NSS

University of New Mexico

The Ingenuity of Neutrosophic Topology via *N*-Topology

[1]M. Lellis Thivagar [2] S. Jafari [3]V. Antonysamy [4]V. Sutha Devi

Abstract: In this paper we desire to extend the neutrosophic topological spaces into N-neutrosophic topological spaces. Also we show that this theory can be deduced to N-intuitionistic and N-fuzzy topological spaces etc. Further we develop not only the concept of classical generalized closed sets into N-neutrosophic topological spaces but also obtain its basic properties. Finally we investigate its continuous function and generalized continuous function.

2010 MSC: 54A05, 54A40, 03E72.

Keywords: N_k-topology, $N_k int(A)$, $N_k cl(A)$, N_k-generalized closed set, N_k-continuous function, N_k-generalized continuous function.

1 Introduction

Set theory is the fundamental concept in mathematics developed by a Russian Mathematician George Cantor in 1877. He showed that the points on two dimensional square has a one to one correspondent with points on different line segment leading to the development of dimensional theory. Frechet and Hausdorff along with others studied general topology. Hausdorff, the German mathematician, following the footsteps of Cantor developed set theory. Set theory enabled us to study various precise concepts in mathematics. But in real life situation we do come across many imprecise concepts or uncertain situation. If a class has fifty students say, to distinguish the taller/stronger students we are left with some short of uncertainty or vagueness. We can overcome the vagueness by fixing the percentage of membership namely the percentage of membership enables us to find out the level of inexactness. This theory is known as fuzzy theory.

The concepts of fuzzy set was established by Zadeh.A [12]. This is an essential tool to analyse imprecise mathematical information. Since 1965, this theory has been greatly acknowledged by the community of mathematicians, scientists, engineers and social scientists [4,9,10,11]. The idea of fuzzy topological space was introduced by Chang.C.L [3]. Atanassov.K introduced the seed of intuitionistic fuzzy set [1] and his colleagues [2] developed it further. Smarandache extended it to a neutrosophic set[7,8]. The notion of neutrosophic crisp sets and topological spaces were the contribution of Salama.A.A and Alblowi.S.A [6]. The geometric existence of N-topology was given by Lellis Thivagar et al. [5] which is a nonempty set equipped with N-arbitrary topologies.

In this paper, we explore the possibility of expanding the classical neutrosophic topological spaces into N-neutrosophic topological spaces and also try to deduce N-intuitionistic and N-fuzzy topological spaces etc. Further we develop the concept of classical generalized closed sets into N-neutrosophic topological spaces and verify its properties. Finally, we investigate the related continuous function and generalized continuous function.

2 Preliminaries

In this section, we discuss some basic definitions and properties of N-topological spaces as well as fuzzy, intuitionistic and neutrosophic topological spaces which are useful in sequel.

Definition 2.1 [5] Let X be a non empty set, then $\tau_1, \tau_2, \ldots, \tau_N$ be N-arbitrary topologies defined on X and the collection $N\tau = \{S \subseteq X : S = (\bigcup_{i=1}^{N} A_i) \cup (\bigcap_{i=1}^{N} B_i),\ A_i, B_i \in \tau_i\}$ is called a N-topology on X if the following axioms are satisfied:

(i) $X, \emptyset \in N\tau$.

(ii) $\bigcup_{i=1}^{\infty} S_i \in N\tau$ for all $\{S_i\}_{i=1}^{\infty} \in N\tau$.

(iii) $\bigcap_{i=1}^{n} S_i \in N\tau$ for all $\{S_i\}_{i=1}^{n} \in N\tau$.

Then $(X, N\tau)$ is called a N-topological space on X. The elements of $N\tau$ are known as $N\tau$-open sets on X and its complement is called as $N\tau$-closed on X.

Definition 2.2 [12] Let X be a non empty set. A fuzzy set A is an object having the form $A = \{(x, \mu_A(x)) : x \in X\}$, where $0 \leq \mu_A(x) \leq 1$ represents the degree of membership of each $x \in X$ to the set A.

Definition 2.3 [1,2] Let X be a non empty set. An intuitionistic set A is of the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x)$ and $\gamma_A(x)$ represent the degree of membership and non membership function respectively of each $x \in X$ to the set A and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$.

