Neutrosophic Invertible Graphs of Neutrosophic Rings

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ABSTRACT

Let $N(R, \mathrm{I})$ be a Neutrosophic ring of a finite commutative classical ring R with non-zero identity. Then the Neutrosophic invertible graph of $N(R, \mathrm{I})$, denoted by $\mathcal{I}_G(\mathrm{N}(R, \mathrm{I}))$ and defined as an undirected simple graph whose vertex set is $N(R, \mathrm{I})$ and two vertices $a+b\mathrm{I}$ and $c+d\mathrm{I}$ are adjacent in $\mathcal{I}_G(\mathrm{N}(R, \mathrm{I}))$ if and only $a+b\mathrm{I}$ is different from $-(c+d\mathrm{I})$ which is equivalent to $c+d\mathrm{I}$ is different from $-(a+b\mathrm{I})$. We begin by considering some properties of the self and mutual additive inverse elements of finite Neutrosophic rings. We then proceed to determine several properties of Neutrosophic invertible graphs and we obtain an interrelation between classical rings, Neutrosophic rings and their Neutrosophic invertible graphs.

KEYWORDS: Classical ring, Neutrosophic ring, Neutrosophic invertible graphs, Neutrosophic Isomorphism,self and additive inverse elements.

1. INTRODUCTION

The investigation of simple undirected graphs associated to finite algebraic structures, namely, rings and fields which are very important in the theory of algebraic graphs. In recent years the interplay between Neutrosophic algebraic structure and graph structure is studied by few researchers. For such kind of study, researchers define a Neutrosophic graph whose vertices are set of elements of a Neutrosophic algebraic structure and edges are defined with respect to a well-defined condition on the pre-defined vertex set. Kandasami and Smarandache (2006) introduced the notion and structure of the Neutrosophic graphs. Also, the authors Kandasami and Smarandache (2006) and Kandasamy, Ilanthenral, & Smarandache (2015) studied the notion and structure of the Neutrosophic graphs of several finite algebraic structures and exhibited them with various examples. Later, Chalapathi and Kiran (2017a) introduced another Neutrosophic graph of a finite group and this work was specifically concerned with finite Neutrosophic multiplicative groups only.

Throughout this paper, we will write $N(R, \mathrm{I})$ be a finite Neutrosophic commutative ring with identity 1 and indeterminacy I. For this Neutrosophic algebraic structure, we denote $S\left(N(R, \mathrm{I})\right)$ and $M\left(N(R, \mathrm{I})\right)$ be the set of self and respectively mutual additive Neutrosophic inverse elements. We may construct a new type of graphs associated with Neutrosophic rings. Our primary goal is to introduce Neutrosophic invertible graphs of finite rings and to study properties of these graphs. Further, we determine the diameter of Neutrosophic invertible graphs and introduce an isomorphic relation between classical rings, Neutrosophic rings and their invertible graphs.

2. BASIC PROPERTIES OF NEUTROSOPHIC RINGS

In this section, for all terminology and notations in graph theory, classical ring theory and Neutrosophic ring theory, we refer (Vitaly & Voloshin, 2009), (Lanski, 2004). and (Agboola, Akinola, & Oyebola. (2011); Agboola, Adeleke, & Akinleye, 2012) respectively. Chalapathi and Kiran (2017b) introduced and studied self and mutual additive inverse elements of finite Neutrosophic rings and illustrated them with few examples in different cases and proposed various results regarding the characterization of the Neutrosophic rings with identity $1 \neq 0$. We will restate some of the results as follows (Chalapathi & Kiran, 2017a; 2017b).

Definition 2.1. Let $(R, +, \cdot)$ be a finite ring. The set $N(R, I) = \langle R \cup I \rangle = \{a + bI : a, b \in R\}$ is called a Neutrosophic finite ring generated by R and I, where I is the Neutrosophic element with the properties $I^2 = I$, $0I = 0$, $I + I = 2I$ and I^{-1} does not exist.

Theorem 2.2. Let R be a finite ring with unity. Then $S(R) = R$ if and only if $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

Theorem 2.3. Let R be a finite Boolean ring with unity. Then $S(R) = R$ and $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

Theorem2.4. Let R and R' be two finite commutative rings with unity. If $R \cong R'$, then $S\left(N(R, \mathrm{I})\right) \cong S\left(N(R', \mathrm{I})\right)$.

