Single Valued Neutrosophic Finite State Machine and Switchboard State Machine

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ABSTRACT

Using single valued neutrosophic set we introduced the notion of single valued neutrosophic finite state machine, single valued neutrosophic successor, single valued neutrosophic subsystem and single valued submachine, single valued neutrosophic switchboard state machine, homomorphism and strong homomorphism between single valued neutrosophic switchboard state machine and discussed some related results and properties.

KEYWORDS: Single valued neutrosophic set, single valued neutrosophic state machine, single valued neutrosophic switchboard state machine, homomorphism and strong homomorphism.

1. INTRODUCTION

Fuzzy set was introduced by Zadeh (1965) which is the generalization of mathematical logic. Fuzzy set is a new mathematical tool to describe the uncertainty. There was so many generalizations of fuzzy set namely interval valued fuzzy set (Turksen, 1986), intuitionistic fuzzy set (Atanassov, 1986, 1989), vague set (Gau, & Buehrer, 1993) etc. Interval valued fuzzy was introduced by Turksons in 1986. Intuitionistic fuzzy set was introduced by Attanasov in 1986. Intuitionistic fuzzy set was the generalization of Zadeh fuzzy set and is provably equivalent to interval valued fuzzy where the lower bound of the interval is called membership degree and upper bound of the interval is non-membership degree. The concept of vague set was given by Gua and Buehrer. Butillo and Bustince show that vague set are intuitionistic fuzzy set (Bustince, & Burillo, 1996). Bipolar fuzzy set was introduced by W. R. Zhang (1998). Jun et al. (2012) introduced the concept of cubic set. Cubic set is an ordered pair of interval-valued fuzzy set and fuzzy set. These all are mathematical modeling to solve the problems in our daily life. These tools have its own inherent problems to solve these types of uncertainty while the cubic set is more informative tool to solve this uncertainty. After the introduction of all fuzzy set extensions Florentin Smarandache (Smarandache, 1998, 1999) introduced the concept of neutrosophy and neutrosophic sets which was the generalization of fuzzy sets, intuitionistic fuzzy sets, interval valued fuzzy set and all extensions of fuzzy sets defined above. The words "neutrosophy" Etymologically, "neutro-sophy" (noun) comes from French neuter Latin neuter, neutral, and Greek sophia, skill/wisdom means knowledge of neutral thought. Neutrosophy is a branch of philosophy introduced by which studies the origin and scope of neutralities, as well as thier interaction with ideational spectra. This theory considers every notion or idea <A> together with its opposite or negation <anti A> and with their spectrum of neutralities <neut A> in between them (i.e. notions or ideas supporting neither < A> nor <anti A>). The <neutA> and <anti A> ideas together are referred to as <nonA>. Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). While a "neutrosophic" (adjective), means having the nature of, or having the characteristic of Neutrosophy. A neutrosophic set A is characterized by a truth membership function T_A, Indeterminacy membership function I_A and Falsity membership function F_A. Where $T_{A,} I_A$ and F_A are real standard and nonstandard subsets of $]^{-}0,1^{+}[$. The neutrosophic sets is suitable for real life problem, but it is difficult to apply in scientific problems. The difference between neutrosophic sets and intuitionistic fuzzy sets is that in neutrosophic sets the degree of indeterminacy is defined independently. To apply neutrosophic set in real life and in scientific problems Wang et al. introduced the concept of single valued neutrosophic set and interval neutrosophic set (Wang et al., 2005, 2010) which are subclasses of neutrosophic set. In which membership function, indeterminacy membership function, falsity membership was taken in the closed interval [0, 1] rather than the nonstandard unit interval. Malik et al. (1994a. 1994b, 1994c, 1997) given the concept and notion of fuzzy finite state machine, submachine of fuzzy finite state machine, subsystem of fuzzy finite state machine, product of fuzzy finite state machine and discussed some related properties. Kumbhojkar & Chaudhari (2002) introduced covering of fuzzy finite state machine. Sato & Kuroki (2002) introduced fuzzy finite switchboard state machine. Jun (2005) generalized the concept of malik et al. (1994a, 1994b, 1994c, 1997) and introduced the concept of intuitionistic fuzzy finite state machine, submachines of intuitionistic fuzzy finite state machine (2006), intuitionistic successor and discussed some related properties (Jun, 2005). Jun (2006) introduced the concept of intuitionistic fuzzy finite switchboard state machine, commutative intuitionistic fuzzy finite state machine and strong homomorphism (Jun, 2006). Jun & Kavikumar (2011) also introduced the concept of bipolar fuzzy finite state machine.

The paper is arranged as follows, section 2 contains preliminaries, section 3 contains the main result single valued neutrosophic finite state machine and related results, section 4 contained Single valued finite switchboard state machine homomorphism, strong homomorphism and related properties. At the end conclusion and references are given.

Section 2. PRELIMINARIES

For basic definition and results the reader should refer to study [10-13, 18].

Definition 1: (Malik et al., 1994a)

A fuzzy finite state machine is a triple $F=(M,U,\lambda)$. Where M and U are finite non-empty sets called the set of states and the set of input symbols respectively, λ is a fuzzy function in $M\times U\times M$ into [0, 1].

Definition 2: (Jun, 2005)

An intuitionistic fuzzy finite state machine is a triple $F=(M,U,C)$. Where M and U are finite non-empty sets called the set of states and the set of input symbols respectively, $C=(\tau_C,\upsilon_C)$ is an intuitionistic fuzzy set in $M\times U\times M$ into [0, 1].

Definition 3: (Malik et al., 1994b)

Let $F=(M,U,\lambda)$ be a fuzzy finite state machine and $r,s\in M$. Then r is called an immediate successor of $s\in M$ if there exists $x\in U$ such that $\lambda(s,x,r)>0$. We say that r is called fuzzy successor of s , if there exists $\lambda^{*}(s,x,r)>0$.

Definition 4: (Wang et al., 2010)

A single valued neutrosophic set N in X is an object of the form

$$N=\{(\chi_N(x),\psi_N(x),\omega_N(x))\forall x\in X\}.$$

Where $\chi_N(x), \psi_N(x), \omega_N(x)$ are functions from X into $[0,1]$.

3. SINGLE VALUED NEUTROSOPHIC FINITE STATE MACHINE

Definition 5. A triple $F=(M,U,N)$ is called single valued neutrosophic finite state machine (SVNFSM) for short), where M and U are finite sets. The elements of M is called states and the elements of U is called input symbols. Where is N is a single valued neutrosophic set in $M \times U \times M$. Let the set of all words of finite length of the elements of U is denoted by U^*. The empty word in U^* is donated by Λ and $|a|$ denote the length of a for every $a \in U^*$.

Definition 6. Let $F=(M,U,N)$ be a SVNFSM. Define a SVNS $N^*=(\chi_{N^*}, \psi_{N^*}, \omega_{N^*})$ in $M \times U^* \times M$ by

$$\chi_{N^*}(u,\Lambda,v)=\begin{cases}1 & if \quad u=v\\ 0 & if \quad u\neq v\end{cases}$$

$$\psi_{N^*}(u,\Lambda,v)=\begin{cases}0 & if \quad u=v\\ 1 & if \quad u\neq v\end{cases}$$

$$\omega_{N^*}(u,\Lambda,v)=\begin{cases}0 & if \quad u=v\\ 1 & if \quad u\neq v\end{cases}$$

$$\chi_{N^*}(u,ab,v)=\vee_{w\varepsilon M}\left\lfloor \chi_{N^*}(u,a,w)\wedge \chi_N(w,b,v)\right\rfloor$$

$$\psi_{N^*}(u,ab,v)=\wedge_{w\varepsilon M}\left\lfloor \chi_{N^*}(u,a,w)\vee \chi_N(w,b,v)\right\rfloor$$

$$\omega_{N^*}(u,ab,v)=\wedge_{w\varepsilon M}\left\lfloor \chi_{N^*}(u,a,w)\vee \chi_N(w,b,v)\right\rfloor$$

for all $u,v \in M$ and $a \in U^*$ and $b \in U$.

