On Neutrosophic α-Supra Open Sets and Neutrosophic α-Supra Continuous Functions

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ABSTRACT

In this paper, we introduce and investigate a new class of sets and functions between supra topological spaces called neutrosophic α-supra open set and neutrosophic α-supra continuous function.

KEYWORDS AND PHRASES: Neutrosophic Supra topological spaces, Neutrosophic α-supra open set, Neutrosophic semi-supraopen set, Neutrosophic α-supraopen set, Neutrosophic pre-supraopen set.

1 INTRODUCTION AND PRELIMINARIES

Zadeh (1965) introduced the concept of a fuzzy set and since its advent invaded almost all branches of mathematics and proved to have applications in many fields such as information theory (Smets (1981)) and control theory (Sugeno (1985)). The theory of fuzzy topological space was introduced and developed by Chang (1968) and since then various notions in classical topology have been extended to fuzzy topological spaces. The idea of "intuitionistic fuzzy set" was first published by Atanassov (1983) and many works by the same author

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and his colleagues appeared in the literature (Atanassov (1986, 1988); Atanassov and Stoeva (1983)). Later, this concept was generalized to "intuitionistic L - fuzzy sets" by Atanassov and Stoeva (1984). Utilizing the notion of intuitionistic fuzzy sets, Coker (1997) introduced the notion of intuitionistic fuzzy topological spaces. In 1983, Mashhour et al. introduced the supra topological spaces and studied s-continuous functions and s*-continuous functions. In 1987, Abd El-Monsef et al. introduced the fuzzy supra topological spaces and studied fuzzy supra-continuous functions and obtained some properties and characterizations. In 1996, Keun Min introduced fuzzy s-continuous, fuzzy s-open and fuzzy s-closed maps and established a number of characterizations. In 2008, Devi et al. introduced the concept of supra α-open set, $s\alpha$-continuous functions and studied some of the basic properties for this class of functions. In 1999 and 2003, Turanl introduced the concept of intuitionistic fuzzy supra topological space. The concepts of neutrosophy and neutrosophic set are introduced by Smarandache (1999, 2000). Afterwards Salama and Alblowi (2012), introduced the concepts of neutrosophic crisp set and neutrosophic crisp topological spaces.

In this paper, we introduce and investigate a new class of sets and functions between supra topological spaces called neutrosophic α-supra open set and neutrosophic α-supra continuous functions.

Definition 1.1. (Salama and Alblowi (2012)) Let X be a nonempty fixed set. A neutrosophic set [NS for short] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

Remark 1.1. (Salama and Alblowi (2012))

(1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.2. (Salama and Alblowi (2012)) Let X be a nonempty set and the neutrosophic sets A and B in the form
$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$; [complement of A]

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\,]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\rangle A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.3. (Salama and Alblowi (2012)) Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

(a) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 1.4. (Salama and Alblowi (2012)) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 1.5. [10] A neutrosophic topology (NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (briefly NTS) and each neutrosophic set in T is called a neutrosophic open set (briefly NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (briefly NCS) in X. Each neutrosophic supra set (briefly, NS) which belongs to (X, T) is called a neutrosophic supra open set (briefly, NSOS) in X. The complement $\overline{A}$ of a NSOS A in X is called a neutrosophic supra closed set (briefly IFSCS) in X.

Definition 1.6. (Dhavaseelan, & Jafari (submitted) Let A be a neutrosophic set in a neutrosophic topological space X. Then

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Definition 1.7. (Dhavaseelan, & Jafari (submitted) Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point (briefly NP) in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

2 NEUTROSOPHIC α-SUPRA OPEN SETS

Definition 2.1. A family T of neutrosophic sets on X is called a neutrosophic supratopology (briefly NST) on X if $0_N \in T$, $1_N \in T$ and T is closed under arbitrary suprema. Then we call the pair (X, T) a neutrosophic supratopological space (briefly NSTS).