Theorem 2.5. Let R and R' be two finite commutative rings with unity. Then $R \cong R'$ if and only if $N(R, \mathrm{I}) \cong N(R', \mathrm{I})$.

Theorem 2.6. Let R be a finite Boolean ring with unity and $|R| > 1$. Then $4 \leq |N(R, \mathrm{I})| \leq |R|^2$

Proof. Since $R = \{0\}$ if and only if $N(R, \mathrm{I}) = \{0\}$. It is clear that $R \neq \{0\}$ implies that $|R| > 1$. Suppose $|R| = 2$. Then, obviously, $R \cong Z_2$. This implies that $N(R, \mathrm{I}) = N(Z_2, \mathrm{I})$

$=\{0, 1, \text{I}, 1+\text{I}\}$, and hence $|N(R, \text{I})| = 4$. It is one extremity of the inequality. For another extremity of the inequality, we set $R^*\text{I} = \{a\text{I}: a \in \text{R}^*\}$, $\text{R}^* + \text{R}^*\text{I} = \{a + b\text{I}: a, b \in \text{R}^*\}$ where R^* $= R - \{0\}$. These sets imply that R, $R^*\text{I}$ and $\text{R}^* + \text{R}^*\text{I}$ are mutually non-empty disjoint subsets of $N(R, \text{I})$. Thus, $N(R, \text{I}) = R \cup R^*\text{I} \cup (R^* + R^*\text{I})$, and clearly the cardinality of $N(R, \text{I})$ is $|N(R, \text{I})| = |R| + |R^*\text{I}| + |R^* + R^*\text{I}| = |R| + (|R|-1) + (|R|-1)^2 = |R|^2$.

Theorem 2.7. For any finite ring R with $|R| > 1$, we have $N(R, \text{I})$ is the disjoint union of $S(N(R, \text{I}))$ and $M(N(R, \text{I}))$.

Proof. By the definition of self and mutual additive inverse elements of the Neutrosophic ring,

$$S(N(R, \text{I})) = \{a + b\text{I}:\ 2a = 0, 2b = 0\}$$

and $M(N(R, \text{I})) = \{c + d\text{I}:\ 2c \neq 0, 2d \neq 0\}$.

Clearly, $S(N(R, \text{I})) \cap M(N(R, \text{I})) = \phi$, and thus $S(N(R, \text{I})) \cup M(N(R, \text{I})) = N(R, \text{I})$.

3. NEUTROSOPHIC INVERTIBLE GRAPHS

In this section, we introduced Neutrosophic invertible graphs and characterized its structural concepts.

Definition3.1. Let R be a finite commutative ring with identity $1 \neq 0$. A graph with its vertex set as $N(R, \text{I})$ and two distinct vertices $a + b\text{I}$ and $c + d\text{I}$ are adjacent if and only $a + b\text{I}$ is different from $-(c + d\text{I})$ which is equivalent to $c + d\text{I}$ is different from $-(a + b\text{I})$ and we denote it by $\mathcal{I}_G$ (N(R, I)).

The following theorem is a consequence of the Definition [3.1].

Theorem3.2. For each $N(R, \text{I}) \neq \{0\}$, there exist Neutrosophic invertible graph $\mathcal{I}_G(\text{N}(R, \text{I}))$.

Further, the aim of this section is to show how Neutrosophic algebraic representation of some philosophical concepts and some real world problems in the society can be modified to the study of algebraic Neutrosophic graphs. So, we shall investigate some important concrete properties of Neutrosophic invertible graphs, and also establish results of these graphs, which we required in the subsequent sections.

We begin with the algebraic graph theoretical properties of $\mathcal{I}_G$ (N(R, I)) , $|R| > 1$. Note that $|R| > 1$ if and only if $4 \leq |N(R, \text{I})| \leq |R|^2$.

Theorem 3.3. The Neutrosophic invertible graph $\mathcal{I}_G$ (N(R, I)) is connected.

Proof. Since $0+0\mathrm{I} \in S\left(N(R, \mathrm{I})\right)$ for any $N(R, \mathrm{I})$, $\left|N(R, \mathrm{I})\right| \geq 4$. So, $\left(a+b\mathrm{I}\right)+\left(0+0\mathrm{I}\right)$ $= a+b\mathrm{I} \neq 0+0\mathrm{I}$, for any non-zero element in $a+b\mathrm{I}$ in $S\left(N(R, \mathrm{I})\right)$.This implies that the vertex $0+0\mathrm{I}$ is adjacent with remaining all the vertices in $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$. It is clear that there is a pathbetween the vertices $0+0\mathrm{I}$ and $a+b\mathrm{I}$ in $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(R,\mathrm{I})\right)$.Hence $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(R,\mathrm{I})\right)$is connected.