Lemma 1. Let $F=(M,U,N)$ be SVNFSM. Then

$$\chi_{N^*}(u,ab,v)=\vee_{w\varepsilon M}\left\lfloor \chi_{N^*}(u,a,w)\wedge \chi_{N^*}(w,b,v)\right\rfloor$$

$$\psi_{N^*}(u,ab,v)=\wedge_{w\varepsilon M}\left\lfloor \chi_{N^*}(u,a,w)\vee \chi_{N^*}(w,b,v)\right\rfloor$$

$$\omega_{N^*}(u,ab,v)=\wedge_{w\varepsilon M}\left\lfloor \chi_{N^*}(u,a,w)\ \vee \chi_{N^*}(w,b,v)\right\rfloor$$

for all $u,v \in M$ and $a,b \in U^*$.

Proof. Let $u,v \in M$ and $a,b \in U^*$. Suppose $|b|=n$. We prove the result by induction. If $n=0$, then $b=\Lambda$ and so $ab=a\Lambda=a$.

$$\vee_{w\varepsilon M}\left\lfloor \chi_{N^*}(u,a,w)\wedge \chi_{N^*}(w,b,v)\right\rfloor$$

$$\wedge_{w\varepsilon M}\left\lfloor \chi_{N^*}(u,a,w)\wedge \chi_{N^*}(w,\Lambda,v)\right\rfloor$$

$$\chi_{N^*}(u,a,v)=\chi_{N^*}(u,ab,v)$$

and

$$\wedge_{w\varepsilon M}\left\lfloor \psi_{N^*}(u,a,w)\vee \psi_{N^*}(w,b,v)\right\rfloor$$

$$\wedge_{w\varepsilon M}\left\lfloor \psi_{N^*}(u,a,w)\vee \psi_{N^*}(w,\Lambda,v)\right\rfloor$$

$$\psi_{N^*}(u,a,v)=\psi_{N^*}(u,ab,v)$$

and

$$\wedge_{w\varepsilon M}\lfloor \omega_{N^*}(u,a,w) \ \vee \omega_{N^*}(w,b,v)\rfloor$$
$$\wedge_{w\varepsilon M}\lfloor \omega_{N^*}(u,a,w) \ \vee \omega_{N^*}(w,\Lambda,v)\rfloor$$
$$\omega_{N^*}(u,a,v)=\omega_{N^*}(u,ab,v)$$

Hence the result is true for $n=0$. Let us assume that the result is true for all $c\in U^*$ such that $|c|=n-1, n>0$. Then $b=cd$, where $c\in U^*$ and $d\in U, |c|=n-1, n>0$. Then

$$\chi_{N^*}(u,ab,v)=\chi_{N^*}(u,acd,v)=\vee_{w\varepsilon M}\lfloor \chi_{N^*}(u,ac,w)\wedge \chi_N(w,d,v)\rfloor$$
$$=\vee_{w\varepsilon M}\lfloor \vee_{z\varepsilon M}\left(\chi_{N^*}(u,a,z) \ \wedge \chi_{N^*}(z,c,w)\right)\wedge \chi_N(w,d,v)\rfloor$$
$$=\vee_{w,z\varepsilon M}\lfloor \chi_{N^*}(u,a,z) \ \wedge \chi_{N^*}(z,c,w)\wedge \chi_N(w,d,v)\rfloor$$
$$=\vee_{z\varepsilon M}\lfloor \chi_{N^*}(u,a,z) \ \wedge (\vee_{w\varepsilon M}\left(\chi_{N^*}(z,c,w)\wedge \chi_N(w,d,v)\right)\rfloor$$
$$=\vee_{z\varepsilon M}\lfloor \chi_{N^*}(u,a,z)\wedge \chi_{N^*}(z,cd,v)\rfloor$$
$$=\vee_{z\varepsilon M}\lfloor \chi_{N^*}(u,a,z)\wedge \chi_{N^*}(z,b,v)\rfloor$$

and

$$\psi_{N^*}(u,ab,v)=\psi_{N^*}(u,acd,v)=\wedge_{w\varepsilon M}\lfloor \psi_{N^*}(u,ac,w)\vee \psi_N(w,d,v)\rfloor$$
$$=\wedge_{w\varepsilon M}\lfloor \wedge_{z\varepsilon M}\left(\psi_{N^*}(u,a,z)\vee \psi_{N^*}(z,c,w)\right)\vee \psi_N(w,d,v)\rfloor$$
$$=\wedge_{w,z\varepsilon M}\lfloor \psi_{N^*}(u,a,z)\vee \psi_{N^*}(z,c,w)\vee \psi_N(w,d,v)\rfloor$$
$$=\wedge_{z\varepsilon M}\lfloor \psi_{N^*}(u,a,z)\vee (\vee_{w\varepsilon M}\psi_{N^*}(z,c,w)\vee \psi_N(w,d,v))\rfloor$$
$$=\wedge_{z\varepsilon M}\lfloor \psi_{N^*}(u,a,z)\vee \psi_{N^*}(z,cd,v)\rfloor$$
$$=\wedge_{z\varepsilon M}\lfloor \psi_{N^*}(u,a,z)\vee \psi_{N^*}(z,b,v)\rfloor$$

and

$$\omega_{N^*}(u,ab,v)=\omega_{N^*}(u,acd,v)=\wedge_{w\varepsilon M}\lfloor \omega_{N^*}(u,ac,w) \ \vee \omega_N(w,d,v)\rfloor$$
$$=\wedge_{w\varepsilon M}\lfloor \wedge_{z\varepsilon M}\left(\omega_{N^*}(u,a,z) \ \vee \omega_{N^*}(z,c,w)\right)\vee \omega_N(w,d,v)\rfloor$$
$$=\wedge_{w,z\varepsilon M}\lfloor \omega_{N^*}(u,a,z) \ \vee \omega_{N^*}(z,c,w)\vee \omega_N(w,d,v)\rfloor$$
$$=\wedge_{z\varepsilon M}\lfloor \omega_{N^*}(u,a,z) \ \vee (\wedge_{w\varepsilon M}(\omega_{N^*}(z,c,w)\vee \omega_N(w,d,v))\rfloor$$
$$=\wedge_{z\varepsilon M}\lfloor \omega_{N^*}(u,a,z) \ \vee \omega_{N^*}(z,cd,v)\rfloor$$
$$=\wedge_{z\varepsilon M}\lfloor \omega_{N^*}(u,a,z) \ \vee \omega_{N^*}(z,b,v)\rfloor$$

Therefore, the result is true for $|b|=n, n>0$.

Definition 7. Let $F=(M,U,N)$ be a SVNFSM and $u,v\in M$. Then v is called single valued neutrosophic immediate successor of u if there exists $x \in U$ such that $\chi_N(u,x,v) > 0, \psi_N(u,x,v) < 1$ and $\omega_N(u,x,v) < 1.$ We say that v is called single valued neutrosophic successor of u if there exists $x \in U$ such that $\chi_{N^*}(u,x,v) > 0, \psi^*_N(u,x,v) < 1$ and $\omega_{N^*}(u,x,v) < 1.$ The set of all single valued neutrosophic successor of u is denoted by $SVNS(u)$. The set of all single valued neutrosophic successor of N is denote by

$$SVNS(N) = \cup\{SNS(u) \mid u \in N\}$$

where N is any subset of M.

Proposition 1. Let $F = (M, U, N)$ be a SVNFSM. For any $u, v \in M$, the following hold:

(i) $u \in SVNS(u)$

(ii) if $u \in SVNS(v)$ and $w \in SVNS(u)$, then $r \in SVNS(v)$.

Proof. (i) Since $\chi_{N^*}(u,\Lambda,u) = 1 > 0$, $\psi_{N^*}(u,\Lambda,u) = 0 < 1$ and $\omega_{N^*}(u,\Lambda,u) = 0 < 1$

(ii) Let $v \in SVNS(u)$ and $w \in SVNS(v)$. Then there exists $a, b \in U^?$ such that $\chi_{N^*}(u,a,v) > 0, \psi_{N^*}(u,a,v) < 1,$ and $\omega_{N^*}(u,a,v) < 1, \chi_{N^*}(v,b,w) > 0, \psi_{N^*}(v,b,w) < 1$ and $\omega_{N^*}(v,b,w) < 1.$ Using lemma (1), we have

$$\begin{aligned}\chi_{N^*}(u,ab,w) &= \vee_{z\varepsilon M}\left\lfloor \chi_{N^*}(u,a,z) \wedge \chi_{N^*}(z,b,w)\right\rfloor \\ &\geq \chi_{N^*}(u,a,v) \wedge \chi_{N^*}(v,b,w) > 0\end{aligned}$$

And

$$\begin{aligned}\psi_{N^*}(u,ab,w) &= \wedge_{z\varepsilon M}\left\lfloor \psi_{N^*}(u,a,z) \vee \psi_{N^*}(z,b,w)\right\rfloor \\ &\leq \psi_{N^*}(u,a,v) \vee \psi_{N^*}(v,b,w) < 1\end{aligned}$$

and

$$\begin{aligned}\omega_{N^*}(u,ab,w) &= \wedge_{z\varepsilon M}\left\lfloor \omega_{N^*}(u,a,z) \ \vee \omega_{N^*}(z,b,w)\right\rfloor \\ &\leq \omega_{N^*}(u,a,v) \ \vee \omega_{N^*}(v,b,w) < 1\end{aligned}$$

Hence $w \in SVNS(v).$

Proposition 2. Let $F = (M, U, N)$ be SVNFSM. For any subsets C and D the following assertions hold.