Definition 2.4 [7] Let X be a non empty set. A neutrosophic set A having the form $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) respectively of each $x \in X$ to the set A. Also $^{-}0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^{+}$ for all $x \in X$.

Remark 2.5 The following definitions can be deduced into fuzzy if the percentages of indeterminacy and non membership are not taken into consideration so also for intuitionistic case the percentage of indeterminacy is not considered.

Definition 2.6 [7] Let X be a non empty neutrosophic set. if $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \sigma_B(x), \gamma_B(x)) : x \in X\}$ are two neutrosophic sets in X, then the following statements hold:

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$.

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(iii) $A^c = \{(x, \gamma_A(x), \sigma_A(x), \mu_A(x)) : x \in X\}$ [Complement of A].

(iv) $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \min\{\sigma_A(x), \sigma_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

(v) $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \max\{\sigma_A(x), \sigma_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

Remark 2.7 Let X be a non empty neutrosophic set. We consider the neutrosophic empty set 0 as $0 = \{(x, 0, 0, 1) : x \in X\}$ and the neutrosophic whole set 1 as $1 = \{(x, 1, 1, 0) : x \in X\}$.

Remark 2.8 By the notion k-set we mean any one of the following sets: fuzzy set, intuitionistic set, neutrosophic set.

Definition 2.9 [6,7] Let X be a non empty set. A k-topology on X is a family ${}_k\tau$ of k-sets in X satisfying the following axioms:

(i) the sets 1 and 0 belong to the family ${}_k\tau$.

(ii) an arbitrary union of sets of the family ${}_k\tau$ belong to ${}_k\tau$.

(iii) the finite intersection of sets of the family ${}_k\tau$ belong to ${}_k\tau$.

Then the ordered pair $(X, {}_k\tau)$ (simply X) is called k-topological space on X. The elements of ${}_k\tau$ are known as k-open sets on X and its complement is called as k-closed on X.

Definition 2.10 [6] The interior and closure of a k-set A of a k-topological space $(X, {}_k\tau)$ are respectively defined as

(i) ${}_kint(A) = \cup\{G : G \subseteq A \text{ and } G \text{ is } k\text{-open in } X\}$.

(ii) ${}_kcl(A) = \cap\{F : A \subseteq F \text{ and } F \text{ is } k\text{-closed in } X\}$.

Corollary 2.11 [7] If A, B, C and D are k-sets in X, then the followings are true:

(i) $A \subseteq B$ and $C \subseteq D \Rightarrow A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$.

(ii) If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$. If $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

(iii) If $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$.

(iv) $(A \cap B)^c = A^c \cup B^c$, $(A \cup B)^c = A^c \cap B^c$ and $(A^c)^c = A$. If $A \subseteq B \Rightarrow B^c \subseteq A^c$.

(v) $1^c = 0$ and $0^c = 1$.

Now, we introduce the notions of image and pre-image of neutrosophic sets. Let us consider X and Y as two non empty sets and $f : X \to Y$ be a function.

Definition 2.12 [6] Let X and Y be two non empty sets, $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ be a neutrosophic set in X and $B = \{(y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y\}$ be a neutrosophic set in Y. Then

(i) the pre-image of B under f, denoted by $f^{-1}(B)$, is the neutrosophic set in X defined by $f^{-1}(B) = \{(x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)) : x \in X\}$.

(ii) the image of A under f, denoted by $f(A)$, is the neutrosophic set in Y defined by $f(A) = \{(y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \gamma_A))(y)) : y \in Y\}$, where

$$f(\mu_A)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$f(\sigma_A)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \sigma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} inf_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

For the sake of simplicity, let us use the symbol $f_-(\gamma_A)$ for $(1 - f(1 - \gamma_A))$.

Corollary 2.13 [6] Let $A_{i \in J}$, $B_{i \in J}$ be k-sets in X and Y respectively and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$.

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

(c) $A_{i \in J} \subseteq f^{-1}(f(A_{i \in J}))$ { If f is injective, then $A_{i \in J} = f^{-1}(f(A_{i \in J}))$}.

