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(α, β, γ)-Equalities of single valued neutrosophic sets

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Abstract: The single valued neutrosophic set (SVNS) is a subclass of neutrosophic set, which can describe and handle indeterminate information and inconsistent information. Since a SVNS is characterized independently by three functions: a truth-membership function, an indeterminacy-membership function, and a falsity-membership function. This paper introduces (α, β, γ)-equalities of SVNS, which contains three parameters corresponding to three characteristic functions of SVNS. Then we show how various operations of single valued neutrosophic sets affect these three parameters.

Keywords: Neutrosophic set, Single valued neutrosophic set, (α, β, γ)-equality.

1 Introduction

Neutrosophic sets introduced by Smarandache [17] are the generalization of fuzzy sets [23] and intuitionistic fuzzy sets [3]. A neutrosophic set is characterized independently by three functions: a truth-membership function, an indeterminacy-membership function, and a falsity-membership function. However, since these three functions are real standard or non-standard subsets of $]^{-}0, 1^{+}[$, it will be difficult to apply in real engineering fields [18]. Thus, Wang et.al [18] introduced the concept of single valued neutrosophic set (SVNS), which membership functions are the normal standard subsets of real unit interval $[0, 1]$. SVNS can deal with indeterminate and inconsistent information and therefore have been applied to many domains [9, 13, 14, 19, 20, 21].

Pappis [16] studied the value approximation of fuzzy systems variables. As a generalization of the work of Pappis, Hong and Hwang [10] discussed the value similarity of fuzzy system variables. Further, Cai introduced the so-called δ-equalities of fuzzy sets and applied them to discuss robustness of fuzzy reasoning. Georgescu [7, 8] generalized δ-equalities of fuzzy sets to (δ, H)-equality of fuzzy sets based on triangular norms. Dai et al. [6] and Jin et al. [11] discussed robustness of fuzzy reasoning based on (δ, H)-equality of fuzzy sets. Zhang et al. [22] studied the δ-equalities of complex fuzzy sets and applied the new concept in a signal processing application. Ngan and Ali [15] studied the δ-equalities of intuitionistic fuzzy sets and applied the new concept the application of medical diagnosis. Ali et al. [2] studied the δ-equalities of neutrosophic sets. Moreover, Ali and Smarandache [1] studied the δ-equalities of complex neutrosophic sets.

However, the concepts in [4, 5, 15, 22, 1, 2] are based on distance measures. Only one parameter is used to measure the degree of equality of fuzzy sets and their extensions. As we know, a SVNS is characterized independently by three functions. For example, from [2] we have $A = (0.2)B$ and $A = (0.2)C$ for $A \equiv (1, 0, 0)$, $B \equiv (1, 0, 0.8)$ and $C \equiv (0.2, 0.8, 0.8)$, i.e., B and C satisfy the same δ-equality with respect to A for $\delta = 0.2$. But B and C are quite different. Based on the above analysis, we find out that the only parameter given in [2] is a little rough to some extent. In view of this, it is more suitable to use three parameters to measure the degree of equality in these three functions respectively.

This paper investigates the concept of (α, β, γ)-equalities between single valued neutrosophic sets by following the work of Smarandache [17], Wang et.al [18] and Cai [4, 5]. Different from the distance based concepts in [1, 4, 5, 22], the new concept uses three parameters to measure the equality degree of three characteristic functions independently.

The rest of this paper is organized as follows: In section 2 ,we first briefly recall the concept of single valued neutrosophic set and its operations. In section 3, we introduce the concept of (α, β, γ)-equalities of single valued neutrosophic sets and its basic properties. Section 4 discusses (α, β, γ)-equalities with respect to operations of single valued neutrosophic sets. Finally, conclusions are stated in section 6.

2 Preliminaries

Definition 1. *[18] Suppose X is a universe containing all related objects. A SVNS A in X is characterized by three functions, i.e., a truth-membership function $T_A : X \to [0, 1]$, an indeterminacy-membership function $I_A : X \to [0, 1]$, and a falsity-membership function $F_A : X \to [0, 1]$. Then, a SVNS A can be defined as follows*

$$A = \{x, T_A(x), I_A(x), F_A(x)|x \in X\},$$

where $T_A(x), I_A(x), F_A(x) \in [0, 1]$ for each $x \in X$.