(i) If $C \subseteq D,$ then $SVNS(C) \subseteq SVNS(D).$

(ii) $C \subseteq SVNS(C).$

(iii) $SVNS(SVNS(C)) = SVNS(C).$

(iv) $SVNS(C \cup D) = SVNS(C) \cup SVNS(D).$

(v) $SVNS(C \cap D) \subseteq SVNS(C) \cap SVNS(D)$

Proof. The proofs of (i),(ii),(iv), and (v) are simple and straightforward.

(iii) Obviously $SVNS(C) \subseteq SVNS(SVNS(C)).$ If $u \in SVNS(SVNS(C)),$ then $u \in SVNS(v)$ for some $v \in SVNS(C)$. From $v \in SVNS(C)$, there exists $w \in C$ such that $v \in SVNS(w)$. it follows from proposition (1) that $u \in SVNS(w) \subseteq SVNS(C)$ so that $SVNS(SVNS(C)) \subseteq SVNS(C)$. Hence (iii) is valid.

Definition 8. Let $F = (M, U, N)$ be SVNFSM. We say that ? satisfies the single valued neutrosophic exchange property if , for all $u, v \in M$ and $G \subseteq M$, whenever $v \in SVNS(G \cup \{u\})$ and $v \notin SVNS(G)$ then $u \in SVNS(G \cup \{v\})$.

Theorem 1. Let $F = (M, U, N)$ be a SVNFSM. Then the following assertions are equivalent.

(i) F satisfies the single valued neutrosophic exchange property.

(ii) $(\text{for all } u, v \in M)(v \in SVNS(u)) \Leftrightarrow u \in VNS(v)$.

Proof. Suppose that ? satisfies the Single valued neutrosophic exchange property. Let $u, v \in M$ be such that $v \in SVNS(u) = SVNS(\varphi \cup \{u\})$. Note that $v \notin SVNS(\varphi)$ and so $u \in SVNS(\varphi \cup \{v\}) = SVNS(v)$. Similarly if $u \in SVNS(v)$ then $v \in SVNS(u)$. Conversely assume that (ii) is valid. Let $u, v \in M$ and $G \subseteq M$. If $v \in SVNS(G \cup \{u\})$, then $v \in SVNS(u)$. It follows from (ii) that

$$u \in SVNS(v) \subseteq SVNS(G \cup \{v\}).$$

Therefore ? satisfies the single valued exchange property.

Definition 9. Let $F = (M, U, N)$ be a SVNFSM. Let $M^* = (\chi_{M^*}, \psi_{M^*}, \omega_{M^*})$ be a single valued neutrosophic set in M. Then (M, M^*, U, N) is called single valued neutrosophic submachine of F if for all $u, v \in M$ and $x \in U$,

$$\chi_{M^*}(u) \geq \chi_{M^*}(v) \wedge \chi_N(v, x, u),$$

$$\psi_{M^*}(u) \leq \psi_{M^*}(v) \vee \psi_N(v, x, u),$$

$$\omega_{M^*}(u) \leq \omega_{M^*}(v) \vee \omega_N(v, x, u)$$

Example 1. Let $M = \{u, v\}$, $U = \{x\}$, $\chi_N(u, x, v) = 0.75, \psi_N(u, x, v)$ and $\omega_N(u, x, v) = 0.5$ for all $u, v \in M$. Let $M^* = (\chi_{M^*}, \psi_{M^*}, \omega_{M^*})$ be given by $\chi_{M^*}(u) = 0.5 = \psi_{M^*}(u)$, $\omega_{M^?}(u) = 0.15$. Then

$$\chi_{M^*}(u) \wedge \chi_N(u, x, v) = 0.5 \wedge 0.75 = 0.5 = \chi_{M^*}(v)$$

$$\psi_{M^*}(u) \wedge \psi_N(u, x, v) = 0.5 \vee 0.75 = 0.75 > \psi_{M^*}(v)$$

$$\omega_{M^*}(u) \vee \omega_N(u, x, v) = 0.15 \vee 0.5 = 0.5 > \omega_{M^*}(v)$$

Therefore M^* is a single valued neutrosophic subsystem.

Theorem 2. Let $F = (M, U, N)$ be a SVNFSM and let $M^* = (\chi_{M^*}, \psi_{M^*}, \omega_{M^*})$ be a single valued neutrosophic set in M. Then M^* is a single valued neutrosophic subsystem of M iff

$$\chi_{M^?}(u) \geq \chi_{M^?}(v) \wedge \chi_{N^?}(v, x, u)$$

$$\psi_{M^?}(u) \leq \psi_{M^?}(v) \vee \psi_{N^?}(v, x, u)$$

$$\omega_{M^?}(u) \le \omega_{M^?}(v) \vee \omega_{N^?}(v,x,u)$$

for all $u,v \in M$ and $x \in M^?$.

Proof. Let us assume that $M^?$ is a single valued neutrosophic subsystem of F. Let $u,v \in M$ and $x \in M^*$. We prove the result by induction on $|x| = n$. If $n = 0$, we have $x = \Lambda$. Now if $v = u$, then

$$\chi_{M^*}(u) \wedge \chi_{N^*}(u,\Lambda,u) = \chi_{M^*}(u)$$

$$\psi_{M^*}(u) \vee \psi_{N^*}(u,\Lambda,u) = \psi_{M^*}(u)$$

and

$$\omega_{M^*}(u) \vee \omega_{N^*}(u,\Lambda,u) = \omega_{M^*}(u)$$

If $u \ne v$, then

$$\chi_{M^*}(v) \wedge \chi_{N^*}(v,x,u) = 0 \le \chi_{M^*}(u)$$

$$\psi_{M^*}(v) \vee \psi_{N^*}(v,x,u) = 1 \ge \psi_{M^*}(u)$$

and

$$\omega_{M^*}(u) \vee \omega_{N^*}(u,\Lambda,u) = 1 \ge \omega_{M^*}(u)$$

Hence for $n = 0$ the result is true. Now let us assume that the result is true for all $b \in M^*$ with $|b| = n-1, n > 0$. Let $x = bc$ with $c \in M$. Then

$$\chi_{M^*}(v) \wedge \chi_{N^*}(v,x,u) = \chi_{M^*}(v) \wedge \chi_{N^*}(v,bc,u)$$

$$= \chi_{M^*}(v) \wedge \left(\vee_{w\varepsilon M} \left[\chi_{N^*}(v,b,w) \ \wedge \chi_N(w,c,u) \right] \right)$$

$$= \vee_{w\varepsilon M} \left(\chi_{M^*}(v) \wedge \chi_{N^*}(v,b,w) \ \wedge \chi_N(w,c,u) \right)$$

$$\le \vee_{w\varepsilon M} [\chi_{M^*}(w) \wedge \chi_N(w,c,u)] \le \chi_{M^*}(v)$$

and

$$\psi_{M^*}(v) \vee \psi_{N^*}(v,x,u) = \psi_{M^*}(v) \vee \psi_{N^*}(v,bc,u)$$

$$= \psi_{M^*}(v) \vee \left(\wedge_{w\varepsilon M} \left[\psi_{N^*}(v,b,w) \vee \psi_N(w,c,u) \right] \right)$$

$$= \wedge_{w\varepsilon M} \left(\psi_{M^*}(v) \vee \psi_{N^*}(v,b,w) \vee \psi_N(w,c,u) \right)$$

$$\ge \wedge_{w\varepsilon M} [\psi_{M^*}(w) \vee \psi_N(w,c,u)] \ge \psi_{M^*}(v)$$

and

$$\omega_{M^*}(v) \vee \omega_{N^*}(v,x,u) = \omega_{M^*}(v) \vee \omega_{N^*}(v,bc,u)$$

$$= \omega_{M^*}(v) \vee \left(\wedge_{w\varepsilon M} \left[\omega_{N^*}(v,b,w) \ \vee \omega_N(w,c,u) \right] \right)$$

$$= \wedge_{w\varepsilon M} \left(\omega_{M^*}(v) \vee \omega_{N^*}(v,b,w) \ \vee \omega_N(w,c,u) \right)$$

$$\ge \wedge_{w\varepsilon M} [\omega_{M^*}(w) \vee \omega_N(w,c,u)] \ge \omega_{M^*}(v)$$

the converse of the above theorem is trivial.