The next few results provide a characterization for all Neutrosophic rings whose invertible graphs are complete.

Theorem3.4. The Neutrosophic invertible graph $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ is complete if and only if $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

Proof. Necessity. Suppose that $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(R,\mathrm{I})\right)$is complete. Then any two vertices $a+b\mathrm{I}$ and $c+d\mathrm{I}$ are adjacent in $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(R,\mathrm{I})\right)$. Consequently,

$(a+b\mathrm{I})+(c+d\mathrm{I}) \neq 0+0\mathrm{I} \Rightarrow 2(a+b\mathrm{I}) = 0$ and $2(c+d\mathrm{I}) = 0$

$$\Rightarrow a+b\mathrm{I},\ \mathrm{c}+d\mathrm{I} \in S\left(N(R, \mathrm{I})\right).$$

This implies that each and every element in $N(R, \mathrm{I})$ is an element of $S\left(N(R, \mathrm{I})\right)$. This shows that $N(R, \mathrm{I}) \subseteq S\left(N(R, \mathrm{I})\right)$. Further, by the Theorem [4.2] (Chalapathi & Kiran, 2017b), $S\left(N(R, \mathrm{I})\right)$ is a Neutrosophic subring of $N(R, \mathrm{I})$. So, $S\left(N(R, \mathrm{I})\right) \subseteq N(R, \mathrm{I})$. Hence, $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$.

Sufficient. Let $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$. Then we have to prove that $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ is complete. Suppose $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ is not complete. Then there exist at least two vertices $a'+b'\mathrm{I}$ and $c'+d'\mathrm{I}$ in $N(R, \mathrm{I})$ such that $(a'+b'\mathrm{I})+(c'+d'\mathrm{I})=0+0\mathrm{I}$. Therefore,

$a'+b'\mathrm{I} = -(c'+d'\mathrm{I}) \Rightarrow a'+b'\mathrm{I},\ c'+d'\mathrm{I} \in M\left(N(R, \mathrm{I})\right)$

$\Rightarrow a'+b'\mathrm{I},\ c'+d'\mathrm{I} \notin S\left(N(R, \mathrm{I})\right)$, by the Theorem [2.7]

$\Rightarrow S\left(N(R, \mathrm{I})\right) \neq N(R, \mathrm{I})$, this is a contradiction to our hypothesis, and hence $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ is complete.

Corollary3.5. The Neutrosophic invertible graph of $N(R, \mathrm{I})$ is complete if and only if $N(R, \mathrm{I})$ is a finite Neutrosophic Boolean ring.

Proof. In view of the Theorem [2.5] and Theorem [3.4], $N(R, \mathrm{I})$ is a Neutrosophic Boolean ring if and only if $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$ if and only if $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ is complete.

Corollary 3.6. For $n \geq 1$, $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(Z_2^n,\mathrm{I}))$ is complete.

Proof. Since $\mathrm{N}(Z_2^n, \mathrm{I})$ is a Neutrosophic Boolean ring with 2^{2n} elements; $(0, 0,..., 0)$, $(1, 0,..., 0)$,..., $(1, 1,..., 1)$, $(\mathrm{I}, 0,..., 0)$,..., $(\mathrm{I}, \mathrm{I},..., \mathrm{I})$. Clearly, it is the vertex set of the graph$\mathcal{I}_G(\mathrm{N}(Z_2^n, \mathrm{I}))$, and the sum of any two vertices in $\mathcal{I}_G(\mathrm{N}(Z_2^n, \mathrm{I}))$is non-zero. This implies that $S\left(N(Z_2{}^n, \mathrm{I})\right) = N(Z_2{}^n, \mathrm{I})$. So, by the Theorem [3.4], $\mathcal{I}_G(\mathrm{N}(Z_2^n, \mathrm{I}))$is complete.

