Topological Skyrme Model and the Nucleus

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Abstract

We study the two-flavour topological Skyrme model with lagrangian $\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4$, and point out that, in spite of all the successes attibuted to it, as to the electric charges, it predicts $Q(proton) = +\frac{1}{2}$ *and* $Q(neutron) = -\frac{1}{2}$. This is in direct conflict with the experimental values of proton and neutron charges. This should be considered a failure of the Skyrme model. The Wess-Zumino anomaly term however, comes to its rescue and provides additional contribution which lead to the the correct charges for baryons as per the standard Gell-Mann-Nishijima expression. But as per conventional understanding, that the Skyrme model gives a conserved atomic mass number A=Z+N, is not fulfilled in the above picture. We suggest a new consistent scenario wherein on quantization, a dual description beyond the above model arises, and which provides a framework which is fully compatible with nuclear physics. This picture finds justfication with respect to the surprising 1949 succcessful calculation by Steinberger for the decay $\pi_0 \to \gamma\gamma$.

Keywords: Topological Skyrme model, Wess-Zumino anomaly, SU(2) isospin, nuclear charge, nuclear structure, neutral pion decay

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The topological Skyrme model [1], has been a successful model of the hadrons and the nucleus [2-6]. In fact the 2-flavour model has been focus of much studies in recent years [7-11]. The Skyrme model has a single topological invariant, which Skyrme identified with the atomic mass number A [12]. The atomic mass number is A = Z + N, where Z is proton number and N neutron number, in a nucleus. Most significantly note that a basic feature of this model is that 'A' is the topological invariant in the Skyrme model. The possible topological nature of the proton number Z and the neutron number N, has been emphasized by Atiyah and Manton recently [12].

So one has to be able to understand as to what the atomic mass number A, the proton number Z, and the neutron number N (in a nucleus), are in the Skyrme model and in the Standard Model of particle physics, with group structure $SU(3_c) \otimes SU(2)_L \otimes U(1)_Y$. Crucial is to distinguish between the atomic mass number A and the baryon number B. It is the latter which occurs in the Gell-Mann-Nishijima (GMN) electric charge formula [6],

$$Q = I_3 + \frac{B}{2} \tag{1}$$

where I_3 is the third component of the SU(2)-isospin group and B is called the baryon number. B does not distinguish between proton and neutron and its value is B=1 for both of them. Actually this formula was suggested in 1953 to account for the large number of particles which were being discovered in the 1950's. So the SU(2) group had to be extended to $SU(2) \otimes U(1)$ and where the generator of U(1) was identified as hypercharge Y=B+S and for nucleon N = $\binom{p}{n}$, S=0 as in the above GMN formula. Later the group was extended to $SU(3)$ with Y identified with the second diagonal generator of the group. Note that 'B' is quite distinct from 'A'.

In this paper we ask the question: vis-a-vis the Skyrme model and the Standard Model of particle physics, what is the significance of the difference between the atomic mass number A and the baryon number B?

In this paper we concentrate mainly on two-flavour Skyrme model. Here we take the Skyrme lagrangian and supplement it with the Wess-Zumino (WZ) anomaly term. As well known, the WZ term vanishes for two-flavours. It does however, contribute to the baryon current and the electric charges for two flavours. As we stated above, within the conventional understanding of it, the Skyrme model provides atomic mass number A as topological. What does our model, the Skyrme lagrangian plus the WZ terms in their wholeness, have to say about the above statement?

The Skyrme Lagrangian is [2-6],

$$L_S = \frac{{f_\pi}^2}{4} Tr(L_\mu L^\mu) + \frac{1}{32e^2} Tr[L_\mu, L_\nu]^2 \tag{2}$$

where $L_\mu = U^\dagger \partial_\mu U$. Here the Skyrme topological current is,

$$W_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} Tr[L_\nu L_\alpha L_\beta] \tag{3}$$

On most general grounds this topological current is conserved, i.e. $\partial^\mu W_\mu = 0$ and giving a conserved topological charge $q = \int W_0 d^3x$. This current is independent of any WZ term and which shall be added below.