Each member of T is called a neutrosophic supraopen set and the complement of a neutrosophic supraopen set is called a neutrosophic supraclosed set. The neutrosophic supr-aclosure of a neutrosophic set A is denoted by s-$Ncl(A)$. Here s-$Ncl(A)$ is the intersection of all neutrosophic supraclosed sets containing A. The neutrosophic suprainterior of A will be denoted by s-$Nint(A)$. Here, s-$Nint(A)$ is the union of all neutrosophic supraopen sets contained in A.

Definition 2.2. Let A be a neutrosophic set in a neutrosophic supratopological space X is called

(a) neutrosophic semi-supraopen set iff $A \subseteq s - Ncl(s - Nint(A))$,

(b) neutrosophic α-supraopen set iff $A \subseteq s - Nint(s - Ncl(s - Nint(A)))$,

(c) neutrosophic pre-supraopen set iff $A \subseteq s - Nint(s - Ncl(A))$.

Definition 2.3. Let f be a function from an ordinary set X into an ordinary set Y. If $B = \{\langle y, \mu_B(y), \sigma_B(y), \gamma_B(y)\rangle : y \in Y\}$ is a neutrosophic supratopology in Y, then the inverse image of B under f is a neutrosophic supratopology defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

The image of neutrosophic supratopology $A = \{\langle y, \mu_A(y), \sigma_A(y), \gamma_A(y)\rangle : y \in Y\}$ under f is a neutrosophic supratopology defined by $f(A) = \{\langle y, f(\mu_A)(y), f(\sigma_A)(y), f(\gamma_A)(y)\rangle : y \in Y\}$.

Definition 2.4. Let (X, T) be a neutrosophic supra topological space. a neutrosophic set A is called a neutrosophic α-supra open set (briefly, NαSOS) if $A \subseteq s - Nint(s - Ncl(s - Nint(A)))$. The complement of a neutrosophic α-supra open set is called a neutrosophic α-supra closed set.

Theorem 2.1. Every neutrosophic supra open set is neutrosophic α-supra open.

Proof. Let A be a neutrosophic supra open set in (X, T). Since $A \subseteq s - Ncl(A)$, we get $A \subseteq s - Ncl(s - Nint(A))$. Then $s - Nint(A) \subseteq s - Nint(s - Ncl(s - Nint(A)))$. Hence $A \subseteq s - Nint(s - Ncl(s - Nint(A)))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.1. Let $X = \{a, b\}$. Define the neutrosophic sets A and B in X as follows:
$A = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5})\rangle$, $B = \langle x, (\frac{a}{0.4}, \frac{b}{0.2}), (\frac{a}{0.4}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.3})\rangle$.
We have $T = \{0_N, 1_N, A, B, A \cup B\}$. Let $C = \langle x, (\frac{a}{0.4}, \frac{b}{0.6}), (\frac{a}{0.4}, \frac{b}{0.6}), (\frac{a}{0.3}, \frac{b}{0.4})\rangle$. Then C is neutrosophic α-supra open but not neutrosophic supra open.

Theorem 2.2. Every neutrosophic α-supra open set is neutrosophic semi-supra open.

Proof. Let A be a neutrosophic α-supra open set in (X, T). Then, $A \subseteq s\text{-}Nint(s\text{-}Ncl(s\text{-}Nint(A)))$. It is obvious that $s\text{-}Nint(s\text{-}Ncl(s\text{-}Nint(A))) \subseteq s\text{-}Ncl(s\text{-}Nint(A))$. Hence $A \subseteq s\text{-}Ncl(s\text{-}Nint(A))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.2. Let $X = \{a, b\}$. Define the neutrosophic sets A and B in X as follows:
$A = \langle x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5})\rangle$, $B = \langle x, (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.4}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.4})\rangle$. We have $T = \{0_N, 1_N, A, B, A \cup B\}$. Let $C = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}), (\frac{a}{0.4}, \frac{b}{0.4}), (\frac{a}{0.5}, \frac{b}{0.4})\rangle$. Then C is neutrosophic semi-supra open but not neutrosophic α-supra open.

Theorem 2.3. Every neutrosophic α-supra open set is neutrosophic pre-supra open.

Proof. Let A be a neutrosophic α-supra open set in (X, T). Then, $A \subseteq s\text{-}Nint(s\text{-}Ncl(s\text{-}Nint(A)))$. It is obvious that $A \subseteq s\text{-}Nint(s\text{-}Ncl((A)))$. □

The converse of the above theorem need not be true as shown by the following example.

