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RETRACT NEUTROSOPHIC CRISP SYSTEM FOR GRAY SCALE IMAGE

A. A. SALAMA[1], HEWAYDA EL GHAWALBY[2] AND ASMAA M. NASR[2*]

[1]Department of Mathematics and Computer Science, Faculty of Science, Port Said University, Egypt.
[2]Department of Physics and Engineering Mathematics, Faculty of Engineering, Port Said University, Egypt.

AUTHORS' CONTRIBUTIONS

This work was carried out in collaboration between all authors. Author AAS designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Authors HEG and AMN managed the analyses of the study. Author AMN managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

In this paper, we aim to develop a new type of neutrosophic crisp set called the retract neutrosophic crisp set and shows a grayscale image in a 2D Cartesian domain with neutrosophic crisp components in the neutrosophic domain. The introduced set is a retraction of any triple structured crisp set. Whereas, the retract set deduced from any neutrosophic crisp set is coincide its corresponding star neutrosophic crisp set defined in by Salama et al. [1]. Hence we construct a new type of neutrosophic crisp topological spaces, called the retract neutrosophic crisp topological space as a retraction of the star neutrosophic topological space. Moreover, we investigate some of its properties.

Keywords: Neutrosophic crisp set; neutrosophic crisp topological space**;** star neutrosophic crisp set.

1 Introduction

Basically, topology is the modern version of geometry, the study of all different sorts of spaces. In 1992, Samarandache introduced the new concept of neutrosophy [2,3]; which is considered as a new branch of philosophy that studies the origin, nature and the scope of their neutralities. Leading to a new era, the neutrosophy had founded a whale family of new mathematical theories which form a generalization for the classical counterpart. As well as the counterparts; founded by Zadah in 1965 [4], such that neutrosophic set theory, Neutrosophic Probability Set and Logic, Neutrosophic Topological Spaces Neutrosophic Crisp Set

**Corresponding author: Email: rsalama44@gmail.com;*

Theory, Generalized Neutrosophic Set and Ultra Neutrosophic Crisp Sets and Relations. For instance, the work done by Salama et al. [5-12]. After the development of fuzzy sets, much attention has been paid to the generalization of basic concepts of classical topology to fuzzy sets and accordingly developing a theory of fuzzy topology [5,9,3]. In 1983, the intuitionistic fuzzy set was introduced by K. Atanassov [13] as a generalization of the fuzzy set, beyond the degree of membership and the degree of non-membership of each element. In 1983, K. Atanassov introduced a generalization for the fuzzy set, the intuitionistic fuzzy set [13]. While the elements in Zadah's fuzzy set have one value, which represents the membership degree of each element; the elements in the intuitionistic fuzzy set have two values, which represent both the membership degree and the non-membership degree. In [10], the authors introduced the concepts of neutrosophic crisp topological spaces as a generalization of the general topology. Neutrosophy was pioneered by Smarandache [2, 3] in 1995. It is a new branch of philosophy, which studies the origin, nature, and scope of neutralities. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. The fuzzy set was introduced by Zadeh [4] in 1965, where each element had a degree of membership. The intuitionistic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [13] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Some neutrosophic concepts have been investigated by Salama et al. [5-12]. This paper is devoted for developing develop a new type of neutrosophic crisp set called the retract neutrosophic crisp set. The introduced set is a retraction of any triple structured crisp set. Whereas, the retract set deduced from any neutrosophic crisp set is coincide its corresponding star neutrosophic crisp set defined in by Salama et al. in [1]. Hence we develop a new type of neutrosophic crisp topological spaces, called the retract neutrosophic crisp topological space as a retraction of the star neutrosophic topological space. Moreover, we investigate some of its properties. The structure of the paper is as follows:

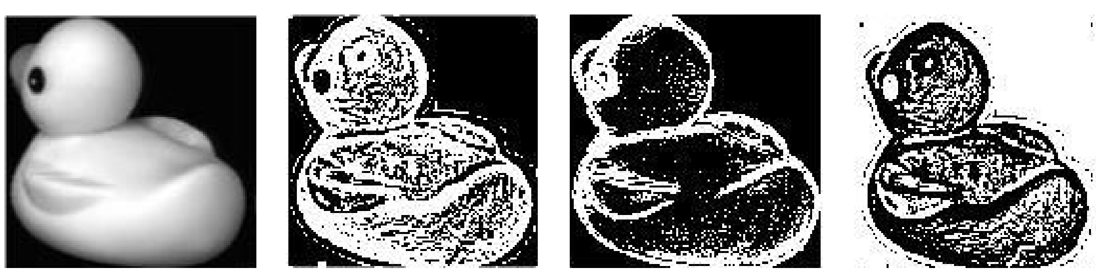
2 Neutrosophic Crisp Sets

In this section, we recall some definitions for essential concepts of neutrosophic crisp sets and its operations, which were introduced by Salama and Smarandachein [5,9].

2.1 Definition [5]

Let X be a non-empty fixed sample space, a neutrosophic crisp set A (NCS for short), can be defined as a triple of the form $\langle A_1, A_2, A_3 \rangle$, where A_1, A_2 and A_3 are crisp subsets of X. The three components represent a classification of the elements of the space X according to the event A; the subset A_1 contains all the elements of X that are supportive to $\mathbf{A}$, A_3 contains all the elements of X that are against $\mathbf{A}$, and A_2 contains all the elements of X that stand in a distance from being with or against A. Consequently, every crisp event $\mathbf{A}$ in X is obviously a NCS having the form $A = \langle A_1, A_2, A_3 \rangle$.

Fig. 1, shows a grayscale image in a 2D Cartesian domain, and its star neutrosophic crisp components in the neutrosophic domain.



a) b)

Fig. 1. a) Grayscale image b) Neutrosophic Crisp Components $\langle A_1, A_2, A_3 \rangle$ respectively

2.2 Definition [9]

The object having the form $A = \langle A_1, A_2, A_3 \rangle$can be classified into three different classes as follows:

(a) A, is said to be neutrosophic crisp set of class 1 (NCS-Class1) if satisfying:

$A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$ and$A_2 \cap A_3 = \emptyset$.

(b) A, is said to beneutrosophic crisp set of class 2 (NCS-Class2) if satisfying:

$A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$, $A_2 \cap A_3 = \emptyset$ and$A_1 \cup A_2 \cup A_3 = X$.

(c) A, is said to be neutrosophic crisp set of class 3 (NCS-Class3) if satisfying

$A_1 \cap A_2 \cap A_3 = \emptyset$ and $A_1 \cup A_2 \cup A_3 = X$.

To fulfill the basic requirements to follow up the rest of this paper we choose the following definitions which were introduced in [2,7].

2.3 Definition [9]

The null (empty) neutrosophic set φ_N, the absolute (universe) neutrosophic set $X_N$and the complement of a neutrosophic crisp set are defined as follows:

1) φ_N may be defined as one of the following two types:

n1:$\varphi_N = \langle \varphi, \varphi, X \rangle$,
n2:$\varphi_N = \langle \varphi, X, X \rangle$.

2) X_N may be defined as one of the following two types:

u1: $X_N = \langle X, X, \varphi \rangle$,
u2: $X_N = \langle X, \varphi, \varphi \rangle$.

3) The complement of a NCS A (co A for short) may be defined as one of the following two types:

c1: $coA = \langle coA_1, coA_2, coA_3 \rangle$
c2: $coA = \langle A_3, coA_2, A_1 \rangle$.

3 Neutrosophic Crisp Sets' Operations

In [2,7], the authors extended the definitions of the crisp set's operations to be defined over neutrosophic crisp sets. In the following definitions we consider anon-empty set X, and any two neutrosophic crisp sets of X, A and B, where $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$.

3.1 Definition [5, 9]

The neutrosophic crisp set A is said to be a neutrosophic crisp subset of the neutrosophic crisp set B ($A \subseteq B$),and to be defined as one of the following two types:

Type 1: $A \subseteq B \iff A_1 \subseteq B_1$, $A_2 \subseteq B_2$ and $A_3 \supseteq B_3$.