Here $U(x)$ is an element of the group $SU(2)_F$,

$$U(x)^{SU(2)} = exp((i\tau^a \phi^a / f_\pi), \ \ (a = 1, 2, 3) \tag{4}$$

The solitonic structure present in the Lagrangian is obtained on making Skyrme ansatz as follows [2-6].

$$U_c(x)^{SU(2)} = exp((i/f_\pi \theta(r) \hat{r}^a \tau^a), \ \ (a = 1, 2, 3) \tag{5}$$

This $U_c(x)$ is called the Skyrmion. But on quantization, the two flavour model Skyrmion has a well known boson-fermion ambiguity [2-6]. This is rectified by going to three flavours. In that case we take,

$$U(x)^{SU(3)} = exp[\frac{i\lambda^a \phi^a(x)}{f_\pi}] \ \ (a = 1, 2..., 8) \tag{6}$$

with ϕ^a the pseudoscalar octet of π, K and η mesons. In the full topological Skyrme model this is supplemented with a Wess-Zumino (WZ) effective action [2-6]

$$\Gamma_{WZ} = \frac{-i}{240\pi^2} \int_\Sigma d^5x \epsilon^{\mu\nu\alpha\beta\gamma} Tr[L_\mu L_\nu L_\alpha L_\beta L_\gamma] \tag{7}$$

on surface Σ. Thus with this anomaly term, the effective action is.

$$S_{eff} = \frac{f_\pi^2}{4} \int d^4x \ Tr \ [L_\mu L^\mu] + n \ \Gamma_{WZ} \tag{8}$$

where the winding number n is an integer $n \in Z$, the homotopy group of mapping being $\Pi_5(SU(3)) = Z$.

Write effective action as,

$$S_{eff} = \frac{f_\pi^2}{4} \int d^4x \ Tr \ [\partial_\mu U \partial^\mu U^\dagger] + n \ \Gamma_{WZ} \tag{9}$$

Taking Q as charge operator, under a local electro-magnetic gauge transformation $h(x) = \exp(i\theta(x)Q)$ with small θ, one finds

$$\Gamma_{WZ} \to \Gamma_{WZ} - \int d^4x \partial_\mu x J^\mu(x) \tag{10}$$

whee J^μ is the Noether current arising from the WZ term. This coupling to the photon field is like,

$$J_\mu = \frac{1}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} Tr[Q(L_\nu L_\alpha L_\beta - R_\nu R_\alpha R_\beta)] \tag{11}$$

where $L_\mu = U^\dagger \partial_\mu U$, $R_\mu = U \partial_\mu U^\dagger$. With the electromagnetic field A_μ present, the gauge invariant form of eqn. (8) is,

$$\hat{S_{eff}} = \frac{f_\pi^2}{4}\int d^4x\; Tr\;[L_\mu L^\mu] + n\;\hat{\Gamma_{WZ}} \tag{12}$$

This means that when replacing the LHS in eqn. (10) by $\hat{\Gamma_{WZ}}$, then the RHS has two new terms involving $F_{\mu\nu}F^{\mu\nu}$. This allows us to interpret J_μ with the current carried by quarks [2-6]. With the charge operator Q, J_μ is found to be isoscalar. To obtain the baryon current from eqn. (11), one replaces Q by $\frac{1}{N_c}$ (where N_c is the number of colours in $SU(N_c)$ - QCD for arbitrary number of colours), which is the baryon charge carried by each quark making up the baryon. For total antisymmetry, N_c number of quarks are needed to make up a baryon. Then $nJ_\mu \rightarrow J_\mu^B$ gives,

$$\begin{aligned} nJ_\mu^B(x) &= \frac{1}{48\pi^2}\left(\frac{n}{N_c}\right)\epsilon^{\mu\nu\alpha\beta}Tr[(L_\nu L_\alpha L_\beta - R_\nu R_\alpha R_\beta)] \\ &= \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}Tr[L_\nu L_\alpha L_\beta] \end{aligned} \tag{13}$$

This is the same as the topological current of Skyrme as given by eqn. (3). Thus the gauged WZ term gives rise to $J_\mu(x)$ which in turn gives the baryon charge. Thus though the WZ term Γ_{WZ} is zero for two-flavour case, but $J_\mu(x)$ still contibutes to the two-flavour case.