Example 2.3. In Example 2.2, let $C = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.4})\rangle$. Here C is neutrosophic pre-supra open but not neutrosophic α-supra open.

Theorem 2.4. (i) Arbitrary union of neutrosophic α-supra open sets is always neutrosophic α-supra open set.

(ii) Finite intersection of neutrosophic α-supra open sets may fail to be neutrosophic α-supra open set.

Proof. (i) Let $\{A_\lambda : \lambda \in \Lambda\}$ be a family of neutrosophic α-supra open set in a topological space X. Then for any $\lambda \in \Lambda$, we have $A_\lambda \subseteq s\text{-}Nint(s\text{-}Ncl(s\text{-}Nint(A_\lambda)))$. Hence $\cup_{\lambda\in\Lambda} A_\lambda \subseteq \cup_{\lambda\in\Lambda}(s\text{-}Nint(s\text{-}Ncl(s\text{-}Nint(A_\lambda)))) \subseteq s\text{-}Nint(\cup_{\lambda\in\Lambda}(s\text{-}Ncl(s\text{-}Nint(A_\lambda)))) \subseteq s\text{-}Nint(s\text{-}Ncl(s\text{-}Nint(\cup_{\lambda\in\Lambda} A_\lambda)))$. Therefore, $\cup_{\lambda\in\Lambda} A_\lambda$ is a neutrosophic α-supra open set.

(ii) Let $X=\{a,b\}$. Define the neutrosophic sets A and B in X as follows:
$A = \langle x,(\frac{a}{0.3},\frac{b}{0.4}),(\frac{a}{0.3},\frac{b}{0.4}),(\frac{a}{0.4},\frac{b}{0.5})\rangle$, $B = \langle x,(\frac{a}{0.2},\frac{b}{0.4}),(\frac{a}{0.2},\frac{b}{0.4}),(\frac{a}{0.5},\frac{b}{0.3})\rangle$ and $T = \{0_N,1_N,A,B,A\cup B\}$. Let $C = \langle x,(\frac{a}{0.4},\frac{b}{0.6}),(\frac{a}{0.4},\frac{b}{0.6}),(\frac{a}{0.3},\frac{b}{0.4})\rangle$. Here B and C are neutrosophic α-supra open but $B\cap C$ is not neutrosophic α-supra open.

□

Theorem 2.5. (i) Arbitrary intersection of neutrosophic α-supra closed sets is always neutrosophic α-supra closed set.

(ii) Finite union of neutrosophic α-supra closed sets may fail to be neutrosophic α-supra closed set.

Proof. (i) The proof follows immediately from Theorem 2.4

(ii) Let $X=\{a,b\}$. Define the neutrosophic sets A and B in X as follows:
$A=\langle x,(\frac{a}{0.3},\frac{b}{0.5}),(\frac{a}{0.3},\frac{b}{0.5}),(\frac{a}{0.6},\frac{b}{0.5})\rangle$, $B=\langle x,(\frac{a}{0.6},\frac{b}{0.3}),(\frac{a}{0.6},\frac{b}{0.3}),(\frac{a}{0.3},\frac{b}{0.4})\rangle$
and $T=\{0_N,1_N,A,B,A\cup B\}$. Let $C=\langle x,(\frac{a}{0.3},\frac{b}{0.5}),(\frac{a}{0.3},\frac{b}{0.5}),(\frac{a}{0.4},\frac{b}{0.5})\rangle$ and $D=\langle x,(\frac{a}{0.3},\frac{b}{0.4}),(\frac{a}{0.3},\frac{b}{0.4}),(\frac{a}{0.6},\frac{b}{0.2})\rangle$. Here C and D are neutrosophic α-supra closed but $C\cup D$ is not neutrosophic α-supra closed.

□

Definition 2.5. The neutrosophic α-supra-closure of a set A is denoted by
$\alpha s\text{-}Ncl(A)=\cup\{G : G$ is a NαSOS in X and $G\subseteq A\}$.
The neutrosophic α-supra-interior of a set A is denoted by $\alpha s\text{-}Nint(A)=\cap\{G : G$ is a NαSCS in X and $G\supseteq A\}$.