(d) $f(f^{-1}(B_{i \in J})) \subseteq B_{i \in J}$ { If f is surjective, then $f(f^{-1}(B_{i \in J})) = B_{i \in J}$}.

(e) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$.

(f) $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$.

(g) $f(\cup A_i) = \cup f(A_i)$.

(h) $f(\cap A_i) \subseteq f(A_i)$ { If f is injective, then $f(\cap A_i) = \cap f(A_i)$}.

(i) $f^{-1}(1) = 1$.

(j) $f^{-1}(0) = 0$.

(k) $f(1) = 1$, if f is surjective.

(l) $f(0) = 0$.

(m) $(f(A_{i\in J}))^c \subseteq f(A^c_{i\in J})$, if f is surjective.

(n) $(f^{-1}(B_{i\in J}))^c = f^{-1}(B^c_{i\in J})$.

3 N_k-Topological Spaces

In this section, we introduce N-fuzzy, N-intuitionistic and N-neutrosophic topological spaces and discuss their properties. Henceforth in this paper by the notion $N_k\tau$ we mean N-fuzzy topology (if $k = f$), N-intuitionistic topology (if $k = i$) and N-neutrosophic topology (if $k = n$).

Definition 3.1 Let X be a non empty set, then ${}_k\tau_1, {}_k\tau_2, \dots, {}_k\tau_N$ be N-arbitrary k topologies defined on X and the collection $N_k\tau = \{G \subseteq X : G = (\bigcup_{i=1}^{N} A_i) \cup (\bigcap_{i=1}^{N} B_i),\ A_i, B_i \in {}_k\tau_i\}$ is called N_k-topology on X if the following axioms are satisfied:

(i) $1, 0 \in N_k\tau$.

(ii) $\bigcup_{i=1}^{\infty} G_i \in N_k\tau$ for all $\{G_i\}_{i=1}^{\infty} \in N_k\tau$.

(iii) $\bigcap_{i=1}^{n} G_i \in N_k\tau$ for all $\{G_i\}_{i=1}^{n} \in N_k\tau$.

Then $(X, N_k\tau)$ is called N_k-topological space on X. The elements of $N_k\tau$ are known as N_k-open sets on X and its complement is called N_k-closed sets on X.

Example 3.2 Let $N = 3$, $X = \{a, b, c\}$. Define the neutrosophic sets $A = \{(x, (\frac{a}{1}, \frac{b}{1}, \frac{c}{1}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}))\}$ and $B = \{(x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}))\}$ in X. Then $A \cup B = \{(x, (\frac{a}{1}, \frac{b}{1}, \frac{c}{1}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}))\}$, $A \cap B = \{(x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}))\}$. Considering ${}_n\tau_1O(X) = \{0, 1, A\}$, ${}_n\tau_2O(X) = \{0, 1, B\}$ and ${}_n\tau_3O(X) = \{0, 1\}$, we get $3_n\tau O(X) = \{0, 1, A, B, A \cup B, A \cap B\}$ which is a tri-neutrosophic topology on X. The pair $(X, 3_n\tau)$ is called a tri-neutrosophic topological space on X.

Remark 3.3 Considering $N = 2$ in definition 3.1 we get the required definition of bi-neutrosophic topology on X. The pair $(X, 2_n\tau)$ is called a bi-neutrosophic topological space on X.

Example 3.4 Let $N = 2$, $X = \{a, b, c\}$. Define the neutrosophic set $A = \{(x, (\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.3}))\}$ in X. If ${}_n\tau_1O(X) = \{0, 1, A\}$ and ${}_n\tau_2O(X) = \{0, 1\}$ are two neutrosophic topologies then we get $2_n\tau O(X) = \{0, 1, A\}$ which is a bi-neutrosophic topology on X.

Definition 3.5 Let $(X, N_k\tau)$ be a N_k-topological space on X and A be a k-set on X then the $N_kint(A)$ and $N_kcl(A)$ are respectively defined as

(i) $N_kint(A) = \cup\{G : G \subseteq A$ and G is a N_k-open set in $X\}$.

(ii) $N_kcl(A) = \cap\{F : A \subseteq F$ and F is a N_k-closed set in $X\}$.