Next we embed the SU(2) Skyrme ansatz into $U(x)^{SU(3)}$ as follows for the SU(3) Skyrmion [13],

$$U_c(x)^{SU(2)} \;\rightarrow\; U_c(x)^{SU(3)} = \begin{pmatrix} U_c(x)^{SU(2)} & \\ & 1 \end{pmatrix} \tag{14}$$

Next we insert the identity,

$$U(\vec{r},t)^{SU(3)} = A(t)U(\vec{r})_c^{SU(3)}A^{-1}(t) \qquad A \in SU(3)_F \tag{15}$$

where A is the collective coordinate. Note that $U(\vec{r},t)$ is invariant under,

$$A \rightarrow Ae^{iY\alpha(t)} \tag{16}$$

where

$$Y = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{17}$$

With this the quantum degrees of freedom manifest themselves in the WZ term (eqn. (7)) as,

$$L_{WZ} = -\frac{1}{2}N_c B(U_c)tr(YA^{-1}A) \tag{18}$$

where $B(U_c)$ is the baryon number (winding number) of the classical configuration U_c. The gauge invariance leads to changing L_{WZ} to

$$L_{WZ} \to L_{WZ} + \frac{1}{3} N_c B(U_c)\dot{\alpha} \tag{19}$$

In the quantized theory A and Y are operators and from Noether's theorem one obtains (with Ψ as allowed quantum state)

$$\hat{Y}\Psi = \frac{1}{3} N_c B \Psi \tag{20}$$

This gives the right-hypercharge,

$$Y_R = \frac{1}{3} N_c B \tag{21}$$

where the baryon number B is necessarily an integer and colour N_c is an integer too. [2,3,13]

With B = 1 and $N_c = 3$ one gets $Y_R = 1$. This identifies the nucleon hypercharge with the body-fixed hypercharge Y_R. This shows that the baryon number is the nucleon number of the subgroup SU(2) of SU(3). In SU(2) the the nucleon is defined as N = $\left(\begin{smallmatrix} p \\ n \end{smallmatrix}\right)$, This interpretation of the baryon number or the nucleon number, B=1 continues to hold for the Skyrme two-flavour model as we see below. Thus there is no atomic mass number A arising in this Skyrme model. This is at variance with what Skyrme had surmised. We discuss this crisis in detail below.

Let us now study the structure of the electric charge in the $SU(2)_F$ model, which as pointed out by Balachandran et. al. [13, p. 176] has not been paid the attention it deserves. This because as we show below, it presents a serious challenge to the Skyrme lagrangian for two flavours. Following Balachandran et. al. [13], we define the electric charge operator in SU(2) as,

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \tag{22}$$

It induces the following transformation,

$$U(x) \to e^{i\epsilon_0 \Lambda Q} U(x) e^{-i\epsilon_0 \Lambda Q} = e^{\frac{i\epsilon_0 \Lambda \tau_3 (q_1 - q_2)}{2}} U(x) e^{\frac{-i\epsilon_0 \Lambda \tau_3 (q_1 - q_2)}{2}} \tag{23}$$

where ϵ_0 is the electromagnetic coupling constant. The Noether current associated with the above symmetry is,

$$\frac{J_\mu^{em}}{\epsilon_0} = \frac{iF_\pi^2}{8}\, Tr\; L_\mu (Q - U^\dagger Q U) - \frac{i}{8\epsilon_0^2}\, Tr\; [L_\nu, Q - U^\dagger Q U][L_\mu, L_\nu] \tag{24}$$

We obtain the gauge theory by replacing

$$\partial_\mu U \to D_\mu U = \partial_\mu U - i\epsilon_0 \Lambda_\mu [Q, U] \tag{25}$$

To obtain constraints on charges in eqn. (22), first expand on pion fields to obtain,

$$J_\mu^{em} = -i\epsilon_0(q_1 - q_2)(\pi_-\partial_\mu\pi_+ - \pi_+\partial_\mu\pi_-) + ... \tag{26}$$

From pion charges one gets

$$(q_1 - q_2) = 1 \tag{27}$$

Next the charges of baryons $N\ and\ \Delta$ with B=1 charge on using eqn. (15),

$$Q = \int d^4x\ J_0^{em}(\vec{x}, t) = \epsilon_0 L_\alpha\ Tr\tau_\alpha Q \tag{28}$$

From eqn. (22) we get,

$$Q = \epsilon_0(q_1 - q_2)L_3 \tag{29}$$

On using eqn. (27),

$$Q = \epsilon_0 L_3 \tag{30}$$

As L_3 is the third component of the isospiun operator, we get (in units of ϵ_0),

$$Q(proton) = +\frac{1}{2}\ \ and\ \ Q(neutron) = -\frac{1}{2} \tag{31}$$

This is in complete disagreement with experiment. Thus the Skyrme Lagrangian eqn. (2) fails to provide correct electric charges to proton and neutron. As such this should be construed to mean that just the Skyrme lagrangian in itself, is not enough to give consistent description of the B=1 nucleon.