Definition 10. Let $F=(M,U,N)$ be a SVNFSM. Let $G\subseteq M$. Let $C=(\chi_C,\psi_C,\omega_C)$ be a single valued neutrosophic set in $G\times U\times G$ and let $\square=(G,U,C)$ be a SVNFSM. Then $\square$ is called single valued neutrosophic submachine of F, if

(i) $N_{|G\times U\times G}=C$ that is $\chi_{N_{|G\times U\times G}}=\chi_C$, $\psi_{N_{|G\times U\times G}}=\psi_C$ and $\omega_{N_{|G\times U\times G}}=\omega_C$

(ii) $SVNS(G)\subseteq G$

We assume that $\varphi=(\varphi,U,C)$ is a single valued neutrosophic submachine of F. Obviously, if ξ is a single valued neutrosophic submachine of $\square$ and $\square$ is single valued submachine of F, Then ξ is a single valued submachine of F.

Definition 11. Let $F=(M,U,N)$ be a SVNFSM. Then it is said to be strongly single valued neutrosophic connected if $v\in SVNS(u)$ for every $u,v\in M$. A single valued neutrosophic submachine $\square=(G,U,C)$ of a $SVNfsm$ is said to be proper if $\square\neq\varphi$ and $\square\neq M$.

Theorem 3. Let $F=(M,U,N)$ be a SVNFSM and let $\square_i=(G_i,U,C_i), i\in I$, be a family of single valued neutrosophic submachine of F. Then we have the following,

(i) $\bigcap_{i\in I}\square_i=(\bigcap_{i\in I}G_i,U,\bigcap_{i\in I}C_i)$ is a single valued neutrosophic submachine of F.

(ii) $\bigcup_{i\in I}\square_i=(\bigcup_{i\in I}G_i,U,D)$ is a single valued neutrosophic submachine of F , where

$D=(\chi_D,\psi_D,\omega_D)$ is given by

$\chi_D=\chi_{N_{|\cup_{i\in I}G\times U\times\cup_{i\in I}G}}$, $\psi_D=\psi_{N_{|\cup_{i\in I}G\times U\times\cup_{i\in I}G}}$ and $\omega_D=\omega_{N_{|\cup_{i\in I}G\times U\times\cup_{i\in I}G}}$

Proof (i) Let $(u,a,v)\in\bigcap_{i\in I}G_i\times U\times\bigcap_{i\in I}G_i$. Then,

$$(\wedge_{i\in I}\chi_{C_i})(u,a,v)=\wedge_{i\in I}\chi_{C_i}(u,a,v)=\wedge_{i\in I}\chi_N(u,a,v)=\chi_N(u,a,v)$$

$$(\vee_{i\in I}\psi_{C_i})(u,a,v)=\vee_{i\in I}\psi_{C_i}(u,a,v)=\vee_{i\in I}\psi_N(u,a,v)=\psi_N(u,a,v)$$

and

$$(\vee_{i\in I}\omega_{C_i})(u,a,v)=\vee_{i\in I}\omega_{C_i}(u,a,v)=\vee_{i\in I}\omega_N(u,a,v)=\omega_N(u,a,v)$$

Therefore $N_{|\cup_{i\in I}G_i\times U\times\cup_{i\in I}G_i}=\bigcap_{i\in I}C_i$. Now

$$SVNS(\bigcap_{i\in I}G_i)\subseteq\bigcap_{i\in I}SVNS(G_i)\subseteq\bigcap_{i\in I}G_i.$$

Hence $\bigcap_{i\in I}\square_i$ is a single valued neutrosophic submachine of F.

(ii) Since $SVNS(\bigcup_{i\in I}G_i)=\bigcup_{i\in I}SVNS(G_i)\subseteq\bigcup_{i\in I}G_i$, $\bigcup_{i\in I}\square_i$ is a submachine of F.

Theorem 4. A $SVNFSM$ $F=(M,U,N)$ is strongly single valved neutrosophic connected iff F has no proper single valued neutrosophic submachine.

Proof. Assume that $F=(M,U,N)$ is strongly single valued neutrosophic connected. Let $\square=(G,U,C)$ be a single valued neutrosophic submachine of F such that $G\neq\varphi$. Then there exists $u\in G$. If $v\in SVNS(u)$ since F is strongly single valued neutrosophic connected. It follows that $v\in SVNS(u)\subseteq SVNS(G)\subseteq G$ so that $G=M$. Hence $\square=F$, that is F has no proper single valued neutrosophic submachine. Conveersely assume that F has no poper single valued neutrosophic submachines. Let $u,v \in M$ nad let $\square = (SVNS(u), U, C)$, where $C = (\chi_C, \psi_C, \omega_C)$ is given by

$\chi_C = \chi_{N|_{SVNS(u)\times U\times SVNS(u)}}$, $\psi_C = \psi_{N|_{SVNS(u)\times U\times SVNS(u)}}$, and $\omega_C = \omega_{N|_{SVNS(u)\times U\times SVNS(u)}}$

Then $\square$ is a single valued neutrosophic submachine of F and $SVNS(u) \neq \varphi,$ and so $SVNS(u) = M.$ Thus $v \in SVNS(u),$ and therefore F is strongly single valued neutrosophic connected.

4. SINGLE VALUED NEUTROSOPHIC FINITE SWITCHBOARD STATE MACHINE

Definition 12. An SVNFSM $M = (N, U, S)$ is said to be switching if it satisfies:

$$\chi_S(r,a,s) = \chi_S(s,a,r),\ \psi_S(r,a,s) = \psi_S(s,a,r)$$

and

$$\omega_S(r,a,s) = \omega_S(s,a,r)$$

for all $r, s \in N$ and $a \in U.$

An SVNFSM $M = (N, U, S)$ is said to be commutative if it satisfies:

$$\chi_S(r,ab,s) = \chi_S(r,ba,s),\ \psi_S(r,ab,s) = \psi_S(r,ba,s)$$

and

$$\omega_S(r,ab,s) = \omega_S(r,ba,s)$$

for all $r, s \in N$ and $a, b \in U.$

If an SVNFSM $M = (N, U, S)$ is both switching and commutative, then it is called single valued neutrosophic finite switchboard state machine (SVNFSSM for short).

Proposition 3. If $M = (N, U, S)$ is a commutative SVNFSM, then

$$\chi_{S^*}(r,ba,s) = \chi_{S^*}(r,ab,s),\ \psi_{S^*}(r,ba,s) = \psi_{S^*}(r,ab,s)$$

and

$$\omega_{S^*}(r,ba,s) = \omega_{S^*}(r,ab,s).$$

for all $r, s \in N$ and $a \in U, b \in U^*.$

Proof. Let $r, s \in N$ and $a, b \in U^*$. We prove the result by induction on $|b| = k.$ If $k = 0,$ then $b = \zeta,$ hence

$$\chi_{S^*}(r,ba,s) = \chi_{S^*}(r,\zeta a,s) = \chi_{S^*}(r,a,s) = \chi_{S^*}(r,a\zeta,s) = \chi_{S^*}(r,ab,s),$$
$$\psi_{S^*}(r,ba,s) = \psi_{S^*}(r,\zeta a,s) = \psi_{S^*}(r,a,s) = \psi_{S^*}(r,a\zeta,s) = \psi_{S^*}(r,ab,s)$$

and

$$\omega_{S^*}(r,ba,s) = \omega_{S^*}(r,\zeta a,s) = \omega_{S^*}(r,a,s) = \omega_{S^*}(r,a\zeta,s) = \omega_{S^*}(r,ab,s)$$

