Newton's First Law Revisited

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Abstract: Newton's first law is expressed in textual form. It states that, *unless acted upon by a net unbalanced force, an object will remain at rest, or move uniformly forward in a straight line*. Accordingly, "inertial motion" means uniform rectilinear motion, while uniform circular motion is considered to be a noninertial, accelerated motion. This differentiation has resulted in the aftermath in different analytical treatments of the two types, both in classical and relativistic physics. In this short note, we show, based on Newton's laws, that, contrary to the conventional differentiation between rectilinear and circular systems of motion, the two are *dynamically equivalent*, such that the set of laws describing the dynamics of one system correspond to an identical set of laws describing the dynamics of the second. An immediate corollary for the special case of uniform motion is that Newtonian physics is inconsistent with his first law and is, instead, consistent with Galileo's definition of inertial motion. To rectify the apparent inconsistency, we propose a natural modification of the first law, which incorporates the case of uniform circular motion. By means of this inclusion, the first law can retain its unique standing, as a separate law that could not be deduced from the second law as a special case. We formulate the modified law textually and mathematically and comment briefly on the implication of the proposed modification to the theory and education of classical, and relativistic physics.

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1. Introduction

Newton's first law exists solely in textual form, thus making it difficult to eliminate the ambiguity regarding what it exactly tells us.[1] Nonetheless, it articulates a basic principle of nature, which, has passed untouched to relativity theories and all modern physics. In a commonly used English translation of Newton's seminal *Principia,* [2] the first law states that *"every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it"* (p. 13). Another common translation of the law states that, *unless acted upon by a net unbalanced force, an object will remain at rest, or move uniformly forward in a straight line.* [3] Yet there are many other translations that diverge from each other due to the inherent vagueness in the original text and the different nuances opted by different translators.[1] However, despite such differences, all statements of the law agree that an inertial motion is *motion in uniform velocity in a straight line*. Accordingly, uniform circular motion is considered to be a noninertial, accelerated motion. This differentiation between uniform rectilinear and uniform circular motions is not semantic. In fact, it is the source of the different analytical treatments of the two types of motion, both in classical and relativistic physics. In special relativity theory (SRT),[4] the formalization of the "relativity axiom" and derivation of the theory transformations assume *constant rectilinear motion* and, thus, are not supposed to apply to other types of motion, such as circular motion, which, according to Newton's first law and Einstein SRT, is a noninertial, accelerated motion.

Here, we challenge this convention by demonstrating that there is a one-to-one correspondence between the dynamics of rectilinear and circular types of motion. In the language of systems analysis, we show that the two types of motion are *equivalent systems*.[5,6] Before we do that it is in place to emphasize that the definition adopted here of the word *dynamics,* differs in a significant way from the customary connotation of the term in physics, where it refers to what was called before the mid-20th century *kinematics*, i.e., the study of motion along with their cause (forces and torques). This customary differentiation, on the bases on force-dependency, was probably needed in adopted because it separates between motion under zero net forces, termed inertial, covered by Newton's first law, and motion under the influence of non-zero net forces (and acceleration), covered by Newton's second law. However, the aspired separation remains unconvincing, because the zero-forces case remains a special case of the second law.

A similar standing is held by many. As example, in one textbook after stressing the 'logical redundancy in Newton's laws', the author proceeds to say that "Since Newton's first law is thus merely a special case of the second, the question may be raised as to why it is separately stated at all. Is it not merely a redundancy? The answer to this is clearly in the affirmative. The second law having once been stated, the first is definitely a redundancy and is not logically required"[7] (p. 130).

Notwithstanding the debates of whether the first law was given a special standing by Newton, or that it was a result of the chronical order in which Newton addressed the topic, we opt for the universal meaning of the word "dynamic", as it is understood in mathematics, and in system analysis, namely the time-dependency of a system's variables, and their functional associations, and not the force-dependency connotation. In addition to its universality, perceiving the dynamics of a physical system as tied to the characteristics of its time dependence, is also more congruent with GRT and similar theories, which abolishes the notion of force.