It needs another term to pull it out of this conundrum. And indeed we have the additional WZ term to do the job. Again let the field U be transformed by an electric charge operator Q as, $U(x) \rightarrow e^{i\Lambda\epsilon_0 Q}U(x)e^{-i\Lambda\epsilon_0 Q}$,

Making $\Lambda = \Lambda(x)$ a local transformation the Noether current is [13]

$$J_\mu{}^{em}(x) = j_\mu{}^{em}(x) + j_\mu{}^{WZ}(x) \tag{32}$$

where the first one is the standard Skyrme term and the second is the Wess-Zumino term

$$j_\mu{}^{WZ}(x) = \frac{\epsilon_0 N_c}{48\pi^2}\epsilon_{\mu\nu\lambda\sigma}TrV^\nu V^\lambda V^\sigma(Q + U^\dagger QU) \tag{33}$$

Remember that even though the WZ term vanishes for two flavours, its resulting contribution to electric chaege does not. This term was of course missing in the original version of the Skyrme Lagrangian (eqn. (2)).

One finally obtains [13, p. 208],

$$j_\mu{}^{WZ}(x) = \frac{\epsilon_0}{2}(q_1 + q_2)N_c J_\mu(x) \tag{34}$$

The WZ term correction to the electric charge is therefore,

$$\frac{\epsilon_0}{2}(q_1 + q_2)N_c \int J_0(x)d^3x \tag{35}$$

Using eqn. (15) above,

$$\frac{\epsilon_0}{2}(q_1 + q_2)N_c B(U_c) \tag{36}$$

Remember the right hypercharge $Y_R = 1$ in eqn. (21) and subsequently B=1 for $N_c = 3$. Note also the baryon in the Skyrme model with B=1 now as per eqn. (13) has three quarks. We thus obtain the charges of $N \ and \ \Delta$ if we put,

$$q_1 + q_2 = \frac{1}{3} \tag{37}$$

Along with eqn. (27), we obtain the charges as,

$$q_1 = \frac{2}{3}\ , \quad q_2 = -\frac{1}{3} \tag{38}$$

These are the charges of u- and d- quarks, which make up the proton of three quarks with baryon number B=1, as per eqn. (13). This gives the proper charges of the baryons as per the GMN charge formula of eqn. (1). There is no atomic mass number A here. As we stated earlier this is at variance with respect to the original surmise of Skyrme as per the topological baryon, which was expected to be the atomic mass number. The model here is reproducing the quark model result of composite baryons of three quarks due to fact that there are three colours. The Skyrme model seems to be giving "quarks without explicit quarks" [2,3]. This is good, but then where is the original Skyrme's atomic mass number A = Z + N baryon?

We have seen that the original Skyrme lagrangian eqn. (2) has a conserved topological charge as given by eqn. (3). This is independent of any quantization brought in by the WZ term. Next we saw thst the WZ term has gives baryon number eqn. (13), which is exactly the Skyrme model topological charge in eqn. (2). This though leads to baryon number B=1 when there are three fractionally charged quarks. This picture is correct as it is reproducing the conventional quark model result with electric charge and baryon number related by the GMN electric charge expression eqn. (1).

But where is the original Skyrme model where baryon number B=1 is actually the atomic mass number A = Z + N [12]? What have we missed out? We seek answer within the WZ term baryon number as obtained in eqn. (13) above. We saw that it requires the winding number and the number of colours be related as,

$$n = N_c \tag{39}$$

This also necessarily used the SU(3) model result as given in eqn. (21), which demanded that $Y_R = 1 \rightarrow N_c = 3 \ and \ B = 1$. Thus from eqn. (39) n=3 in eqn. (13).

However, if we take $N_c = 1$ then by eqn. (39), n=1 and we get a different baryon number from eqn. (13). This baryon number will reproduce the same Skyrme topological current as given in eqn. (3).

But $N_c = 1$ means baryon built of only one flavour of quark. However the conventional Standard Model says that this quark would be a u-quark with a fractional charge of 2/3 [14]. This is not acceptable.