Therefore the result is true for $k = 0.$ Suppose that the result is true for $|c| = k-1.$ That is for all $c \in U^*$ with $|c| = k-1, k > 0.$ Let $d \in U$ be such that $b = cd.$ Then

$$\begin{aligned}\chi_{S^*}(r,ba,s) = \chi_{S^*}(r,cda,s) &= \vee_{v\in N}\left\lfloor \chi_{S^*}(r,c,v)\wedge \chi_{S^*}(v,da,s)\right\rfloor \\ &= \vee_{v\in N}\left[\chi_{S^*}(r,c,v)\wedge \chi_{S^*}(v,ad,s)\right] \\ &= \chi_{S^*}(r,cad,s) \\ &= \vee_{v\in N}\left[\chi_{S^*}(r,ca,v)\wedge \chi_{S}(v,d,s)\right] \\ &= \vee_{v\in N}\left[\chi_{S^*}(r,ac,v)\wedge \chi_{S}(v,d,s)\right] \\ &= \chi_{S^*}(r,acd,s) = \chi_{S^*}(r,ab,s),\end{aligned}$$

$$\begin{aligned}\psi_{S^*}(r,ba,s) = \psi_{S^*}(r,cda,s) &= \wedge_{v\in N}\left[\psi_{S^*}(r,c,v)\vee \psi_{S^*}(v,da,s)\right] \\ &= \wedge_{v\in N}\left[\psi_{S^*}(r,c,v)\vee \psi_{S^*}(v,ad,s)\right] \\ &= \psi_{S^*}(r,cad,s) \\ &= \wedge_{v\in N}\left[\psi_{S^*}(r,ca,v)\vee \psi_{S}(v,d,s)\right] \\ &= \wedge_{v\in N}\left[\psi_{S^*}(r,ac,v)\vee \psi_{S}(v,d,s)\right] \\ &= \psi_{S^*}(r,acd,s) = \psi_{S^*}(r,ab,s)\end{aligned}$$

and

$$\begin{aligned}\omega_{S^*}(r,ba,s) = \omega_{S^*}(r,cda,s) &= \wedge_{v\in N}\left\lfloor \omega_{S^*}(r,c,v)\vee \omega_{S^*}(v,da,s)\right\rfloor \\ &= \wedge_{v\in N}\left[\omega_{S^*}(r,c,v)\vee \omega_{S^*}(v,ad,s)\right] \\ &= \omega_{S^*}(r,cad,s) \\ &= \wedge_{v\in N}\left[\omega_{S^*}(r,ca,v)\vee \omega_{S}(v,d,s)\right] \\ &= \wedge_{v\in N}\left[\omega_{S^*}(r,ac,v)\vee \omega_{S}(v,d,s)\right] \\ &= \omega_{S^*}(r,acd,s) = \omega_{S^*}(r,ab,s)\end{aligned}$$

Hence the result is true for $|b|=k.$ Thus completes the proof.

Proposition 4. If $M=(N,U,S)$ is an SVNFSSM, then

$$\chi_{S^*}(r,a,s)=\chi_{S^*}(s,a,r), \psi_{S^*}(r,a,s)=\psi_{S^*}(s,a,r)$$

and

$$\omega_{S^*}(r,a,s)=\omega_{S^*}(s,a,r).$$

for all $r,s\in N$ and $a\in U^*.$

Proof. Let $r,s\in N$ and $a\in U^*$. We prove the result by induction on $|a|=k.$ If $k=0,$ then $b=\zeta,$ hence

$$\chi_{S^*}(r,a,s)=\chi_{S^*}(r,\zeta,s)=\chi_{S^*}(s,\zeta,r)=\chi_{S^*}(s,a,r),$$
$$\psi_{S^*}(r,a,s)=\psi_{S^*}(r,\zeta,s)=\psi_{S^*}(s,\zeta,r)=\psi_{S^*}(s,a,r)$$

and

$$\omega_{S^*}(r,a,s)=\omega_{S^*}(r,\zeta,s)=\omega_{S^*}(s,\zeta,r)=\omega_{S^*}(s,a,r)$$

Therefore the result is true for $k=0.$ Assume that the result is true for $|b|=k-1.$ That is for all $b\in U^*$ with $|b|=k-1, k>0,$ we have

$$\chi_{S^*}(r,b,s)=\chi_{S^*}(s,b,r),\psi_{S^*}(r,b,s)=\psi_{S^*}(s,b,r)$$

and

$$\omega_{S^*}(r,b,s)=\omega_{S^*}(s,b,r).$$

Let $x\in U$ and $b\in U^*$ be such that $a=bx.$ Then

$$\begin{aligned}\chi_{S^*}(r,a,s)=\chi_{S^*}(r,bx,s)&=\vee_{v\in N}\left\lfloor \chi_{S^*}(r,b,v)\wedge\chi_S(v,x,s)\right\rfloor\\&=\vee_{v\in N}\left[\chi_{S^*}(v,b,r)\wedge\chi_S(s,x,r)\right]\\&=\vee_{v\in N}\left[\chi_{S^*}(v,b,r)\wedge\chi_{S^*}(s,x,r)\right]\\&=\vee_{v\in N}\left[\chi_{S^*}(s,x,r)\wedge\chi_{S^*}(r,b,v)\right]\\&=\chi_{S^*}(s,xb,r)=\chi_{S^*}(s,bx,r)=\chi_{S^*}(s,a,r),\end{aligned}$$

$$\begin{aligned}\psi_{S^*}(r,a,s)=\psi_{S^*}(r,bx,s)&=\wedge_{v\in N}\left[\psi_{S^*}(r,b,v)\vee\psi_S(v,x,s)\right]\\&=\wedge_{v\in N}\left[\psi_{S^*}(v,b,r)\vee\psi_S(s,x,r)\right]\\&=\wedge_{v\in N}\left[\psi_{S^*}(v,b,r)\vee\psi_{S^*}(s,x,r)\right]\\&=\wedge_{v\in N}\left[\psi_{S^*}(s,x,r)\vee\psi_{S^*}(r,b,v)\right]\\&=\psi_{S^*}(s,xb,r)=\psi_{S^*}(s,bx,r)=\psi_{S^*}(s,a,r)\end{aligned}$$

and

$$\begin{aligned}\omega_{S^*}(r,a,s)=\omega_{S^*}(r,bx,s)&=\wedge_{v\in N}\left\lfloor \omega_{S^*}(r,b,v)\vee\omega_S(v,x,s)\right\rfloor\\&=\wedge_{v\in N}\left[\omega_{S^*}(v,b,r)\vee\omega_S(s,x,r)\right]\\&=\wedge_{v\in N}\left[\omega_{S^*}(v,b,r)\vee\omega_{S^*}(s,x,r)\right]\\&=\wedge_{v\in N}\left[\omega_{S^*}(s,x,r)\vee\omega_{S^*}(r,b,v)\right]\\&=\omega_{S^*}(s,xb,r)=\omega_{S^*}(s,bx,r)=\omega_{S^*}(s,a,r)\end{aligned}$$

This shows that the result is true for $|b|=k.$

Proposition 5. If $M=(N,U,S)$ is an SVNFSSM, then

$$\alpha_{S^*}(r,ab,s)=\alpha_{S^*}(r,ba,s),\ \beta_{S^*}(r,ab,s)=\ \beta_{S^*}(r,ba,s)$$

and

$$\gamma_{S^*}(r,ab,s)=\ \gamma_{S^*}(r,ba,s).$$

for all c and $a,b\in U^*.$

Proof. Let $r,s\in N$ and $a,b\in U^*$. We prove the result by induction on $|b|=k.$ If $k=0,$ then

$b=\zeta$, hence

$$\chi_{S^*}(r,ab,s)=\chi_{S^*}(r,a\zeta,s)=\chi_{S^*}(r,a,s)=\chi_{S^*}(r,\zeta a,s)=\chi_{S^*}(r,ba,s),$$
$$\psi_{S^*}(r,ab,s)=\psi_{S^*}(r,a\zeta,s)=\psi_{S^*}(r,a,s)=\psi_{S^*}(r,\zeta a,s)=\psi_{S^*}(r,ba,s)$$

and

$$\omega_{S^*}(r,ab,s)=\omega_{S^*}(r,a\zeta,s)=\omega_{S^*}(r,a,s)=\omega_{S^*}(r,\zeta a,s)=\omega_{S^*}(r,ba,s)$$