We use the notation $SVN(X)$ to denote the set of all single valued neutrosophic sets of X.

Suppose A and B are two single valued neutrosophic sets of X, then the following relations and operations are defined as follows [18, 21].

(i) $A \subseteq B$ if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x), \forall x \in X$.

(ii) $A = B$ if and only if $A \subseteq B, B \subseteq A$.

(iii) $A^c = \{x, F_A(x), 1 - I_A(x), T_A(x) | x \in X\}$.

(iv) $A \cup B = \{x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x) | x \in X\}$.

(v) $A \cap B = \{x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x) | x \in X\}$.

(vi) $A + B = \{x, T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) | x \in X\}$.

(vii) $A \times B = \{x, T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) | x \in X\}$.

(viii) $\lambda A = \{x, 1-(1-T_A(x))^\lambda, I_A^\lambda(x), F_A^\lambda(x) | x \in X\}$, $\lambda > 0$.

(ix) $A^\lambda = \{x, T_A^\lambda(x), 1 - (1 - I_A(x))^\lambda, 1 - (1 - F_A(x))^\lambda | x \in X\}$, $\lambda > 0$.

To facilitate future discussion, we review the following two lemmas.

Lemma 2. *[10] Let f and g be bounded, real valued functions on a set X. Then*

(i) $|\bigvee_{x\in X} f(x) - \bigvee_{x\in X} g(x)| \leq \bigvee_{x\in X} |f(x) - g(x)|$,

(ii) $|\bigwedge_{x\in X} f(x) - \bigwedge_{x\in X} g(x)| \leq \bigvee_{x\in X} |f(x) - g(x)|$.

Lemma 3. *[12] Let $a, b \in [0, 1]$ and $\lambda > 0$. Then*

(i) *If $0 < \lambda \leq 1$, then $|a^\lambda - b^\lambda| \leq |a - b|^\lambda$;*

(ii) *If $\lambda \geq 1$, then $|a^\lambda - b^\lambda| \geq |a - b|^\lambda$.*

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Definition 4. *[2] Suppose A and B are two neutrosophic sets and $\delta \in [0, 1]$, then A and B are said to be δ-equal, if and only if, the following properties hold*

$$\bigvee_{x\in X} \big|T_A(x) - T_B(x)\big| \leq 1 - \delta,$$
$$\bigvee_{x\in X} \big|I_A(x) - I_B(x)\big| \leq 1 - \delta,$$
$$\bigvee_{x\in X} \big|F_A(x) - F_B(x)\big| \leq 1 - \delta.$$

It is denoted by $A = (\delta)B$.

Definition 5. *Suppose A and B are two single valued neutrosophic sets and $\alpha, \beta, \gamma \in [0, 1]$, then A and B are said to be (α, β, γ)-equal, if and only if, the following properties hold*

$$\bigvee_{x\in X} \big|T_A(x) - T_B(x)\big| \leq 1 - \alpha, \tag{1}$$
$$\bigvee_{x\in X} \big|I_A(x) - I_B(x)\big| \leq 1 - \beta, \tag{2}$$
$$\bigvee_{x\in X} \big|F_A(x) - F_B(x)\big| \leq 1 - \gamma. \tag{3}$$

It is denoted by $A = (\alpha, \beta, \gamma)B$.

Remark 6.

(i) *In Definition 4, if two single valued neutrosophic sets A and B are 1-equal, then $A = B$ holds and vice versa, i.e., $A = (1)B$ iff $A = B$. However, when we consider the case $A = (\delta)B$ for $\delta \neq 1$. See the example in the Introduction section, let $A \equiv (1, 0, 0)$, $B \equiv (1, 0, 0.8)$ and $C \equiv (0.2, 0.8, 0.8)$, then it follows from [2] that B and C satisfy the same δ-equality with respect to A for $\delta = 0.2$. Note that B and C are quite different. Using Definition 5, we have $A = (1, 1, 0.2)B$, and $A = (0.2, 0.2, 0.2)C$. These are consistent with the fact that B is close to A while C is far from A.*