Remark 2.1. It is clear that $\alpha s\text{-}Nint(A)$ is a neutrosophic α-supra open set and $\alpha s\text{-}Ncl(A)$ is a neutrosophic α-supra closed set.

Theorem 2.6. Let X be a neutrosophic supratopological spaces. If A and B are two subsets of X, then

(i) $\overline{\alpha s - Nint(A)}=\alpha s\text{-}Ncl(\overline{A})$

(ii) $\overline{\alpha s - Ncl(A)}=\alpha s\text{-}Nint(\overline{A})$

(iii) If $A\subseteq B$, then $\alpha s\text{-}Ncl(A)\subseteq\alpha s\text{-}Ncl(B)$ and $\alpha s\text{-}Nint(A)\subseteq\alpha s\text{-}Nint(B)$

Proof. It is obvious. □

Theorem 2.7. Let X be a neutrosophic supratopological spaces. If A and B are two neutrosophic subsets of X, then

(i) $\alpha s\text{-}Nint(A)\cup\alpha s\text{-}Nint(B)\subseteq\alpha s\text{-}Nint(A\cup B)$

(ii) $\alpha s\text{-}Nint(A\cap B)\subseteq\alpha s\text{-}Nint(A)\cap\alpha s\text{-}Nint(B)$

(iii) If $A \subseteq B$, then $\alpha s\text{-}Ncl(A) \subseteq \alpha s\text{-}Ncl(B)$ and $\alpha s\text{-}Nint(A) \subseteq \alpha s\text{-}Nint(B)$

Proof. It is obvious. □

Theorem 2.8. (i) The intersection of a neutrosophic -supra open set and a neutrosophic α-supra open set is neutrosophic α-supra open.

(ii) The intersection of a neutrosophic α-supra open set and a neutrosophic pre-supra open set is neutrosophic pre-supra open.

Proof. It is obvious. □

3 NEUTROSOPHIC α-SUPRA CONTINUOUS FUNCTIONS

Definition 3.1. Let (X, T) and (Y, S) be two neutrosophic α-supra topological spaces. A map $f : (X, T) \to (Y, S)$ is called neutrosophic α-supra continuous function if the inverse image of each neutrosophic open set in Y is a neutrosophic α-supra open set in X.

Theorem 3.1. Every neutrosophic supra continuous function is neutrosophic α-supra continuous function.

Proof. Let $f : (X, T) \to (Y, S)$ be a neutrosophic supra continuous function and A is a neutrosophic open set in Y. Then $f^{-1}(A)$ is a neutrosophic open set in X. Therefore, $f^{-1}(A)$ is a neutrosophic supra open set in X which is a neutrosophic α supra open set in X. Hence f is a neutrosophic α-supra continuous function. □

Remark 3.1. Every neutrosophic α-supra continuous function need not be neutrosophic supra continuous function.

Theorem 3.2. Let (X, T) and (Y, S) be two neutrosophic supra topological spaces. Let f be a map from X into Y. Then the following are equivalent:

(i) f is a neutrosophic supra α-continuous function.

(ii) the inverse image of a closed sets in Y is a neutrosophic supra α-closed set in X.

(iii) $\alpha s\text{-}Ncl(f^{-1}(A)) \subseteq f^{-1}(Ncl(A))$ for every neutrosophic set A in Y.

(iv) $f(\alpha s\text{-}Ncl(A)) \subseteq Ncl(f(A))$ for every neutrosophic set A in X.

(v) $f^{-1}(Nint(B)) \subseteq \alpha s\text{-}Nint(f^{-1}(B))$ for every neutrosophic set B in Y.