Proposition 3.6 Let $(X, N_k\tau)$ be any N_k-topological space. If A and B are any two k-sets in $(X, N_k\tau)$, then the N_k-closure operator satisfy the following properties:

(i) $A \subseteq N_kcl(A)$.

(ii) $N_kint(A) \subseteq A$.

(iii) $A \subseteq B \Rightarrow N_kcl(A) \subseteq N_kcl(B)$.

(iv) $A \subseteq B \Rightarrow N_kint(A) \subseteq N_kint(B)$.

(v) $N_kcl(A \cup B) = N_kcl(A) \cup N_kcl(B)$.

(vi) $N_kint(A \cap B) = N_kint(A) \cap N_kint(B)$.

(vii) $(N_kcl(A))^c = N_kint(A)^c$.

(viii) $(N_kint(A))^c = N_kcl(A)^c$.

Proof

(i) $N_kcl(A) = \cap\{G : G$ is a N_k-closed set in X and $A \subseteq G\}$. Thus, $A \subseteq N_kcl(A)$.

(ii) $N_kint(A) = \cup\{G : G$ is a N_k-open set in X and $G \subseteq A\}$. Thus, $N_kint(A) \subseteq A$.

(iii) $N_kcl(B) = \cap\{G : G$ is a N_k-closed set in X and $B \subseteq G\} \supseteq \cap\{G : G$ is a N_k-closed set in X and $A \subseteq G\} \supseteq N_kcl(A)$. Thus, $N_kcl(A) \subseteq N_kcl(B)$.

(iv) $N_kint(B) = \cup\{G : G$ is a N_k-open set in X and $B \supseteq G\} \supseteq \cup\{G : G$ is a N_k-open set in X and $A \supseteq G\} \supseteq N_kint(A)$. Thus, $N_kint(A) \subseteq N_kint(B)$.

(v) $N_kcl(A \cup B) = \cap\{G : G$ is a N_k-closed set in X and $A \cup B \subseteq G\} = (\cap\{G : G$ is a N_k-closed set in X and $A \subseteq G\}) \cup (\cap\{G : G$ is a N_k-closed set in X and $B \subseteq G\}) = N_kcl(A) \cup N_kcl(B)$. Thus, $N_kcl(A \cup B) = N_kcl(A) \cup N_kcl(B)$.

(vi) $N_kint(A \cap B) = \cup\{G : G$ is a N_k-open set in X and $A \cap B \supseteq G\} = (\cup\{G : G$ is a N_k-open set in X and $A \supseteq G\}) \cap (\cup\{G : G$ is a N_k-open set in X and $B \supseteq G\}) = N_kint(A) \cap N_kint(B)$. Thus, $N_kint(A \cap B) = N_kint(A) \cap N_kint(B)$.

(vii) $N_kcl(A) = \cap\{G : G$ is a N_k-closed set in X and $A \subseteq G\}, (N_kcl(A))^c = \cup\{G^c : G^c$ is a N_k-open set in X and $A^c \supseteq G^c\} = N_kint(A)^c$. Thus, $(N_kcl(A))^c = N_kint(A)^c$.

(viii) $N_kint(A) = \cup\{G : G$ is a N_k-open set in X and $A \supseteq G\}, (N_kint(A))^c = \cap\{G^c : G^c$ is a N_k-closed set in X and $A^c \supseteq G^c\} = N_kcl(A)^c$. Thus, $(N_kint(A))^c = N_kcl(A)^c$.

4 Generalized Closed Sets in N_k-topology

We introduce here the generalized closed sets in N_k-topological spaces and investigate their properties.

Definition 4.1 Let $(X, N_k\tau)$ be a N_k-topological space. A k-set A in $(X, N_k\tau)$ is said to be a N_k-generalized closed set if $N_kcl(A) \subseteq G$, whenever $A \subseteq G$ and G is a N_k-open set. The complement of a N_k-generalized closed set is called a N_k-generalized open set.