(ii) *The new concept is a generalization of the existing concepts in [2, 4, 15]. We note that $A = (\alpha, \beta, \gamma)B \Rightarrow A = (\delta)B$, where $\delta = \min(\alpha, \beta, \gamma)$. When A and B are two intuitionistic fuzzy sets, i.e, $T_A(x) + I_A(x) + F_A(x) = 1$ and $T_B(x) + I_B(x) + F_B(x) = 1$ for all $x \in X$, then it follows from [15] that A and B are δ-equal for $\delta = \min(\alpha, \gamma)$. When A and B are two fuzzy sets, i.e, $T_A(x) + F_A(x) = 1$ and $T_B(x) + F_B(x) = 1$ for all $x \in X$, then we have $\alpha = \gamma, \beta = 1$ from $A = (\alpha, \beta, \gamma)B$, it follows from [4] that A and B are δ-equal for $\delta = \alpha$.*

Example 7. *Let $X = \{x_1, x_2\}$ and two single valued neutrosophic sets defined as*

$$A = \big\{ < x_1, 0.1, 0.2, 0.9 >, < x_2, 0.1, 0.2, 1.0 > \big\},$$
$$B = \big\{ < x_1, 0.2, 0.2, 0.1 >, < x_2, 0.1, 0.1, 0.1 > \big\}.$$

It is easy to know that $A = (0.9, 0.9, 0.1)B$.

If we consider the degree of equality based on the single valued neutrosophic distance measure, we only obtain one value for the degree of equality between single valued neutrosophic sets. For instance, if we use the following distance of single valued neutrosophic sets

$$d(A, B) = \max\Big\{ \bigvee_{x\in X} |T_A(x) - T_B(x)|, \bigvee_{x\in X} |I_A(x) - I_B(x)|, \bigvee_{x\in X} |F_A(x) - F_B(x)| \Big\} \tag{4}$$

then we have $d(A,B) = 0.9 = 1 - 0.1$. However, 0.1 is not a rational estimation of degree of equality for truth-membership function and indeterminacy-membership function in this example. We note that $A = (\delta)B \Leftrightarrow d(A,B) \le 1-\delta$. Based on an overall consideration of three characteristic functions, there parameters have been used accordingly.

And it is easy to know that $A = (\alpha,\beta,\gamma)B$ implies $d(A,B) \le 1-\alpha\wedge\beta\wedge\gamma$.

Theorem 8. *Suppose A, B and C are single valued neutrosophic sets, then the following hold*

(i) $A = (0,0,0)B$;

(ii) $A = (1,1,1)B$ *if and only if* $A = B$;

(iii) $A = (\alpha,\beta,\gamma)B$ *if and only if* $B = (\alpha,\beta,\gamma)A$;

(iv) $A = (\alpha_1,\beta_1,\gamma_1)B$ *and* $\alpha_2 \le \alpha_1$, $\beta_2 \le \beta_1$ *and* $\gamma_2 \le \gamma_1$, *then* $A = (\alpha_2,\beta_2,\gamma_2)B$;

(v) *If* $A = (\alpha_1,\beta_1,\gamma_1)B$ *and* $B = (\alpha_2,\beta_2,\gamma_2)C$, *then* $A = (\alpha_1*\alpha_2,\beta_1*\beta_2,\gamma_1*\gamma_2)C$,

where $a*b = (a+b-1)\vee 0$ *for any* $a,b\in[0,1]$

Proof. Properties (i)(iv) can be proved easily. We only prove (v).

Since $A = (\alpha_1,\beta_1,\gamma_1)B$, then

$$\bigvee_{x\in X} \big|T_A(x) - T_B(x)\big| \le 1-\alpha_1, \tag{5}$$

$$\bigvee_{x\in X} \big|I_A(x) - I_B(x)\big| \le 1-\beta_1, \tag{6}$$

$$\bigvee_{x\in X} \big|F_A(x) - F_B(x)\big| \le 1-\gamma_1. \tag{7}$$

From $B = (\alpha_2,\beta_2,\gamma_2)C$, we obtain

$$\bigvee_{x\in X} \big|T_B(x) - T_C(x)\big| \le 1-\alpha_2, \tag{8}$$

$$\bigvee_{x\in X} \big|I_B(x) - I_C(x)\big| \le 1-\beta_2, \tag{9}$$

$$\bigvee_{x\in X} \big|F_B(x) - F_C(x)\big| \le 1-\gamma_2. \tag{10}$$

Then from (5) and (8),

$$\begin{aligned}
&\bigvee_{x\in X} \big|T_A(x) - T_C(x)\big| \\
=\ &\bigvee_{x\in X} \big|T_A(x) - T_B(x) + T_B(x) - T_C(x)\big| \\
\le\ &\bigvee_{x\in X} \big|T_A(x) - T_B(x)\big| + \bigvee_{x\in X} \big|T_B(x) - T_C(x)\big| \\
\le\ & 1-\alpha_1 + 1 - \alpha_2 \\
=\ & 1-(\alpha_1+\alpha_2-1).
\end{aligned}$$