However, note that a proper variant of the Standard Model (SM) is the Quantized Charge Standsrd model (QCSM) [15,16] which is also consistent with the three-flavour Skyrme model [17]. See Appendix below for details and for salient points about the QCSM and the SM. In QCSM the charges are quantized consistently and the quark charges are given as,

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c}) \quad ; \quad Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c}) \tag{40}$$

Note the remarkable fact that these quark charges are colour dependent, giving the experimental charges of 2/3 and -1/3 for three colours. This is the crucial difference with respect to the static chages of the SM.

Note that for $N_c = 1$ the charges are: $Q(u) = 1$, $Q(d) = 0$, which are the proton and the neutron charges respectively. Also as the colour is an odd number here, these baryons are fermionic as well. We carry this further.

For two flavours we saw above that for the electric charge defined in eqn. (22) as Q $= \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}$, and with the pion charges giving $q_1 - q_2 = 1$ in eqn. (27), the Skyrme lagrangian (with no WZ term) gives the charges in eqn. (31) as $Q(p) = \frac{1}{2}$, $Q(n) = -\frac{1}{2}$. This shows that the total proton and neutron charges are eigenstates of the third component of isospin only. Hence the charge that the Skyrme lagrangian provides is pure isovector only.

Including correction arising from the WZ term we saw the electric charge current in eqn. (32) as ${J_\mu}^{em}(x) = {j_\mu}^{em}(x) + {j_\mu}^{WZ}(x)$. Further calculations showed that the WZ contribution in eqn. (36), $\frac{\epsilon_0}{2}(q_1 + q_2)N_c B(U_c)$. For three colours we saw that fractional quark charges arose when both baryons N *and* Δ played their part. But now with $N_c = 1$, we can fix the charge of nucleon only (with no Δ) by putting,

$$q_1 + q_2 = 1 \tag{41}$$

This WZ term contibutes a pure isoscalar form of electric charge of proton and neutron. So,

$$Q(p) = \left(\frac{Z=1}{2}\right)_{isovector} + \left(\frac{Z=1}{2}\right)_{isoscalar}$$
$$Q(n) = -\left(\frac{N=1}{2}\right)_{isovector} + \left(\frac{N=1}{2}\right)_{isoscalar} \tag{42}$$

Hence this model gives right away the charge of a nucleus for arbitrary number of Z protons and N neutrons as,

$$Q = \frac{Z-N}{2} + \frac{Z+N}{2} \quad = \quad T_3 + \frac{A}{2} \tag{43}$$

where the atomic mass number comes in a natural manner to provide the correct charges of the nucleus. Thus we have the proper and correct description of the nuclear charges as per this Skyrme model. Also we see that as per what Atiyah and Manton had asked [12] about the possible topological character of atomic mass number A, the proton number Z, and the neutron number N; we emphasize here that indeed all these are of topological nature in nuclei, as shown above.

Now how about isospin symmetry? For $N_c = 3$ case the quark charges were fractional as per eqn. (38). Now we know that for $N_c = 2k + 1$, proton is made up of k+1 u-quark and k d-quarks. SU(2)-isospin symmetry dictates that neutron has k u-quarks and k+1 d-quarks. The resulting composite proton and neutron develop a subsequent SU(2) isospin symmetry as a result of the basic SU(2) symmetry of u- and d-quarks.

However now for $N_c = 1$, proton and neutron correspond to a single u- and d- quark respectively, and thus there is no SU(2)-isospin symmetry, Hence the proton and neutron that we get for $N_c = 1$ have no isospin symmetry associaied with it as well. Thus protons and neutrons are distinguishable fermions in this model. This is a most important distinction of baryons with respect to those arising in the $N_c = 3$ picture.

Now the above version of proton and neutron as made up of three quarks and which then are indistighuishable with good SU(2)-isopin symmetry. We know that this is the basis of the currently most successful Independent Particle Model(IPM) of the nucleus [18]. The Brueckner-Hartree-Fock model provides theoretical basis for a nucleus made up Z protons and N neutrons with an IPM shell structure. Charge independence and a Generalized Pauli Exclusion Principle plays its basic role in this model of the nucleus.

In our new picture of distinguishable protons and neutrons, then there should be a duality of description within the nuclear models; meaning that while within the well known structure of the nucleus made up up of indistinguishable protons and neutrons holds good, it should be still possible to describe the nucleus simultaneously as constituted of distinguishable protons and neutrons. Before dismissing the above as nonsence, please note that upto about 1950's and 1960's the standard nuclear physics calculations were perforemed only within distinguishable protons and neutrons scenario, see Blatt and Weisskopf [19, p. 153-156]! So what we have stated above should make sense, and would correspond to a basic structural reality of a dual nature of the nucleus.