Definition 4.2 Let $(X, N_k\tau)$ be a N_k-topological space and A be a k-set in X. Then the N_k-generalized closure and N_k-generalized interior of A are defined as:

(i) $N_kGcl(A) = \cap\{G : G$ is a N_k-generalized closed set in X and $A \subseteq G\}$

(ii) $N_kGint(A) = \cup\{G : G$ is a N_k-generalized open set in X and $G \subseteq A\}$.

Proposition 4.3 Let $(X, N_k\tau)$ be any N_k-topological space. If A and B are any two k-sets in $(X, N_k\tau)$, then the N_k-generalized closure operator satisfies the following properties:

(i) $A \subseteq N_kGcl(A)$.

(ii) $N_kGint(A) \subseteq A$.

(iii) $A \subseteq B \Rightarrow N_kGcl(A) \subseteq N_kGcl(B)$.

(iv) $A \subseteq B \Rightarrow N_kGint(A) \subseteq N_kGint(B)$.

(v) $N_kGcl(A \cup B) = N_kGcl(A) \cup N_kGcl(B)$.

(vi) $N_kGint(A \cap B) = N_kGint(A) \cap N_kGint(B)$.

(vii) $(N_kGcl(A))^c = N_kGint(A)^c$.

(viii) $(N_kGint(A))^c = N_kGcl(A)^c$.

Proof The proof is analogous to Proposition 3.6.

Proposition 4.4 Let $(X, N_k\tau)$ be a N_k-topological space. If B is a N_k-generalized closed set and $B \subseteq A \subseteq N_kcl(B)$, then A is a N_k- generalized closed set.

Proof. Let G be a N_k-open set in $(X, N_k\tau)$ such that $A \subseteq G$. Since $B \subseteq A, B \subseteq G$. Now, B is a N_k-generalized closed set and $N_kcl(B) \subseteq G$. But $N_kcl(A) \subseteq N_kcl(B)$. Since $N_kcl(A) \subseteq N_kcl(B) \subseteq G, N_kcl(A) \subseteq G$. Hence, A is a N_k-generalized closed set.

Proposition 4.5 Let $(X, N_k\tau)$ be a N_k-topological space. Then A is a N_k-generalized open set if and only if $B \subseteq N_kint(A)$, whenever B is an N_k-closed set and $B \subseteq A$.

Proof. Let A be a N_k-generalized open set and B a N_k-closed set such that $B \subseteq A$. Now, $B \subseteq A \Rightarrow A^c \subseteq B^c$ and since A^c is a N_k-generalized closed set, then $N_k cl(A^c) \subseteq B^c$. This means that $B = (B^c)^c \subseteq (N_k cl(A^c))^c$. But $(N_k cl(A^c))^c = N_k int(A)$. Hence, $B \subseteq N_k int(A)$. Conversely, suppose that A is a k-set such that $B \subseteq N_k int(A)$, whenever B is a N_k-closed set and $B \subseteq A$. Now, $A^c \subseteq B \Rightarrow B^c \subseteq A$. Hence by assumption, $B^c \subseteq N_k int(A)$. That is, $(N_k int(A))^c \subseteq B$. But $(N_k int(A))^c = N_k cl(A)^c$. Hence, $N_k cl(A)^c \subseteq B$. This means that A is a N_k-generalized closed set. Therefore, A is a N_k-generalized open set.

Proposition 4.6 If $N_k int(A) \subseteq B \subseteq A$ and A is a N_k-generalized open set, then B is also a N_k-generalized open set.
Proof. Now, $A^c \subseteq B^c \subseteq (N_k int(A))^c = N_k cl(A)^c$. Since A is a N_k-generalized open set, then A^c is a N_k-generalized closed set. By Proposition 3.6, B^c is a N_k-generalized closed set. That is, B is a N_k-generalized open set.

5 Continuous Functions in N_k-Topology

In this section, we generalize continuous functions in N-neutrosophic topological spaces and also establish its relationship with other existing continuous functions.

Definition 5.1 Let $(X, N_k\tau))$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. A map $f : (X, N_k\tau) \to (Y, N_k\sigma)$ is said to be N_k-continuous if the inverse image of every N_k-closed set in $(Y, N_k\sigma)$ is a N_k-closed set in $(X, N_k\tau)$. Equivalently if the inverse image of every N_k-open set in $(Y, N_k\sigma)$ is a N_k-open set in $(X, N_k\tau)$.