And from the definition 1, $1-(\alpha_1+\alpha_2-1)\in[0,1]$. Thus, $\bigvee_{x\in X}|T_A(x) - T_C(x)| \le 1-\alpha_1*\alpha_2$.

Similarly, we can get $\bigvee_{x\in X}\big|I_A(x) - I_C(x)\big| \le 1-\beta_1*\beta_2$ from (6) and (9), and $\bigvee_{x\in X}\big|F_A(x) - F_C(x)\big| \le 1-\gamma_1*\gamma_2$ from (7) and (10). Thus, $A = (\alpha_1*\alpha_2,\beta_1*\beta_2,\gamma_1*\gamma_2)C$. □

4 (α,β,γ)-equalities with respect to operations

Theorem 9. *If $A = (\alpha,\beta,\gamma)B$, then $A^c = (\gamma,\beta,\alpha)B^c$.*

Proof. Since

$$\bigvee_{x\in X} \big|T_{A^c}(x) - T_{B^c}(x)\big| = \bigvee_{x\in X} \big|F_A(x) - F_B(x)\big| \le 1-\gamma,$$
$$\bigvee_{x\in X} \big|F_{A^c}(x) - F_{B^c}(x)\big| = \bigvee_{x\in X} \big|T_A(x) - T_B(x)\big| \le 1-\alpha,$$

and

$$\begin{aligned}
\bigvee_{x\in X} \big|I_{A^c}(x) - I_{B^c}(x)\big| &= \bigvee_{x\in X} \big|1 - I_A(x) - (1-I_B(x))\big| \\
&= \bigvee_{x\in X} \big|I_A(x) - I_B(x)\big| \\
&\le 1-\beta.
\end{aligned}$$

Then, $A^c = (\gamma,\beta,\alpha)B^c$. □

Remark 10. *In [2], we have $A = (\delta)B \Leftrightarrow A^c = (\delta)B^c$. However, by using Definition 5 we have $A = (\alpha,\beta,\gamma)B \Leftrightarrow A^c = (\gamma,\beta,\alpha)B^c$, where $(\alpha,\beta,\gamma)\ne(\gamma,\beta,\alpha)$. It is consistent with the fact that $A(x) = \big(T_A(x), I_A(x), F_A(x)\big) \Rightarrow A^c(x) = \big(F_A(x), I_A(x), T_A(x)\big)$.*

Example 11. *Let A, B be two single valued neutrosophic sets defined in Example 1, then*

$$A^c = \big\{ <x_1, 0.9, 0.8, 0.1>, <x_2, 1.0, 0.8, 0.1> \big\},$$
$$B^c = \big\{ <x_1, 0.1, 0.8, 0.2>, <x_2, 0.1, 0.9, 0.1> \big\}.$$

It is easy to know that $A^c = (0.1, 0.9, 0.9)B^c$, whereas $A = (0.9, 0.9, 0.1)B$.

However, if we use the distance defined in (4), we obtain $d(A^c, B^c) = d(A,B) = 0.9 = 1-0.1$. Thus we have $A = (0.1)B$ and $A^c = (0.1)B^c$ from Definition 4. This is difficult to know the changes of single valued neutrosophic sets by using the complement operation.

Theorem 12. *If $A_1 = (\alpha_1,\beta_1,\gamma_1)B_1$ and $A_2 = (\alpha_2,\beta_2,\gamma_2)B_2$, then*

$$A_1\cup A_2 = (\alpha_1\wedge\alpha_2, \beta_1\wedge\beta_2, \gamma_1\wedge\gamma_2)B_1\cup B_2, \tag{11}$$
$$A_1\cap A_2 = (\alpha_1\wedge\alpha_2, \beta_1\wedge\beta_2, \gamma_1\wedge\gamma_2)B_1\cap B_2, \tag{12}$$
$$A_1 + A_2 = (\alpha_1*\alpha_2, \beta_1*\beta_2, \gamma_1*\gamma_2)B_1 + B_2, \tag{13}$$
$$A_1\times A_2 = (\alpha_1*\alpha_2, \beta_1*\beta_2, \gamma_1*\gamma_2)B_1\times B_2. \tag{14}$$