But as we have discussed above, the $SU(2)_I$ model is today a good and a successful model of the nucleus. But the earlier results with distinguishable protons and neutrons were equally good too [19]. Thus there is actually a duality of models here. Therefor a nucleus can be described well in an $SU(2)_I$ model (where (p-n) are indistinguishable) and in another independent picture where the pair (p-n) is treated as made up of distinguishable fermions. Lawson in his text-book [20, p. 107-122] has shown, in a complete section entitled "Isospin and non-isospin methods of calculation", that these two independent methods yield essentially identical results in the nucleus. If the first model based on SU(2) isopin symmetry stands for the IPM model, then we surmise that the second and the new model proposed here, should stand for the liquid drop model character of the nucleus.

The relationship between the two formalisms here is discussed at many places [19,20,21]. These demonstrate that it is merely a formal requirement to move from one formalism to another. So taking the Pauli Exclusion Principle for the proton and neutron separately in a conventional manner or by requiring antisymmetry under the exchange of two nucleons in isospin formalism, one is able to build an antisymmetric wave function from the conventional wave functions [19, 21 p. 16-18].

Thus our picture of a dual description of hsdronic reality arising from the Skyrme lagrangian plus the WZ anomaly term, topologically provindes a dual description of nuclear reality and which seems to be consistent with the overall understanding of the nucleus. So the empirical structure supports our picture. However we know thsat the $N_c = 3$ case is consistent with the successful quark model structures, but the second case of $N_c = 1$ may appear puzzling. The question we ask, does the reality associated with $N_c = 1$ manifest itself in any other successful theoretical structure?

Indeed, it does! It comes from the study of decay $\pi^0 \to \gamma\gamma$ which results from a triangle diagram with internal quark loops attached to one external isovector axial current and two external electromagnetic currents. The decay rate is [22],

$$\Gamma\left(\pi^0 \to \gamma\gamma\right) = N_c^2 (Q^2(u) - Q^2(d))^2 \frac{\alpha^2 m_{\pi 0}^2}{64\ \pi^3 F_\pi^2} \tag{44}$$

where Q_u and Q_d are the $u-$ and $d-$ quark charges, N_c is the number of colours, m_{π^0} is the neutral pion mass, $\alpha = \frac{e^2}{4\pi}$ and F_π, the pion decay constant $\sim$ 91MeV.

The experimental value of the decay rate is $\sim$ 7.8eV [22]. Take quark charges as in the Standard Model as $Q(u) = \frac{2}{3}$ and $Q(d) = -\frac{1}{3}$. If there were no colours ($N_c = 1$), then from the above formulae one obtains the decay rate of 0.84 eV. This is much too low a value. Thus one is forced to include $N_c = 3$ and then the fit is good. This is taken as a Standard Model proof of the evidence of 3-colours in particle physics and is well documented in the current literature [23].

However with the colour dependent charges eqn. (40) of the Quantized Charge Standard Model, the relevant factors in eqn. (44) give,

$$N_c^2 (Q_u^2 - Q_d^2)^2 = N_c^2 \left[\left\{ \frac{1}{2} \left(1 + \frac{1}{N_c}\right) \right\}^2 - \left\{ \frac{1}{2} \left(-1 + \frac{1}{N_c}\right) \right\}^2 \right]^2 = 1 \tag{45}$$

And hence overall there is no N_c- dependence left in the decay rate of $\pi^0 \to \gamma\gamma$ and the subsequent result matches the experiment well. So when correct colour dependent electric charges for quarks are taken, the decay rate is actually independent of colour degrees of freedom as per QCSM.

Hence the same would hold true for $N_c = 1$ also - whence u-quark charge is that of a proton and d-quark charge is that of a neutron in QCSM. This was emphasized by Baer and Wiese [24]. Now this result is exactly the same as the successful result of Steinberger in 1949 [25] with no quarks and only a charged proton going around the triangle loop. That success has often been dubbed as a conincidence, or a lucky chance calculation. But now as per QCSM, $N_c = 1$ u-quark behaving as a proton, is what leads to the successful Steinberger result [24]. This gives an independent support to our topological Skyrme model calculation which gave a successful description of the nucleus. Hence it is bringing out an unrecognised hidden reality from a basic topological point of view.