Therefore the result is true for $k=0.$ Suppose that the result is true for $|c|=k-1.$ That is for all $c\in U^*$ with $|c|=k-1, k>0.$ Let $d\in U$ be such that $b=cd.$ Then

$$\begin{aligned}
\chi_{S^*}(r,ab,s) &= \chi_{S^*}(r,acd,s)=\vee_{v\in N}\left[\chi_{S^*}(r,ac,v)\wedge\chi_S(v,d,s)\right]\\
&= \vee_{v\in N}\left[\chi_{S^*}(r,ca,v)\wedge\chi_S(v,d,s)\right]\\
&= \vee_{v\in N}\left[\chi_{S^*}(v,ca,r)\wedge\chi_S(s,d,v)\right]\\
&= \vee_{v\in N}\left[\chi_S(s,d,v)\wedge\chi_{S^*}(v,ca,r)\right]\\
&= \chi_{S^*}(s,dca,r)=\vee_{v\in N}\left[\chi_{S^*}(s,dc,v)\wedge\chi_{S^*}(v,a,r)\right]\\
&= \vee_{v\in N}\left[\chi_{S^*}(s,cd,v)\wedge\chi_{S^*}(v,a,r)\right]=\chi_{S^*}(s,cda,r)\\
&= \chi_{S^*}(r,cda,s)=\chi_{S^*}(r,ba,s),\\
\psi_{S^*}(r,ab,s) &= \psi_{S^*}(r,acd,s)=\wedge_{v\in N}\left[\psi_{S^*}(r,ac,v)\vee\psi_S(v,d,s)\right]\\
&= \wedge_{v\in N}\left[\psi_{S^*}(r,ca,v)\vee\psi_S(v,d,s)\right]\\
&= \wedge_{v\in N}\left[\psi_{S^*}(v,ca,r)\vee\psi_S(s,d,v)\right]\\
&= \wedge_{v\in N}\left[\psi_S(s,d,v)\vee\psi_{S^*}(v,ca,r)\right]\\
&= \psi_{S^*}(s,dca,r)=\wedge_{v\in N}\left[\psi_{S^*}(s,dc,v)\vee\psi_{S^*}(v,a,r)\right]\\
&= \wedge_{v\in N}\left[\psi_{S^*}(s,cd,v)\vee\psi_{S^*}(v,a,r)\right]=\psi_{S^*}(s,cda,r)\\
&= \psi_{S^*}(r,cda,s)=\psi_{S^*}(r,ba,s)
\end{aligned}$$

and

$$\begin{aligned}
\omega_{S^*}(r,ab,s) = \omega_{S^*}(r,acd,s) &= \wedge_{v\in N}\left\lfloor \omega_{S^*}(r,ac,v)\vee\omega_S(v,d,s)\right\rfloor \\
&= \wedge_{v\in N}\left[\omega_{S^*}(r,ca,v)\vee\omega_S(v,d,s)\right] \\
&= \wedge_{v\in N}\left[\omega_{S^*}(v,ca,r)\vee\omega_S(s,d,v)\right] \\
&= \wedge_{v\in N}\left[\omega_S(s,d,v)\vee\omega_{S^*}(v,ca,r)\right] \\
&= \omega_{S^*}(s,dca,r) = \wedge_{v\in N}\left[\omega_{S^*}(s,dc,v)\vee\omega_{S^*}(v,a,r)\right] \\
&= \wedge_{v\in N}\left[\omega_{S^*}(s,cd,v)\vee\omega_{S^*}(v,a,r)\right] = \omega_{S^*}(s,cda,r) \\
&= \omega_{S^*}(r,cda,s) = \omega_{S^*}(r,ba,s)
\end{aligned}$$

This shows that the result is true for $|b|=k.$

Definition 13. Let $M_S=(N_1,U_1,S)$ and $M_T=(N_2,U_2,T)$ be two SVNFSMs. A pair (α,β) of mappings $\alpha : N_1\to N_2$ and $\beta : U_1\to U_2$ is called homomorphism, written as $(\alpha,\beta):\ M_S\to M_T$, if it satisfies:

$$\chi_S(r,a,s)\le\chi_T(\alpha(r),\beta(a),\alpha(s)),\psi_S(r,a,s)\ge\psi_T(\alpha(r),\beta(a),\alpha(s))$$

and

$$\omega_S(r,a,s)\ge\omega_T(\alpha(r),\beta(a),\alpha(s))$$

for all $r,s\in N_1$ and $a\in U_1.$

Definition 14. Let $M_S=(N_1,U_1,S)$ and $M_T=(N_2,U_2,T)$ be two SVNFSMs. A pair (α,β) of mappings $\alpha : N_1\to N_2$ and $\beta : U_1\to U_2$ is called a strong homomorphism, written as $(\alpha,\beta):\ M_S\to M_T$, if it satisfies:

$$\chi_T(\alpha(r),\beta(a),\alpha(s))=\vee\{\chi_S(r,a,v)\mid v\in N_1,\alpha(v)=\alpha(s)\},$$
$$\psi_T(\alpha(r),\beta(a),\alpha(s))=\wedge\{\psi_S(r,a,v)\mid v\in N_1,\alpha(v)=\alpha(s)\}$$

and

$$\omega_T(\alpha(r),\beta(a),\alpha(s))=\wedge\{\omega_S(r,a,v)\mid v\in N_1,\alpha(v)=\alpha(s)\}$$

for all $r,s\in N_1$ and $a\in U_1.$ If $U_1=U_2$ and β is the identity map, then we simply write $\alpha : M_S\to M_T$ and say that α is a homomorphism or strong homomorphism accordingly. If (α,β) is a strong homorphism with α is one-one, then

$$\chi_T(\alpha(r),\beta(a),\alpha(s))=\chi_S(r,a,s),\psi_T(\alpha(r),\beta(a),\alpha(s))=\psi_S(r,a,s)$$

and

$$\omega_T(\alpha(r),\beta(a),\alpha(s))=\omega_S(r,a,s)$$

for all $r,s\in N_1$ and $a\in U_1.$

Theorem 5. Let $M_S=(N_1,U_1,S)$ and $M_T=(N_2,U_2,T)$ be two SVNFSMs. Let $(\alpha,\beta):\ M_S\to M_T$ be an onto strong homomorphism. If M_S is a commutative, then so is M_T.

Proof. Let $r_2,s_2\in N_2.$ Then there are $r_1,s_1\in N_1$ such that $\alpha(r_1)=r_2$ and $\alpha(s_1)=s_2.$ Let $x_2,y_2\in U_2.$ Then there exists $x_1,y_1\in U_1$ such that $\beta(x_1)=x_2$ and $(y_1)=y_2.$ Since M_S is commutative , we have

$$\begin{aligned}
\chi_{T^*}(r_2, x_2y_2, s_2) &= \chi_{T^*}(\alpha(r_1), \beta(x_1)\beta(y_1), \alpha(s_1)) \\
&= \chi_{T^*}(\alpha(r_1), \beta(x_1, y_1), \alpha(s_1)) \\
&= \vee\{\chi_{S^*}(r_1, x_1y_1, v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(s_1)\} \\
&= \vee\{\chi_{S^*}(r_1, y_1x_1, v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(s_1)\} \\
&= \chi_{T^*}(\alpha(r_1), \beta(y_1x_1), \alpha(s_1)) \\
&= \chi_{T^*}(r_2, y_2x_2, s_2), \\
\psi_{T^*}(r_2, x_2y_2, s_2) &= \psi_{T^*}(\alpha(r_1), \beta(x_1)\beta(y_1), \alpha(s_1)) \\
&= \psi_{T^*}(\alpha(r_1), \beta(x_1, y_1), \alpha(s_1)) \\
&= \wedge\{\psi_{S^*}(r_1, x_1y_1, v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(s_1)\} \\
&= \wedge\{\psi_{S^*}(r_1, y_1x_1, v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(s_1)\} \\
&= \psi_{T^*}(\alpha(r_1), \beta(y_1x_1), \alpha(s_1)) \\
&= \psi_{T^*}(\alpha(r_1), \beta(y_1)\beta(x_1), \alpha(s_1)) \\
&= \psi_{T^*}(r_2, y_2x_2, s_2)
\end{aligned}$$

and

$$\begin{aligned}
\omega_{T^*}(r_2, x_2y_2, s_2) &= \omega_{T^*}(\alpha(r_1), \beta(x_1)\beta(y_1), \alpha(s_1)) \\
&= \omega_{T^*}(\alpha(r_1), \beta(x_1, y_1), \alpha(s_1)) \\
&= \wedge\{\omega_{S^*}(r_1, x_1y_1, v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(s_1)\} \\
&= \wedge\{\omega_{S^*}(r_1, y_1x_1, v_1) \mid v_1 \in N_1, v(v_1) = \alpha(s_1)\} \\
&= \omega_{T^*}(\alpha(r_1), \beta(y_1x_1), \alpha(s_1)) \\
&= \omega_{T^*}(\alpha(r_1), \beta(y_1)\beta(x_1), \alpha(s_1)) \\
&= \omega_{T^*}(r_2, y_2x_2, s_2)
\end{aligned}$$

Hence M_T is a commutative SVNFSM. This completes the proof.