Remark 5.2 By considering $N = 2$ in definition 5.1 we obtain bi-neutrosophic continuous function.

The following properties can be extended to N-fuzzy and N-intuitionistic topological spaces too.

Proposition 5.3 Let $(X, N_k\tau))$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. Let $f : (X, N_k\tau) \to (Y, N_k\sigma)$ be a N_k- continuous function. Then for every k-set A in X, $f(N_k cl(A)) \subseteq N_k cl(f(A))$.

Proof. Let A be a k-set in $(X, N_k\tau)$. Since $N_k cl(f(A))$ is a N_k-closed set and f is a N_k-continuous function, $f^{-1}(N_k cl(f(A))$ is a N_k-closed set and $f^{-1}(N_k cl(f(A))) \supseteq A$. Now, $N_k cl(A) \subseteq f^{-1}(N_k cl(f(A)))$. Therefore, $f(N_k cl(A)) \subseteq N_k cl(f(A))$.

Proposition 5.4 Let $(X, N_k\tau)$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. Let $f : (X, N_k\tau) \to (Y, N_k\sigma)$ be a N_k-continuous function. Then for every N_k-set A in Y, $N_k cl(f^{-1}(A)) \subseteq f^{-1}(N_k cl(A))$.

Proof. Let A be a N_k-set in $(Y, N_k\sigma)$. Let $B = f^{-1}(A)$. Then, $f(B) = f(f^{-1}(A)) \subseteq A$. By Proposition 5.3, $f(N_k cl(f^{-1}(A))) \subseteq N_k cl(f(f^{-1}(A)))$. Thus, $N_k cl(f^{-1}(A)) \subseteq f^{-1}(N_k cl(A))$.

Definition 5.5 Let $(X, N_k\tau))$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. A map $f : (X, N_k\tau) \to (Y, N_k\sigma)$ is said to be N_k-generalized continuous if the inverse image of every N_k-closed set in $(Y, N_k\sigma)$ is a N_k-generalized closed set in $(X, N_k\tau)$. Equivalently if the inverse image of every N_k-open set in $(Y, N_k\sigma)$ is a N_k-generalized open set in $(X, N_k\tau)$.

Remark 5.6 For $N = 2$ in the above definition we aquire the needed definition of bi-generalized neutrosophic continuous function.

Proposition 5.7 Let $(X, N_k\tau))$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. Let $f : (X, N_k\tau) \to (Y, N_k\sigma)$ be a N_k-generalized continuous function. Then for every N_k-set A in $X, f(N_kGcl(A)) \subseteq N_kcl(f(A))$.

Proof. Let A be a N_k-set in $(X, N_k\tau)$. Since $N_kcl(f(A))$ is a N_k-closed set and f is a N_k-continuous function, $f^{-1}(N_kcl(f(A))$ is a N_k-generalized closed set and $f^{-1}(N_kcl(f(A))) \supseteq A$. Now, $N_kGcl(A) \subseteq f^{-1}(N_kcl(f(A)))$. Therefore, $f(N_kGcl(A)) \subseteq N_kcl(f(A))$.

Proposition 5.8 Let $(X, N_k\tau)$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. Let $f : (X, N_k\tau) \to (Y, N_k\sigma)$ be a N_k-generalized continuous function. Then for every N_k-set A in Y, $N_kGcl(f^{-1}(A)) \subseteq f^{-1}(N_kcl(A))$.

Proof. Let A be a N_k-set in $(Y, N_k\sigma)$. Let $B = f^{-1}(A)$. Then, $f(B) = f(f^{-1}(A)) \subseteq A$. By Proposition 5.7, $f(N_kGcl(f^{-1}(A))) \subseteq N_kcl(f(f^{-1}(A)))$. Thus, $N_kGcl(f^{-1}(A)) \subseteq f^{-1}(N_kcl(A))$.

Proposition 5.9 Let $(X, N_k\tau)$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. If $f : (X, N_k\tau) \to (Y, N_k\sigma)$ is a N_k-continuous function, then it is a N_k-generalized continuous function.