Example3.7. By the definition of Neutrosophic ring, the Neutrosophic ring of Gaussian integers $N(Z_2[i], \mathrm{I})$ of modulo 2 is defined as $\{0, 1, i, 1+i, \mathrm{I}, i\mathrm{I}, (1+i)\mathrm{I}, 1+\mathrm{I}, i+\mathrm{I}, (1+i)+\mathrm{I}, (1+i)+i\mathrm{I}, 1+i\mathrm{I}, i+i\mathrm{I}, i+(1+i)\mathrm{I}, 1+(1+i)\mathrm{I}, (1+i)+(1+i)\mathrm{I}\}$. The Neutrosophic invertible graph of $N(Z_2[i], \mathrm{I})$ is a complete graph because $S\left(N(Z_2[i], \mathrm{I})\right) = N(Z_2[i], \mathrm{I})$, but it is not a Neutrosophic Boolean ring, since $(i+\mathrm{I})^2 \neq (i+\mathrm{I})$,where $i^2 = -1$ and $\mathrm{I}^2 = \mathrm{I}$.

The Example [3.7] explains that the completeness property of the Neutrosophic invertible graph depends on the $S\left(N(R, \mathrm{I})\right) = N(R, \mathrm{I})$, but not the Boolean property.

Theorem 3.8. The graph$\mathcal{I}_G(\mathrm{N}(R, \mathrm{I}))$ is not complete if and only if $S\left(N(R, \mathrm{I})\right) \neq N(R, \mathrm{I})$.

Proof. Follows from the Theorem [3.4].

Theorem3.9. Let p be an odd prime. Then, the Neutrosophic invertible graph of a Neutrosophic field of order p^{2n} is never complete.

Proof. Let $\pi(x)$ be an irreducible polynomial of degree n over the classical field Z_p. Then, the Neutrosophic field of order p^{2n} is isomorphic to $N\left(\frac{Z_p[x]}{\langle \pi(x) \rangle}, \mathrm{I}\right)$. Now to show that its invertible graph is never complete. For this let $u = \frac{p-1}{2}x + \frac{p-1}{2}x\mathrm{I}$, $v = \frac{p+1}{2}x + \frac{p+1}{2}x\mathrm{I}$ be two vertices in $N\left(\frac{Z_p[x]}{\langle \pi(x) \rangle}, \mathrm{I}\right)$, then clearly, $u + v = px + px\mathrm{I} \equiv 0 (\mathrm{mod}\, \mathrm{p})$ .This means that u and v are not adjacent. Hence the proof.

Again we recall that the result $4 \leq |N(R, \mathrm{I})| \leq |R|^2$ for each $|R| > 1$. So the immediate results ensures that the Neutrosophic invertible graph has at least one $3-$ cycle when $|N(R, \mathrm{I})| \geq 4$.

Theorem3.10. Let $|N(R, \mathrm{I})| \geq 4$. Then, $\mathcal{I}_G(\mathrm{N}(R, \mathrm{I}))$ has at least one cycle of length 3.

Proof. Let $N(R, \mathrm{I})$ be a finite Neutrosophic ring with $1 \neq 0$ and $|N(R, \mathrm{I})| = 4$. Then clearly $N(R, \mathrm{I}) \cong N(Z_2, \mathrm{I})$, and its invertible graph has a cycle $1 - \mathrm{I} - (1{+}\mathrm{I}) - 1$ of length 3 because

$1 \quad I \neq +0$, $I(+1+I) \neq 0$ and $(1+I) \quad 1 \neq +0$ so in this case the result is true. Now consider $|N(R, \mathrm{I})| > 4$. Then there exist the following two cases.

Case. (i) Suppose $S(N(R, \mathrm{I})) = N(R, \mathrm{I})$. Then, by the Theorem [3.4], the result is trivial.

Case. (ii) Suppose $S(N(R, \mathrm{I})) \neq N(R, \mathrm{I})$. There is at least one element $s + t\mathrm{I}$ in $S(N(R, \mathrm{I}))$ and $m + n\mathrm{I}$ in $M(N(R, \mathrm{I}))$ such that $(s + t\mathrm{I}) + (m + n\mathrm{I}) \neq 0$. It is clear that there is a cycle $0 - (s + t\mathrm{I}) - (m + n\mathrm{I}) - 0$ of length 3 in $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$.

In the area of graph theory, a simple graph G is bipartite if its vertex set $V(G)$ can be partitioned into two disjoint subsets V_1 and V_2 such that no vertices both in V_1 or both in V_2 are connected. In 1931, the Kőnig's theorem provided by KőnigDénes (Dénes, 1931), it describes the relation between bipartite graph and its odd cycles.