2. On the equivalence between rectilinear and circular motion

Demonstration of the equivalence between the rectilinear and circular forms motion is trivial. Consider a dynamical system of any type (physical, biological, social, etc.), which could be completely defined by a set of dynamical parameters p_i ($i = 1, 2, .. , n$), and a set of equations R defined in (1):

$$R = \{p_2 = \dot{p}_1, p_3 = \ddot{p}_1, p_5 = p_3\, p_4, p_6 = \int p_5 dp_1, p_7 = \frac{1}{2} p_4\, p_2{}^2\}. \qquad (1)$$

If we think of p_1, p_2, p_3 as representing rectilinear position x, velocity v, and acceleration a, respectively, and of p_4, p_5, p_6, p_7 as mass m, rectilinear force F, work W, and kinetic energy E, respectively; then, the dynamical system defined by R gives a full description of a classical *rectilinear motion* (see Table 1). Alternatively, if we think of p_1, p_2, p_3 as representing angular position θ, velocity w, and acceleration α, respectively, and of p_4, p_5, p_6, p_7 as radial inertia I, torque τ, work W, and kinetic energy E, respectively (see Table 1), then the dynamical system defined by R gives a full description of a classical *circular motion* (Q.E.D.).

A strong support for of our conclusion regarding the equivalence between the circular and rectilinear types of motion is provided by experiments conducted by Wang and his colleagues on the "generalized Sagnac effect". As well known, the Sagnac effect is a phase shift observed between two beams of light traveling in opposite directions along the same closed path around a moving object[8-11]. The circular Sagnac effect is a special case of the general Sagnac effect, which has crucial applications in fiber-optic gyroscopes (FOGs)[12-14] and in navigation systems such as GPS [11,15]. Wang and his colleagues[16-18] conducted experiments demonstrating that an *identical* Sagnac effect, to the one found in circular motion, exists in rectilinear uniform motion. Using an optical fiber conveyor, the authors measured the travel-time difference between two counter propagating light beams in a uniformly moving fiber. Their finding revealed that the travel-time difference in a fiber segment of length Δl moving at a speed v, was equal to $\Delta t = 2v\Delta l/c2$, whether the segment was moving uniformly in rectilinear or circular motion.

In fact, a rectilinear inertial motion, according to Newton's definition, cannot exist in reality because there are always forces acting on a body with mass. The closest approximation of a rectilinear inertial motion is the motion of a body on a perfectly horizontal and frictionless surface, like a billiard ball on a pool table. However, given the curvature of all planets, even in terrestrial settings natural rectilinear motion is an approximation. In contrast, natural circular motion is a good approximation of the motion of almost all bodies, from rotating galaxies to spinning particles. In fact, the rectilinear path could be thought of as the limit of the circular path as its radius approaches infinity. Since the nature of the circular motion is independent of the circle's radius (as long as the centripetal force equation of the circular motion is respected), and since there are no singularities in the dynamical equations of circular motion, we may conclude that there should not be any difference in any dynamical aspect, between the limit case of rectilinear motion, and the continuum of circular motion. This conclusion adds further support to the formal proof of the equivalence between the circular and rectilinear types of motion.

3. Implications to the case of uniform motion

The proven equivalence between rectilinear and circular dynamical motion holds in general, and is not restricted to the special case of uniform types of motion. However, the uniform motion is particularly interesting due to its intimate relation to Newton's first law and to special relativity theory. An immediate corollary of the above proven equivalence is that Newton's first law, as articulated by Newton and adopted by Einstein and others, is inconsistent with Newtonian mechanics, which shows without any doubt that the kinetics of circular uniform motion are identical to the kinetics of rectilinear uniform motion; thus, if the latter is inertial, then so is the former. In fact, we can "translate" Newton's first law from its linear coordinates to radial coordinates simply by replacing, in the original statement of the law, the words "straight line" by the word "circle," thus yielding the following law:

Every body continues in its state of uniform rotation in a circle, unless it is compelled to change that state of motion.