Proof. $(i) \Rightarrow (ii)$: Let A be a neutrosophic closed set in Y, then $\overline{A}$ is neutrosophic open in Y. Thus, $f^{-1}(\overline{A}) = \overline{f^{-1}(A)}$ is neutrosophic αs-open in X. It follows that $f^{-1}(A)$ is a neutrosophic αs-closed set of X.
$(ii) \Rightarrow (iii)$: Let A be any subset of X. Since $Ncl(A)$ is closed in Y, then it follows that

$f^{-1}(Ncl(A))$ is neutrosophic αs-closed in X.
Therefore, $f^{-1}(Ncl(A)) = \alpha s\text{-}Ncl(f^{-1}(Ncl(A))) \supseteq \alpha s\text{-}Ncl(f^{-1}(A))$.
$(iii) \Rightarrow (iv)$: Let A be any neutrosophic subset of X. By (iii) we obtain, $f^{-1}(Ncl(f(A))) \supseteq \alpha s\text{-}Ncl(f^{-1}(f(A))) \supseteq \alpha s\text{-}Ncl(A)$ and hence $f(\alpha s\text{-}Ncl(A)) \subseteq Ncl(f(A))$.
$(iv) \Rightarrow (v)$: Let $f(\alpha s\text{-}Ncl(A)) \subseteq Ncl(f(A))$ for every neutrosophic set A in X.
Then $\alpha s\text{-}Ncl(A) \subseteq f^{-1}(Ncl(f(A)))$, $X - \alpha s\text{-}Ncl(A) \supseteq \overline{f^{-1}(Ncl(f(A)))}$ and $\alpha s\text{-}Nint(\overline{A}) \supseteq f^{-1}(Nint(\overline{f(A)}))$. Then $\alpha s\text{-}Nint(f^{-1}(B)) \supseteq f^{-1}(Nint(B))$. Therefore $f^{-1}(Nint(B)) \subseteq s\text{-}Nint(f^{-1}(B))$, for every B in Y.
$(v) \Rightarrow (i)$: Let A be a neutrosophic open set in Y. Therefore, $f^{-1}(Nint(A)) \subseteq \alpha s\text{-}Nint(f^{-1}(A))$, hence $f^{-1}(A) \subseteq \alpha s\text{-}Nint(f^{-1}(A))$. But by other hand, we know that, $\alpha s\text{-}Nint(f^{-1}(A)) \subseteq f^{-1}(A)$. Then $f^{-1}(A) = \alpha s\text{-}Nint(f^{-1}(A))$. Therefore, $f^{-1}(A)$ is a neutrosophic αs-open set.

□

Theorem 3.3. If a function $f : (X, T) \to (Y, S)$ is neutrosophic αs-continuous and $g : (Y, S) \to (Z, R)$ is continuous, then $(g \circ f)$ is αs-continuous.

Proof. Obvious. □

Theorem 3.4. Let $f : (X, T) \to (Y, S)$ be a neutrosophic αs-continuous function, if one of the following holds:

(i) $f^{-1}(\alpha s\text{-}Nint(A)) \subseteq Nint(f^{-1}(A))$ for every neutrosophic set A in Y.

(ii) $Ncl(f^{-1}(A)) \subseteq f^{-1}(\alpha s\text{-}Ncl(A))$ for every neutrosophic set A in Y.

(iii) $f(Ncl(B)) \subseteq \alpha s\text{-}Ncl(f(B))$ for every neutrosophic set B in X.

Proof. Let A be any open set of Y. If condition (i) is satisfied, then $f^{-1}(\alpha s\text{-}Nint(A)) \subseteq Nint(f^{-1}(A))$. We have $f^{-1}(A) \subseteq Nint(f^{-1}(A))$. Therefore $f^{-1}(A)$ is a neutrosophic supra open set. Every neutrosophic supra open set is neutrosophic supra α-open set. Hence f is a neutrosophic αs-continuous function. If condition (ii) is satisfied, then we can easily prove that f is a neutrosophic αs-continuous function. If condition (iii) is satisfied, and A is any open set of Y. Then $f^{-1}(A)$ is a set in X and $f(Ncl(f^{-1}(A))) \subseteq \alpha s\text{-}Ncl(f(f^{-1}(A)))$. This implies $f(Ncl(f^{-1}(A))) \subseteq \alpha s\text{-}Ncl(A)$. This is nothing but condition (ii). Hence f is a neutrosophic αs-continuous function. □

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