Theorem 3.11. A simple graph is bipartite if and only if it does not have an odd length cycle.

Now we are in a position to determine precisely when $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is bipartite or not. Note that $N(R, \mathrm{I}) \cong N(Z_2, \mathrm{I})$ if and only if the graph $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(Z_2, \mathrm{I}))$ is isomorphic to the complete graph K_4 of order 4. It is clear that the following result is hold in view of the Theorem [3.10].

Theorem3.12. Every Neutrosophic invertible graph is never a bipartite graph.

Already we proved that the graph $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is connected for any finite Neutrosophic ring $N(R, \mathrm{I})$. Therefore, $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ has a diameter. Now, we immediate compute the diameter of $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ for any $N(R, \mathrm{I})$ such that $4 \leq |N(R, \mathrm{I})| \leq |R|^2$.

Theorem 3.13. The diameter of $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is at most 2.

Proof. Let $N(R, \mathrm{I})$ be a finite Neutrosophic ring with unity 1 and indeterminacy I. Then we consider the following two cases for finding diameter of $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$. Note that,

$$\mathrm{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))) = \min \{d(u, v) : u, v \in N(R, \mathrm{I})\},$$

where $d(u, v)$ is the length of the shortest path between the vertices u and v.

Case. (i) Suppose $S(N(R, \mathrm{I})) = N(R, \mathrm{I})$. Then, by the Theorem [3.4], $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is complete, so in this case $\mathrm{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))) = 1$.

Case. (ii) Suppose $S(N(R, \mathrm{I})) \neq N(R, \mathrm{I})$. Then, by the Theorem [3.8], $\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))$ is never a complete graph. Therefore, $\mathrm{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))) \neq 1$. This implies that $\mathrm{diam}(\mathcal{J}_{\mathrm{G}}(\mathrm{N}(R, \mathrm{I}))) >$

1. So, there exist a path $(s+t\mathrm{I})-0-(m+n\mathrm{I})$ in $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$, which is smallest. Therefore, $d\left(s+t\mathrm{I},\ m+n\mathrm{I}\right)=2$, this implies that $\mathrm{diam}(\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I})))=2$.

From case (i) and (ii) we conclude that the diameter of $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ is at most 2 .

4.ISOMORPHIC PROPERTIES OF NEUTROSOPHIC INVERTIBLE GRAPHS

In this section, we compute an interrelation between classical rings, their Neutrosophic rings and their Neutrosophic invertible graphs. Refer the definitions of isomorphism of two classical rings, two Neutrosophic rings and two simple graphs from (Chalapathi &Kiran, (2017b).

Theorem 4.1. Let R and R' be two finite rings with unities. Then the following implications holds.

$$R\cong R'\Rightarrow N(R,\mathrm{I})\cong N(R',\mathrm{I})\Rightarrow \mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))\cong \mathcal{I}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I})).$$

Proof. The implication $R\cong R'\Rightarrow N(R,\mathrm{I})\cong N(R',\mathrm{I})$ follows from Theorem [2.4]. To complete the proof, it is enough to show that the second implication of the result. For any finite rings R and R' , suppose $N(R,\mathrm{I})\cong N(R',\mathrm{I})$. Then by the definition of Neutrosophic isomorphism, there exist a bijection f from $N(R,\mathrm{I})$ onto $N(R',\mathrm{I})$ such that $R\cong R'$ and $f(\mathrm{I})=\mathrm{I}$ where $\mathrm{I}^2=\mathrm{I}$.Now to show that $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))\cong \mathcal{I}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$. For this we define a map $\varphi:\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))\rightarrow \mathcal{I}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$as

$$\text{(i). } \varphi(a+b\mathrm{I})= f(a+b\mathrm{I}) \text{ and}$$

$$\text{(ii). } \varphi\left((a+b\mathrm{I},\ \mathrm{c+dI})\right)=\left(f(a+b\mathrm{I}),\ f(\mathrm{c+dI})\right).$$

