delta\cos\eta, -\cos\delta\sin\eta, \sin\delta], \tag{A8}$$

while the second is found through

$$\hat{e}_3 = \hat{q} \times \hat{e}_2 = [\sin\eta, -\cos\eta, 0]. \tag{A9}$$

The circle of radius r in space is set of points P such that $\vec{FP}$ is given by,

$$\vec{FP} = \vec{FC} + r\left(\cos\alpha\,\hat{e}_2 + \sin\alpha\,\hat{e}_3\right), \text{ for } \alpha \in [0, 2\pi], \tag{A10}$$

which can be written coordinate-wise as

$$
\begin{aligned}
x &= x_C - r\cos\alpha\cos\delta\cos\eta + r\sin\alpha\sin\eta, \\
y &= y_C - r\cos\alpha\cos\delta\sin\eta + r\sin\alpha\cos\eta, \\
z &= z_C + r\cos\alpha\sin\delta.
\end{aligned} \tag{A11}
$$

Under the simplifying assumption that the radius of the circle is much smaller then the distance between the circle and the camera, $z_C \gg r$, the intersection of $\vec{FP}$ with the camera image plane at the focal distance $z' \equiv f$ from F becomes,

$$
\begin{aligned}
x' &= f/z_C\,(x_C - r\cos\alpha\cos\delta\cos\eta + r\sin\alpha\sin\eta), \\
y' &= f/z_C\,(y_C - r\cos\alpha\cos\delta\sin\eta + r\sin\alpha\cos\eta).
\end{aligned} \tag{A12}
$$

This is an ellipse with radii $(f\,r/z_C)$ and $\cos\delta(f\,r/z_C)$ centered at $x'_c = f\,x_C/z_C$ and $y'_c = f\,y_C/z_C$. The direction of the center of the circle is found from Eq. (A5), in terms of angles θ and ϕ. These steps are illustrated in Fig. 13.

Further simplification occurs if δ is small, that is, if the image plane is more-or-less parallel with the circle. Then, $\cos\delta \approx 1$, and the image of the circular object of known radius r is easily fitted to a circle centered at $[x'_c, y'_c]$ and of average radius r'. The distance to the object's plane reads,

$$
z_C = \frac{r}{r'} f, \tag{A13}
$$

from the camera, at the distance

$$
d = \frac{r}{r'}\sqrt{c'^2 + f^2}, \tag{A14}
$$

where $c' = \sqrt{{x'_c}^2 + {y'_c}^2}$. Similarly, the approximate aperture σ is given as

$$
\sigma = \arctan(\frac{c' + r'}{f}) - \arctan(\frac{c' - r'}{f}), \tag{A15}
$$

3. Image Processing

Only the images containing Earth or the S-IVB were processed. Processing comprised of extracting the edges of the object of interest and storing the pairs of values of their pixel coordinates $\{X_i, Y_i\}_{i=1,N}$ locally to a file. These were then rescaled to positions in the camera plane $\{x'_i, y'_i\}_{i=1,N}$ as follows,

$$\begin{aligned} x'_i &= \tfrac{h}{V}\left(X_i - \tfrac{H}{2}\right) \\ y'_i &= \tfrac{v}{V}\left(Y_i - \tfrac{V}{2}\right), \end{aligned} \tag{A16}$$

where the size of the camera frame (active area of the film) is h (horizontal width) by v (vertical height), while the digitized medium has H pixels in the horizontal direction, and V pixels in the vertical.

The Earth edge was fitted to a special conic section with the parameters θ, ϕ, σ (direction of the Earth center in the coordinate system of the camera, and the Earth aperture), cf. Eq. (A4). The least squares fitting was performed using the so called implicit method, where data $[X_i, Y_i]_{i=1,N}$ is fitted to an implicit function $f(X,Y;P) = 0$ with parameters P being sought in the least-squares sense. The special form of the conic section coefficients prevents zeros from being the best-fit parameters, as would have been the case if the data were fitted to the most general quadratic form. Once the Earth aperture was known this was converted to the distance (from the center) or the height (from the surface) using Eq. (A3) as shown in Fig. 4.

As for the S-IVB, its outline was fitted to a circle to find its center and radius. This allowed direction of the circle in terms of the angles θ and ϕ to be determined from Eq. (A6), and the distance from Eq. (A14). As is known in the literature, fitting data to an ellipse is complicated because of somewhat arbitrary constraints that have to be imposed upon the most general quadratic form coefficients in order to get a result. Luckily, for the purposes of this report it was safe to assume that $\delta \simeq 0$ so $\cos\delta$ was the second order small quantity. To see that consider set of data points $\{x'_i, y'_i\}_{i=1,N}$ read from the image that uniformly covers the edge, and which describes an ellipse with axes r' and $r' \cos\delta$. It is not difficult to see that when fitted to a circle, this results in slightly smaller best-fit radius r'' given by

$$r'' = r' \sqrt{\frac{2}{1 + \frac{1}{\cos^2\delta}}} \approx (1 - \frac{\delta^2}{4})\, r' \approx r', \tag{A17}$$

and where, most importantly, the center of the ellipse and the fitted circle coincide.