In summary, the classical baryon number of the Skyrme model is given in eqn. (3). Quantization is done with the WZ anomaly term. The presence of N_c, the number of colours, provides a degree of freedom to quantize consistently and independently both with $N_c = 3$ and $N_c = 1$. The first one gives a picture with indistinguishable proton-neutron pair, while the second one gives a picture with distinguishable proton-neutron pair. Both should hold good for a complete description of the nucleus. Indeed, this picture is supported empirically in theoretical model studies of the nucleus.

Hence, it turns out the the Skyrme model with modification brought in by the Wess-Zumino anomaly term, leads to a consistent dual description of the nucleus. Hence it should give support to the prediction of a very heavy scalar neson in particle physics [25]

APPENDIX

I. Electric charge in the Standard Model (SM)

The beginning of the Standard Model may be traced back to 1961, when Glashow in studying the weak interaction, sought to incorporate electric charge in a larger electro-weak group of $SU(2)_W \otimes U(1)_W$, He just copied the Gell-Mann-Nishijima definition of the electric charge of the strong interaction group $SU(2) \otimes U(1)$ in vogue at that time,

$$Q = {T_3}^W + \frac{Y_W}{2} \tag{46}$$

Here Y_W is called weak-hypercharge indicating its different origin from the strong hypercharge. Glashow gave the representation for the first generation particles (it is detailed below in part II, where we replace quarks (u,d) for hadrons (p,n) known at that time when quarks were not known). He then fixed the values of the weak-hypercharge Y_W to fit the various charges of matter particles, Also he had no Englert-Brout-Higgs field which came much later in 1967, through the work of Salam and Weinberg.

The weak-hypercharge defined as a generator of $U(1)_{Y_W}$ is not constrained to give any quantized value for Y_W. It can be any number and as indicated above , Glashow had to fix it by hand to give proper charges to the matter particles. Hence the electric charge is not quantized in the electro-weak model.

Now what we call the Standard Model (SM) is an extension of the above electro-weak group with the strong interaction colour group included to give $SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y_W}$. The above definitions of the electric charge however are carried over in toto to the Standard Model. The electric charge is thus not quantized in the Standard Model and is considered a major weakness of it.

Besides being arbitray and unquantized, these charge already exist in the SM prior to any Spontaneous Symmetry Beaking (SSB) mechanism through Englert-Brout-Higgs (EBH) field. It is immune or independent of colour degree of freedom of the the strong-colour group $SU(N)_c$, i.e. it is rigid of fixed values 2/3 and -1/3. Also anomalies play no role other than being trivially satisfied by the above pre-fixed values of the hypercharge and the charges in the SM.

II. Electric charge in the Quantized Charge Standard Model (QCSM)

We therefore have to go beyond the above SM to get quantized charges [15,16,6]. We take the same generation structure as that in the SM and the same Englert-Brout-Higgs (EBH) field as an $SU(2)_L$ group doublet, $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

However, major differences with respect to the above SM are that: (1) We have no a priori electric charge existing above the electro-weak symmetry breaking scale. (2) We start with the complete group structure of $SU(N_c) \otimes SU(2)_L \otimes U(1)_{Y_W}$ where $N_c = 3$. (2) We take the most general definition of the electric charge in terms of the diagonal generators of the above group structure (3) First we study the effects of SSB through the above EBH field, (4) Next the role of anomaly cancellations is studied in the most general manner. (5) Then we ensure that all massless matter particles acquire mass through Yukawa couplings.

The first generation fermions (the other genrations are repetitive in what we do below) are assigned to the following representations for the group $SU(N_c) \otimes SU(2)_L \otimes U(1)_{Y_W}$.

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, (N_c, 2, Y_q) \quad ; u_R, (N_c, 1, Y_u) \quad ; d_R, (N_c, 1, Y_d)$$

$$l_R = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, (1, 2, Y_l) \quad ; e_R, (1, 1, Y_e) \tag{47}$$

There are five unknown hypercharges above. Including the unknown Y_ϕ, it becomes six unknown hypercharges for the first generation of particles.