Quite interestingly, we found that our view of what defines an inertial system is in complete agreement with Galileo's interpretation of inertia. In Galileo's words: "All external impediments removed, a heavy body on a spherical surface concentric with the earth will maintain itself in that state in which it has been; if placed in movement toward the west (for example), it will maintain itself in that movement."[19]. This notion, which is termed "circular inertia" or "horizontal circular inertia" by historians of science, is a precursor to Newton's notion of rectilinear inertia.[20,21] The inclusion of both uniform rectilinear and circular motions under the umbrella of "inertial" motion. Was also advocated by Aristotle, who classified local motion to *natural* and *violent*, considering the *celestial* uniform circular motion, and the *terrestrial* uniform rectilinear motion, as the only natural forms of natural motion, whereas all other forms of motion were classified by him as violent[22].

4. Restatement and formalization of Newton's first law

Encouraged by the agreement between our view of inertial motion and Galileo's view, we dare to put forward the following formal definition of an inertial motion, which encompasses both the rectilinear and the circular types of motion. According to the proposed definition:

A rigid body is said to be in a state of inertial motion if and only if the scalar product between the sum of all the forces acting on the body and its velocity vector is always equal to zero.

Or in mathematical notation:

$$\left(\sum \overrightarrow{\boldsymbol{F}_{\iota}}(t)\right) \cdot \vec{\boldsymbol{v}}(t) = 0, \quad \text{for all } t. \tag{2}$$

Note that the condition in Eq. (2) is satisfied (under ideal conditions) only by a state of rest, as well as by uniform rectilinear and circular types of motion. According to equality in Eq. (2), an inertial state requires either that $\|\vec{\boldsymbol{v}}(t)\| = 0$, which describes a state of rest, or when $\|\vec{\boldsymbol{v}}(t)\| \neq 0$, but $\left\|\sum \overrightarrow{\boldsymbol{F}_{\iota}}(t)\right\| = 0$, which describes the case of uniform rectilinear motion with zero net force, and also when $\|\vec{\boldsymbol{v}}(t)\| \neq 0$, and $\left\|\sum \overrightarrow{\boldsymbol{F}_\iota}(t)\right\| \neq 0$, but the net force acting on the body is always orthogonal to the velocity vector, which is the case of uniform circular motion. As far as classical systems are concerned, we conjecture that what seems to characterize inertial motion, whether in circular or motion, is that the kinetic energy of the moving body in both cases is constant in time.

It is worth stressing that by including under its umbrella the uniform circular motion, the first law can retain its unique standing as a separate law that could not be deduced from the second law as a special case.

5. Some implications to classical and relativistic physics

The above restatement of Newton's first law has significant implications to classical and relativistic physics. For example, in classical analysis of many body systems in circular motion, as in fiber-optic gyroscopes[14,15], or in approximation of planetary dynamics [24,25], one can simplify the analysis by first solving a rectilinear identical system and then "translating" the solution from one systems of coordinates to another, simply by using a "dictionary" like the one depicted in Table 1, i.e., simply by replacing in the derived solution the variables x with θ, v with w, and so forth.

Another implication to classical physics concerns the teaching of rotational motion, where a fictitious centrifugal force, which has no actor, is usually added to balance the real centripetal force acting on the body[26,27]. Our proposed definition in Eq. (2) makes such artificial and nonphysical addition superfluous.

With regard to relativistic physics, the justified inclusion of uniform circular motion as another form of inertial motion, creates a serious problem for SRT, because it is founded of the customary narrow definition of Newton's the first law, which limits the its scope only to uniform rectilinear motion. But since we have proven the identity between circular and rectilinear motion, SRT should also apply to the former type of motion. But this poses a serious problem, as it sharpens the debated contradiction between SRT and the Sagnac effect, by making it possible to pit the two against each other – not only in rectilinear motion, but also in circular motion.

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Table 1

Dynamical equations of rectilinear and circular systems

Variable	Rectilinear	Circular	General
Position	x	θ	p_1
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$	$p_2 = \frac{dp_1}{dt}$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$	$p_3 = \frac{dp_2}{dt}$
Mass/Inertia	M	I	p_4
Newton's second law	$F = ma$	$\tau = I\,\alpha$	$p_5 = p_4\, p_3$
Work	$W = \int F dx$	$W = \int \tau\, d\theta$	$p_6 = \int p_5 dp_1$
Kinetic energy	$E = \frac{1}{2} mv^2$	$E = \frac{1}{2} I\, \omega^2$	$p_7 = \frac{1}{2} p_4\, {p_2}^2$
	…….	…….	…….