Proof. We only give the proof of (11). From lemma 1, we obtain

$$\begin{aligned}
& \bigvee_{x\in X} \left|T_{A_1\cup A_2}(x) - T_{B_1\cup B_2}(x)\right| \\
= & \bigvee_{x\in X} \left|T_{A_1}(x)\vee T_{A_2}(x) - T_{B_1}(x)\vee T_{B_2}(x)\right|, \\
\leq & \max\Big\{\bigvee_{x\in X}\left|T_{A_1}(x) - T_{B_1}(x)\right|, \bigvee_{x\in X}\left|T_{A_2}(x) - T_{B_2}(x)\right|\Big\} \\
\leq & (1-\alpha_1)\vee(1-\alpha_2) \\
\leq & 1-\alpha_1\wedge\alpha_2.
\end{aligned}$$

$$\begin{aligned}
& \bigvee_{x\in X} \left|I_{A_1\cup A_2}(x) - I_{B_1\cup B_2}(x)\right| \\
= & \bigvee_{x\in X} \left|I_{A_1}(x)\wedge I_{A_2}(x) - I_{B_1}(x)\wedge I_{B_2}(x)\right| \\
\leq & \max\Big\{\bigvee_{x\in X}\left|I_{A_1}(x) - I_{B_1}(x)\right|, \bigvee_{x\in X}\left|I_{A_2}(x) - I_{B_2}(x)\right|\Big\} \\
\leq & (1-\beta_1)\vee(1-\beta_2) \\
\leq & 1-\beta_1\wedge\beta_2.
\end{aligned}$$

and

$$\begin{aligned}
& \bigvee_{x\in X} \left|F_{A_1\cup A_2}(x) - F_{B_1\cup B_2}(x)\right| \\
= & \bigvee_{x\in X} \left|F_{A_1}(x)\wedge F_{A_2}(x) - F_{B_1}(x)\wedge F_{B_2}(x)\right| \\
\leq & \max\Big\{\bigvee_{x\in X}\left|F_{A_1}(x) - F_{B_1}(x)\right|, \bigvee_{x\in X}\left|F_{A_2}(x) - F_{B_2}(x)\right|\Big\} \\
\leq & (1-\gamma_1)\vee(1-\gamma_2) \\
\leq & 1-\gamma_1\wedge\gamma_2.
\end{aligned}$$

Thus, $A_1\cup A_2 = (\alpha_1\wedge\alpha_2, \beta_1\wedge\beta_2, \gamma_1\wedge\gamma_2)B_1\cup B_2$. □

Corollary 13. *If $A_k = (\alpha_k, \beta_k, \gamma_k)B_k$ and $k = 1, 2, ..., n$, then*

$$\bigcup_{k=1}^{n} A_k = (\alpha,\beta,\gamma)\bigcup_{k=1}^{n} B_k, \tag{15}$$

$$\bigcap_{k=1}^{n} A_k = (\alpha,\beta,\gamma)\bigcap_{k=1}^{n} B_k, \tag{16}$$

$$\sum_{k=1}^{n} A_k = (\alpha',\beta',\gamma')\sum_{k=1}^{n} B_k, \tag{17}$$

$$\prod_{k=1}^{n} A_k = (\alpha',\beta',\gamma')\prod_{k=1}^{n} B_k, \tag{18}$$

where $\alpha = \bigwedge_{k-1}^{n}\alpha_k$, $\beta = \bigwedge_{k-1}^{n}\beta_k$, $\gamma = \bigwedge_{k-1}^{n}\gamma_k$, $\alpha' = \alpha_1 * \alpha_2 * \cdots * \alpha_n$, $\beta' = \beta_1 * \beta_2 * \cdots * \beta_n$ and $\gamma' = \gamma_1 * \gamma_2 * \cdots * \gamma_n$.