Proposition 6. Let $M_S = (N_1, U_1, S)$ and $M_T = (N_2, U_2, T)$ be two SVNFSMs. Let $(\alpha, \beta):\ M_S \to M_T$ be a strong homomorphism. Then

$$\begin{aligned}
&(\forall u, v \in N_1)(\forall a \in U_1)(\chi_T(\alpha(u), \beta(a), \alpha(v)) > 0 \\
&\quad \Rightarrow (\exists w \in N_1)(\chi_S(u, a, v) > 0, \alpha(w) = v(v)), \\
&(\forall u, v \in N_1)(\forall a \in U_1)(\psi_T(\alpha(u), \beta(a), \alpha(v)) < 1 \\
&\quad \Rightarrow (\exists w \in N_1)(\psi_S(u, a, v) < 1, \alpha(w) = v(v)),
\end{aligned}$$

and

$$\begin{aligned}
&(\forall u, v \in N_1)(\forall a \in U_1)(\omega_T(\alpha(u), \alpha(a), \alpha(v)) < 1 \\
&\quad \Rightarrow (\exists w \in N_1)(\omega_S(u, a, v) < 1, v(w) = \alpha(v)).
\end{aligned}$$

Moreover,

$$(\forall z \in N_1)(\alpha(z) = v(u) \Rightarrow \chi_S(u, a, w) \geq \chi_S(z, a, r),$$
$$\psi_S(u, a, w) \leq \psi_S(z, a, r) \ and\ \omega_S(u, a, w) \leq \omega_S(z, a, r).$$

Proof. Let $u, v, z \in N_1$ and $a \in U_1$. Assume that $\chi_T(\alpha(u), \beta(a), \alpha(v)) > 0$, $(\psi_T(\alpha(u), \beta(a), \alpha(v)) < 1$ and $(\omega_T(\alpha(u), \beta(a), \alpha(v)) < 1$. Then

$$\vee\{\chi_S(u,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = v(v)\} > 0$$
$$\wedge\{\psi_S(u,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = v(v)\} < 1$$

and

$$\wedge\{\omega_S(u,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(v)\} < 1$$

Since N_1 is finite , it follows that there exists $w \in N_1$ such that $\alpha(w) = \alpha(v)$,

$$\chi_S(u,a,w) = \vee\{\chi_S(u,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(w)\} > 0,$$
$$\psi_S(u,a,v) = \wedge\{\psi_S(u,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(w)\} < 1$$

and

$$\omega_S(u,a,v) = \wedge\{\omega_S(u,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(w)\} < 1$$

Now suppose that $\alpha(z) = \alpha(u)$ for every $z \in N_1$. Then

$$\chi_S(u,a,w) = \chi_T(\alpha(u), \beta(a), \alpha(v)) = \chi_T(\alpha(z), \beta(a), \alpha(v))$$
$$= \vee\{\chi_S(z,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(v)\} \geq \chi_S(z,a,v),$$

$$\psi_S(u,a,w) = \psi_T(\alpha(u), \beta(a), \alpha(v)) = \psi_T(\alpha(z), \beta(a), \alpha(v))$$
$$= \wedge\{\psi_S(z,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(v)\} \leq \psi_S(z,a,v)$$

and

$$\omega_S(u,a,w) = \omega_T(\alpha(u), \beta(a), \alpha(v)) = \omega_T(v(z), \beta(a), \alpha(v))$$
$$= \wedge\{\omega_S(z,a,v_1) \mid v_1 \in N_1, \alpha(v_1) = \alpha(v)\} \leq \omega_S(z,a,v)$$

Which is the required proof.

Lemma 2. Let $M_S = (N_1, U_1, S)$ and $M_T = (N_2, U_2, T)$ be two SVNFSMs. Let $(\alpha, \beta): M_S \to M_T$ be a homomorphism. Define a mapping $\beta^* : U_1^* \to U_2^*$ by $\beta^*(\zeta) = \zeta$ and $\beta^*(xy) = \beta^*(x)\beta^*(y)$ for all $x \in U_1^*$ and $y \in U_1$. Then $\beta^*(ab) = \beta^*(a)\beta^*(b)$ for all $a, b \in U_1^*$.

Proof Let $a, b \in U_1^*$. We prove the result by induction on $|b| = k$. If $k = 0$, then $b = \zeta$. Therefore $ab = a\zeta = a$. Hence

$$\beta^*(ab) = \beta^*(a) = \beta^*(a)\zeta = \beta^*(a)\beta^*(\zeta) = \beta^*(a)\beta^*(b)$$

Which shows that the result is true for $k = 0$. Let us assume that the result is true for each $c \in U_1^*$ such that $|c| = k-1$. That is

$$\beta^*(ab) = \beta^*(a)\beta^*(b)$$

Let $b = cd$, where $c \in U_1^*$ and $d \in U_1$ be such that $|c| = k-1, k > 0$. Then

$$\beta^*(ab) = \beta^*(acd) = \beta^*(ac)\beta(d) = \beta^*(a)\beta^*(c)\psi(d) = \beta^*(a)\beta^*(cd) = \beta^*(a)\beta^*(b).$$

Therefore, the result is true for $|b| = k$.

Theorem 6. Let $M_S = (N_1, U_1, S)$ and $M_T = (N_2, U_2, T)$ be two SVNFSMs. Let $(\alpha, \beta): M_S \to M_T$ be a homomorphism. Then

$$\chi_{S^*}(r,a,s) \leq \chi_{T^*}(\alpha(r), \beta^*(a), \alpha(s)),\ \psi_{S^*}(r,a,s) \geq \psi_{T^*}(\alpha(r), \beta^*(a), \alpha(s))$$

and

$$\psi_{S^*}(r,a,s) \geq \psi_{T^*}(\alpha(r), \beta^*(a), \alpha(s))$$

for all $r, s \in N_1$ and $a \in U_1^*$.

Proof. Let $r, s \in N_1$ and $a \in U_1^*$. We prove the result by induction on $|a| = k$. If $k = 0$, then $a = \zeta$ and so $\psi^*(a) = \psi^*(\zeta) = \zeta$. If $r = s$, then

$$\chi_{S^*}(r,a,s) = \chi_{S^*}(r,\zeta,s) = 1 = \chi_{T^*}(\alpha(r), \zeta, \alpha(s)) = \chi_{T^*}(\alpha(r), \beta^*(a), \alpha(s)),$$

$$\psi_{S^*}(r,a,s) = \psi_{S^*}(r,\zeta,s) = 0 = \psi_{T^*}(\alpha(r), \zeta, \alpha(s)) = \psi_{T^*}(\alpha(r), \beta^*(a), \alpha(s))$$

and

$$\omega_{S^*}(r,a,s) = \omega_{S^*}(r,\zeta,s) = 0 = \omega_{T^*}(\alpha(r), \zeta, \alpha(s)) = \omega_{T^*}(\alpha(r), \beta^*(a), \alpha(s))$$

If $r \neq s$, then

$$\chi_{S^*}(r,a,s) = \chi_{S^*}(r,\zeta,s) = 0 \leq \chi_{T^*}(\alpha(r), \beta^*(a), \alpha(s)),$$

$$\psi_{S^*}(r,a,s) = \psi_{S^*}(r,\zeta,s) = 1 \geq \psi_{T^*}(\alpha(r), \beta^*(a), \alpha(s))$$

and

$$\omega_{S^*}(r,a,s) = \omega_{S^*}(r,\zeta,s) = 1 \geq \omega_{T^*}(\alpha(r), \beta^*(a), \alpha(s))$$