Proof. Let A be a N_k-open set in $(Y, N_k\sigma)$. Since f is a N_k-continuous function, $f^{-1}(A)$ is a N_k-open set in $(X, N_k\tau)$. Every N_k-open set is a N_k-generalized open set. Now, $f^{-1}(A)$ is a N_k-generalized open set in $(X, N_k\tau)$. Hence, f is a N_k-generalized continuous function.
The converse of Proposition 5.9 need not be true as it is shown in the following example.

Example 5.10 Let $N = 2$, $X = \{a, b, c\}$ and $Y = \{p, q, r\}$. Define the neutrosophic sets $A = \{(x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.3}))\}$ in X and $B = \{(y, (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}), (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}))\}$ in Y. Considering ${}_n\tau_1O(X) = \{0, 1, A\}$ and ${}_n\tau_2O(X) = \{0, 1\}$ we get $2_n\tau O(X) = \{0, 1, A\}$. Also by considering ${}_n\sigma_1O(Y) = \{0, 1\}$ and ${}_n\sigma_2O(Y) = \{0, 1, B\}$ we get $2_n\sigma O(Y) = \{0, 1, B\}$. Thus, $(X, 2_n\tau)$ and $(Y, 2_n\sigma)$ are bi-neutrosophic topological space on X and Y, respectively. Define $f : X \to Y$ as $f(a) = q, f(b) = p, f(c) = r$. Then f is bi-generalized neutrosophic continuous but not bi-neutrosophic continuous.

Conclusion

Neutrosophic topology is well equipped to deal with imprecise data. By employing neutrosophic set in spacial data models, we can express the vagueness of the object as expected. This paper has gone a step forward in extending the theory to N-neutrosophic topology that can be used to determine the uncertain situation effectively. Further we also extended the same to N-Fuzzy and N-Intuitionistic topologies and discussed not only the relations but also its properties.

References

[1] Atanassov. K, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20, 1986, 87-96.

[2] Atanassov. K. and Stoeva. S, *Intuitionistic fuzzy sets*, in Polish Syrup. on Interval & Fuzzy Mathematics, Poznan, 1983, 23-26.

[3] Chang. C.L, *Fuzzy topological spaces*, J. Math. Anal. Appl., 24, 1968, 182-190.

[4] Dhavaseelan. R, Roja. E and Uma. M. K, *Generalized intuitionistic fuzzy closed sets*, Advances in Fuzzy Mathematics, 5, 2010, 152-172.

[5] Lellis Thivagar. M, Ramesh.V, Arockia Dasan. M, *On new structure of N-topology*, Cogent Mathematics (Taylor and Francis),3, 2016:1204104.

[6] Salama. A. A. and Albowi. S. A, *Neutrosophic Set and Neutrosophic Topological Spaces*, IOSR Journal of Mathematics, 3(4), 2012, 31-35.

[7] Smarandache. F, *Neutrosophy and Neutrosophic Logic*, First International Conference on Neutrosophy, Neutrosophic Logic , Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002).

[8] Smarandache. F, *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability*, American Research Press, Rehoboth, NM, 1999.

[9] Smets. P, *The degree of belief in a fuzzy event*, Information sciences 25, 1981, 1-19.

[10] Sugeno. M, *An Introductory survey of fuzzy control*, Information sciences, 36, 1985, 59-83.

[11] Thakur. S. S and Chaturvedi. R, *Generalized continuity in intuitionistic fuzzy topological spaces*, NIFS, 12(1), 2006, 38-44.

[12] Zadeh. L. A, *Fuzzy sets*, Inform. and Control, 8, 1965, 338-353.

[1,3,4]School of Mathematics
Madurai Kamaraj University
Madurai - 625 021, Tamil Nadu, INDIA
[2]Department of Mathematics
College of Vestsjaelland South
Herrestraede 11, 4200 Slagelse, Denmark.

[1]E-mail: mlthivagar@yahoo.co.in
[2]E-mail: jafaripersia@gmail.com
[3]E-mail: tonysamsj@yahoo.com
[4]E-mail: vsdsutha@yahoo.co.in

Received : February 19, 2018. **Accepted** : March 12, 2018.