Proof. It can be proven from Theorem 12. □

Theorem 14. *Let A, B be two single valued neutrosophic sets, the following properties hold*

(i) *If $A = (\alpha, \beta, \gamma)B$ and $0 < \lambda \leq 1$, then*

$$\lambda A = (\alpha',\beta',\gamma')\lambda B, \tag{19}$$

$$A^{\lambda} = (\alpha',\beta',\gamma')B^{\lambda}, \tag{20}$$

where $\alpha' = 1-(1-\alpha)^{\lambda}$, $\beta' = 1-(1-\beta)^{\lambda}$ and $\gamma' = 1-(1-\gamma)^{\lambda}$.

(ii) *If $\lambda A = (\alpha, \beta, \gamma)\lambda B$ for some $\lambda \geq 1$, then*

$$A = (\alpha',\beta',\gamma')B, \tag{21}$$

where $\alpha' = 1-(1-\alpha)^{1/\lambda}$, $\beta' = 1-(1-\beta)^{1/\lambda}$ and $\gamma' = 1-(1-\gamma)^{1/\lambda}$.

(iii) *If $A^{\lambda} = (\alpha, \beta, \gamma)B^{\lambda}$ for some $\lambda \geq 1$, then*

$$A = (\alpha',\beta',\gamma')B, \tag{22}$$

where $\alpha' = 1-(1-\alpha)^{1/\lambda}$, $\beta' = 1-(1-\beta)^{1/\lambda}$ and $\gamma' = 1-(1-\gamma)^{1/\lambda}$.

Proof. We only give the proof of (i). From lemma 1 and lemma 2(i), we obtain

$$\begin{aligned}
& \bigvee_{x\in X} \left|T_{\lambda A}(x) - T_{\lambda B}(x)\right| \\
= & \bigvee_{x\in X} \left|1-(1-T_A(x))^{\lambda} - (1-(1-T_B(x))^{\lambda})\right| \\
= & \bigvee_{x\in X} \left|(1-T_A(x))^{\lambda} - (1-T_B(x))^{\lambda}\right| \\
\leq & \bigvee_{x\in X} \left|(1-T_A(x)) - (1-T_B(x))\right|^{\lambda} \\
= & \bigvee_{x\in X} \left|T_A(x) - T_B(x)\right|^{\lambda} \\
\leq & (1-\alpha)^{\lambda} = 1-(1-(1-\alpha)^{\lambda}).
\end{aligned}$$

$$\begin{aligned}
& \bigvee_{x\in X} \left|I_{\lambda A}(x) - I_{\lambda B}(x)\right| \\
= & \bigvee_{x\in X} \left|I_A(x)^{\lambda} - I_B(x)^{\lambda}\right| \\
\leq & \bigvee_{x\in X} \left|I_A(x) - I_B(x)\right|^{\lambda} \\
\leq & (1-\beta)^{\lambda} = 1-(1-(1-\beta)^{\lambda}).
\end{aligned}$$

and

$$
\begin{aligned}
& \bigvee_{x \in X} \left| F_{\lambda A}(x) - F_{\lambda B}(x) \right| \\
= & \bigvee_{x \in X} \left| F_A(x)^\lambda - F_B(x)^\lambda \right| \\
\leq & \bigvee_{x \in X} \left| F_A(x) - F_B(x) \right|^\lambda \\
\leq & (1-\gamma)^\lambda = 1 - (1 - (1-\gamma)^\lambda).
\end{aligned}
$$

Thus, $\lambda A = (\alpha', \beta', \gamma')\lambda B$, where $\alpha' = 1 - (1-\alpha)^\lambda$, $\beta' = 1 - (1-\beta)^\lambda$ and $\gamma' = 1 - (1-\gamma)^\lambda$. □

5 Conclusions

Since a SVNS is characterized by three functions independently, this paper introduced (α, β, γ)-equalities corresponding to characteristic functions of SVNS. The new concept is more comprehensive than the traditional method based distance measure. Firstly, three parameters in the new concept can measure the degree of equality for different characteristic functions (See Example 1). Secondly, the new concept describe the changes of degree of equality with respect to operations more accurate and detailed (See Example 2). Thirdly, since $A = (\alpha, \beta, \gamma)B$ implies $d(A, B) \leq 1 - \alpha \wedge \beta \wedge \gamma$, we can obtain the traditional distance-based parameter by $\delta = \alpha \wedge \beta \wedge \gamma$.

As future work , we can consider the soundness of neutrosophic logic systems and the reliability of neutrosophic fault diagnosis.

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