Therefore the result is true for $k = 0$. Let us assume that the result is true for all $b \in U_1^*$ such that $|b| = k-1, k > 0$. Let $a = bc$ where $b \in U_1^*, c \in U_1$ and $|b| = k-1$. Then

$$\begin{aligned}
\chi_{S^*}(r,a,s) &= \chi_{S^*}(r,bc,s) = \vee_{v \in N_1}[\chi_{S^*}(r,b,v) \wedge \chi_{S^*}(v,c,s)] \\
&\leq \vee_{v \in N_1}[\chi_{T^*}(\alpha(r), \beta^*(b), \alpha(v)) \wedge \chi_{T^*}(\alpha(v), \beta(c), \alpha(s))] \\
&\leq \vee_{v^\circ \in N_1}[\chi_{T^*}(\alpha(r), \beta^*(b), v^\circ) \wedge \chi_{T^*}(v^\circ, \beta(c), \alpha(s))] \\
&= \chi_{T^*}(\alpha(r), \beta^*(b)\beta(c), \alpha(s)) \\
&= \chi_{T^*}(\alpha(r), \beta^*(bc), \alpha(s)) \\
&= \chi_{T^*}(\alpha(r), \beta^*(a), \alpha(s)), \\
\psi_{S^*}(r,a,s) &= \psi_{S^*}(r,bc,s) = \wedge_{v \in N_1}[\psi_{S^*}(r,b,v) \vee \psi_{S^*}(v,c,s)] \\
&\geq \wedge_{v \in N_1}[\psi_{T^*}(\alpha(r), \beta^*(b), \alpha(v)) \vee \psi_{T^*}(\alpha(v), \beta(c), \alpha(s))] \\
&\geq \wedge_{v^\circ \in N_1}[\psi_{T^*}(\alpha(r), \beta^*(b), v^\circ) \vee \psi_{T^*}(v^\circ, \beta(c), v(s))] \\
&= \psi_{T^*}(\alpha(r), \beta^*(b)\beta(c), \alpha(s)) \\
&= \psi_{T^*}(\alpha(r), \beta^*(bc), \alpha(s)) \\
&= \psi_{T^*}(\alpha(r), \beta^*(a), \alpha(s))
\end{aligned}$$

and

$$\begin{aligned}\omega_{S^*}(r,a,s) = \omega_{S^*}(r,bc,s) &= \wedge_{v\in N_1}[\omega_{S^*}(r,b,v)\vee \omega_{S^*}(v,c,s)] \\ &\geq \wedge_{v\in N_1}[\omega_{T^*}(\alpha(r),\beta^*(b),\alpha(v))\vee \omega_{T^*}(\alpha(v),\beta(c),\alpha(s))] \\ &\geq \wedge_{v^\circ\in N_1}[\omega_{T^*}(\alpha(r),\beta^*(b),v^\circ)\vee \omega_{T^*}(v^\circ,\beta(c),\alpha(s))] \\ &= \omega_{T^*}(\alpha(r),\beta^*(b)\beta(c),\alpha(s)) \\ &= \omega_{T^*}(\alpha(r),\beta^*(bc),\alpha(s)) \\ &= \omega_{T^*}(\alpha(r),\beta^*(a),\alpha(s))\end{aligned}$$

Which is the required proof.

Theorem 7. Let $M_S=(N_1,U_1,S)$ and $M_T=(N_2,U_2,T)$ be two SVNFSMs. Let $(\alpha,\beta): M_S\to M_T$ be a strong homomorphism. If α is one-one, then

$$\chi_{S^*}(r,a,s)=\chi_{T^*}(\alpha(r),\beta^*(a),\alpha(s)),\ \psi_{S^*}(r,a,s)=\psi_{T^*}(\alpha(r),\beta^*(a),\alpha(s))$$

and

$$\omega_{S^*}(r,a,s)=\omega_{T^*}(\alpha(r),\beta^*(a),\alpha(s))$$

for all $r,s\in N_1$ and $a\in U_1^*$.

Proof. Let us assume that α is 1-1 and for $r,s\in N_1$ and $a\in U_1^*$. Let $|a|=k$. We prove the result by induction on $|a|=k$. If $k=0$, then $a=\zeta$ and $\beta^*(\zeta)=\zeta$. Since $\alpha(r)=\alpha(s)$ if and only if $r=s$, we get

$$\chi_{S^*}(r,a,s)=\chi_{S^*}(r,\zeta,s)=1$$

if and only if

$$\chi_{T^*}(\alpha(r),\beta^*(a),\alpha(s))=\chi_{T^*}(\alpha(r),\beta^*(\zeta),\alpha(s))=1,$$
$$\psi_{S^*}(r,a,s)=\psi_{S^*}(r,\zeta,s)=0$$

if and only if

$$\psi_{T^*}(\alpha(r),\beta^*(a),\alpha(s))=\psi_{T^*}(\alpha(r),\beta^*(\zeta),\alpha(s))=0,$$

and

$$\omega_{S^*}(r,a,s)=\omega_{S^*}(r,\zeta,s)=0$$

if and only if

$$\omega_{T^*}(\alpha(r),\beta^*(a),\alpha(s))=\omega_{T^*}(\alpha(r),\beta^*(\zeta),\alpha(s))=0$$

Let us assume that the result is true for all $b\in U_1^*$ such that $|b|=k-1, k>0$. Let $a=bc$, where $|b|=k-1,k>0$ and $b\in U_1^*, c\in U_1$. Then

$$\begin{aligned}
\chi_{T^*}(\alpha(r),\beta^*(a),\alpha(s)) &= \chi_{T^*}(\alpha(r),\beta^*(bc),\alpha(s)) = \chi_{T^*}(\alpha(r),\beta^*(b)\beta(c),\alpha(s)) \\
&= \vee_{v\in N_1}[\chi_{T^*}(\alpha(r),\beta^*(b),\alpha(v)) \wedge \chi_T(\alpha(v),\beta(c),\alpha(s))] \\
&= \vee_{v\in N_1}[\chi_{S^*}(r,b,v) \wedge \chi_S(v,c,s)] \\
&= \chi_{S^*}(r,bc,s) = \chi_{S^*}(r,a,s), \\
\psi_{T^*}(\alpha(r),\beta^*(a),\alpha(s)) &= \psi_{T^*}(\alpha(r),\beta^*(bc),\alpha(s)) = \psi_{T^*}(\alpha(r),\beta^*(b)\beta(c),\alpha(s)) \\
&= \wedge_{v\in N_1}[\psi_{T^*}(\alpha(r),\beta^*(b),\alpha(v)) \vee \psi_T(\alpha(v),\beta(c),\alpha(s))] \\
&= \wedge_{v\in N_1}[\psi_{S^*}(r,b,v) \vee \psi_S(v,c,s)] \\
&= \psi_{S^*}(r,bc,s) = \psi_{S^*}(r,a,s)
\end{aligned}$$

and

$$\begin{aligned}
\omega_{T^*}(\alpha(r),\beta^*(a),\alpha(s)) &= \omega_{T^*}(\alpha(r),\beta^*(bc),\alpha(s)) = \omega_{T^*}(\alpha(r),\beta^*(b)\beta(c),\alpha(s)) \\
&= \wedge_{v\in N_1}[\omega_{T^*}(\alpha(r),\beta^*(b),\alpha(v)) \vee \omega_T(\alpha(v),\beta(c),\alpha(s))] \\
&= \wedge_{v\in N_1}[\omega_{S^*}(r,b,v) \vee \omega_S(v,c,s)] \\
&= \omega_{S^*}(r,bc,s) = \omega_{S^*}(r,a,s)
\end{aligned}$$

Which is the required proof.

CONCLUSION

Using the notion of single valued neutrosophic set we introduced the notion of single valued neutrosophic finite state machine, single valued neutrosophic successors, single valued neutrosophic subsystem, and single valued neutrosophic submachines. which are the generalization of fuzzy finite state machine and intuitionistic fuzzy finite state machine. We also defined single valued neutrosophic switchboard state machine, homomorphism and strong homomorphism between single valued neutrosophic switchboard state machine and discussed some related results and properties.

In future we shall apply the concept of neutrosophic set to automata theory.

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