For the group $SU(2)_L \otimes U(1)_{Y_W}$ we define the electric charge operator in the most general way in terms of the diagonal generators of the groups as,

$$Q = T_3 + b\,Y \tag{48}$$

In $SU(2)_L \otimes U(1)_{Y_W}$, symmetry provides three massless generators W_1, W_2, W_3 of $SU(2)_L$ and of $U(1)_Y$. To provide mass to the gauge particles for the weak interaction, the symmetry was broken using an EBH doublet ϕ above by Salam and Weinberg. This provided mass to the $W^\pm$ and Z^0 gauge particles while ensuring zero mass for photons γ. Thus $U(1)_{em}$ is the exact consequent symmetry in the process $SU(2)_L \otimes U(1)_{Y_W} \to U(1)_{em}$. Let the $T_3 = -\frac{1}{2}$ component corresponding to the EBH field develop a nonzero vacuum expectation value $< \phi >_0$. As per the EBH mechanism for SSB, to ensure that one of the four generators $(W_1 W_2 W_3, X)$ is thereby left unbroken (meaning that what we ensure a massless photon as a generator of the $U(1)_{em}$ group), we demand:

$$Q < \phi >_0 = 0 \quad \to \quad T_3^\phi + b < \phi_0 >= 0 \tag{49}$$

This fixes the unknown b and the electric charge is:

$$Q = T_3 + (\frac{1}{2Y_\phi})Y \tag{50}$$

Now separate generation-wise cancellation of anomalies brings in the requirement for the satisfaction of the following three constraints:

$$(a) TrY[SU(N_C)]^2 = 0 \;\; which\ yields \;\; 2Y_q = Y_u + Y_d \tag{51}$$

$$(b) TrY[SU(2)_L]^2 = 0 \;\; which\ gives \;\; 2^2 Y_l + N_c[2^2 Y_q] = 0 \tag{52}$$

$$and\ thus \qquad Y_q = -\frac{Y_l}{N_c} \tag{53}$$

$$(c) \qquad Tr[Y^3] = 0 \tag{54}$$

$$giving \quad 2N_c Y_q^3 - N_c Y_u^3 - N_c Y_d^3 + 2Y_l^3 - Y_e^3 = 0 \tag{55}$$

We still need to have terms for Y_u, Y_d, Y_e, in addition to Y_q.

Now we know that before SSB the matter particles are massless. We have to make them massive through this process of SBB. The is done through Yukawa couplings,

$$\mathcal{L} = -\phi^\dagger \bar{q}_L u_R + \phi q_L \bar{d}_R + \phi e_L \bar{e}_R \tag{56}$$

On demanding gauge invariance the above yields,

$$Y_u = Y_q + Y_\phi \tag{57}$$

$$Y_d = Y_q - Y_\phi \tag{58}$$

$$Y_e = Y_l - Y_\phi \tag{59}$$

Now substituting Y_q and Y_u,Y_d,Y_e from above one obtains:

$$(Y_l + Y_\phi)^3 = 0 \ \rightarrow \ \ Y_l = -Y_\phi \tag{60}$$

and putting this above,

$$Y_q = \frac{Y_\phi}{N_c} \tag{61}$$

These yield,

$$Y_u = Y_\phi(\frac{1}{N_c} + 1) \tag{62}$$

And similarly for Y_d and Y_e. Finally, substituting these expressions above we get, quantized electric charges in the Quantized Charge Standard Model as,

$$Q(u) = \frac{1}{2}(1 + \frac{1}{N_c}) \ \ ; \ Q(d) = \frac{1}{2}(-1 + \frac{1}{N_c})$$
$$Q(\nu_e) = 0 \ \ \ ; \ Q(e) = -1 \tag{63}$$

For $N_c = 3$ these yield the correct electric charges of quarks, and for all the particles in the first generation. Note that inspite of the fact that $U(1)_{em}$ does not know of colour, the electric charges are actually dependent upon colour itself. Also note that this charge quantisation is independent of the EBH field hypercharge Y_ϕ

Note the fact that the electric charge of the quark has colour dependence built into itself, is a significant new result of the Quantized Charge Standard Model. However this is in direct conflict with the charges obtained in the Standard Model. These charges were rigidly always $Q(u) = \frac{2}{3}$ and $Q(d) = -\frac{1}{3}$.

In fact, it has been shown that [15] the colour dependent charges of the Quantized Charge Standard Model are the correct ones, while the static charges $Q(u) = \frac{2}{3}$ and $Q(d) = -\frac{1}{3}$ of the Standard Model are the wrong ones.

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