Trivially, φ is a bijection since f is bijection. Further, we claim that each edge of $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ with end vertices $a+b\mathrm{I}$ and $c+d\mathrm{I}$ is mapped to an edge in $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$ with end vertices $f(a+b\mathrm{I})$ and $f(\mathrm{c+dI})$. So, we have

$$\left(a+b\mathrm{I},\ c+d\mathrm{I}\right)\in E\left(\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(R,\mathrm{I})\right)\right)\Leftrightarrow (a+b\mathrm{I})+(c+d\mathrm{I})\neq 0\Leftrightarrow \varphi\left((a+b\mathrm{I})+(c+d\mathrm{I})\right)\neq\varphi(0)$$

$$\Leftrightarrow \varphi\left((a+c)+(b+d)\mathrm{I}\right)\neq 0\Leftrightarrow f\left((a+c)+(b+d)\mathrm{I}\right)\neq 0\Leftrightarrow f((a+c))+f((b+d)\mathrm{I})\neq 0$$

$$\Leftrightarrow f(a)+f(c)+f(b)\mathrm{I}+f(d)\mathrm{I}\neq 0\Leftrightarrow\left(f(a)+f(b)\mathrm{I}\right)+\left(f(c)+f(d)\mathrm{I}\right)\neq 0$$

$$\Leftrightarrow f(a+b\mathrm{I})+f(c+d\mathrm{I})\neq 0\Leftrightarrow\left(f(a+b\mathrm{I}),\ f(c+d\mathrm{I})\right)\in E(\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))).$$

Similarly we can show that φ maps non-adjacent vertices in $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(R,\mathrm{I})\right)$to non-adjacent vertices in $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$. Thus, φ is a graph isomorphism from $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))$ onto $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$, and hence $\mathcal{I}_{\mathrm{G}}(\mathrm{N}(R,\mathrm{I}))\cong \mathcal{I}_{\mathrm{G}}(\mathrm{N}(R',\mathrm{I}))$.

By the Theorem [2.4], two classical rings are isomorphic, so their Neutrosophic rings are isomorphic and consequently their Neutrosophic invertible graphs are also isomorphic, but converse of these implication, in general, not true. The next results provide such a class. First we state the following results due to isomorphism of two simple graphs. The proof of the following results is essentially contained in Bondy and Murty (2008).

Theorem 4.2. Two simple graphs G and G' are isomorphic if and only if their complement graphs $\overline{G}$ and $\overline{G'}$.

Recall from Mullen and Panario (2013) that F_{p^n} is a field of order p^n and Z_{p^n} is a com mutative ring of order p^n, where p is a prime and $n>1$. Note that F_{p^n} is not isomorphic to Z_{p^n} because the characteristic of F_{p^n} is p and the characteristic of Z_{p^n} is p^n.

Theorem 4.3. Let $p > 2$ be a prime. Then the Neutrosophic invertible graphs of order p^{2n} are isomorphic.

Proof. For each odd prime p, we have $N(F_{p^n}, \mathrm{I})$ is a Neutrosophic field of modulo p. $N(Z_{p^n}, \mathrm{I})$ is a Neutrosophic commutative ring of modulo p^n, clearly these Neutrosophic rings not isomorphic. Now it remains to show that the graphs $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(F_{p^n},\mathrm{I})\right)$ and $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(Z_{p^n},\mathrm{I})\right)$ are isomorphic. For this we shall show that their complement graphs are isomorphic. By the definition of complement graph, $\overline{\mathcal{I}_{\mathrm{G}}}\left(\mathrm{N}(F_{p^n},\mathrm{I})\right) = \left|M\left(N(F_{p^n}, \mathrm{I})\right)\right| K_2 \cup \left|S\left(N(F_{p^n}, \mathrm{I})\right)\right| K_1$ $= \left(\dfrac{p^{2n}-1}{2}\right) K_2 \cup K_1 \cong \overline{\mathcal{I}_{\mathrm{G}}}\left(\mathrm{N}(Z_{p^n},\mathrm{I})\right)$, so due to Theorem [4.2], we get the required result.

Corollary 4.4. For each $n>1$, the Neutrosophic invertible graphs of order 2^{2n} are isomorphic.

Proof. Follows from $\mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(F_{2^n},\mathrm{I})\right) \cong \left|S\left(N(F_{2^n}, \mathrm{I})\right)\right| K_1 \cong 2^{2n}\ K_1 \cong N_{2n} \cong \mathcal{I}_{\mathrm{G}}\left(\mathrm{N}(Z_{2^n},\mathrm{I})\right)$, where N_{2n} is totally disconnected graph of order 2^{2n}. It is clear that $F_{2^n} \ncong Z_{2^n}$ and $N(F_{p^n}, \mathrm{I}) \ncong N(Z_{p^n}, \mathrm{I})$ but their Neutrosophic invertible graphs are isomorphic.

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