Type 2: $A \subseteq B \iff A_1 \subseteq B_1$, $A_2 \supseteq B_2$ and $A_3 \supseteq B_3$.

3.2 Proposition [5, 9]

For any neutrosophic crisp set A , the following properties hold:

$a)$ $\varphi_N \subseteq A$ and$\varphi_N \subseteq \varphi_N$.
b) $A \subseteq X_N$ and $X_N \subseteq X_N$.

3.3 Definition [5, 9]

The neutrosophic intersection and neutrosophic union of any two neutrosophic crisp sets A and B is to be defined as follows:

1.The neutrosophic intersection,$A \cap B$,may be defined as one of the following two types:

Type 1:$A \cap B = \langle A_1 \cap B_1 , A_2 \cap B_2 , A_3 \cup B_3 \rangle$.
Type 2:$A \cap B = \langle A_1 \cap B_1 , A_2 \cup B_2 , A_3 \cup B_3 \rangle$.

2. The neutrosophic intersection,$A \cup B$, may be defined as one of the following two types:

Type 1:$A \cup B = \langle A_1 \cup B_1 , A_2 \cup B_2 , A_3 \cap B_3 \rangle$.
Type 2:$A \cup B = \langle A_1 \cup B_1 , A_2 \cap B_2 , A_3 \cap B_3 \rangle$.

3.4 Proposition [5, 9]

For any two neutrosophic crisp sets A and Bin X, we have that:

$co(A \cap B) = coA \cup coB$ and $co(A \cup B) = coA \cap coB$.

Proof

We can easily prove that the two statements are true for both the complement operators (c1, c2) defined in **Definition 2.3**, as well as for the two types of the neutrosophic union and the neutrosophic intersection operators as defined in **Definition 3.3**.

3.5 Proposition [5, 9]

For any arbitrary family $\{A_j : j \in J\}$, of neutrosophic crisp subsets of$X$, a generalization for the neutrosophic intersection and for the neutrosophic union given in **Definition 3.3** can be defined as follows:

1) $\bigcap_{j \in J} A_j$ may be defined as one of the following two types:

Type 1:$\bigcap_{j \in J} A_j = < \bigcap_{j \in J} A_{j1} , \bigcap_{j \in J} A_{j2} , \bigcup_{j \in J} A_{j3} >$,
Type 2:$\bigcap_{j \in J} A_j = < \bigcap_{j \in J} A_{j1} , \bigcup_{j \in J} A_{j2} , \bigcup_{j \in J} A_{j3} >$..

2)$\bigcup_{j \in J} A_j$may be defined as one of the following two types:

Type 1:$\bigcup_{j \in J} A_j = < \bigcup_{j \in J} A_{j1} , \bigcup_{j \in J} A_{j2} , \bigcap_{j \in J} A_{j3} >$,
Type 2:$\bigcup_{j \in J} A_j = < \bigcup_{j \in J} A_{j1} , \bigcap_{j \in J} A_{j2} , \bigcap_{j \in J} A_{j3} >$.

4 Neutrosophic Crisp Topological Spaces

In [8,9], Salama et al., have introduced the definition of the neutrosophic crisp sets to the concepts of topological space and intuitionistic topological space in order to construct a new kind of topological spaces, namely, the neutrosophic crisp topological space.

4.1 Definition [10, 11]

A neutrosophic crisp topology (NCT) in a non-empty set X is a family τ of neutrosophic crisp subsets of X satisfying the following axioms:

NO1: $\emptyset_N, X_N \in \tau$
NO2: $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
NO3 : $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

The pair (X, τ), is called a neutrosophic crisp topological space (NCTS) in X. The elements in τ are called neutrosophic crisp open sets (NCOSs) in X. A neutrosophic crisp setF, is closed if and only if its complement (co F) is a neutrosophic crisp open set.

4.2 Remark [10,11]

The neutrosophic crisp topological spaces are considered to beg generalizations of the topological spaces and the intuitionist topological spaces. Moreover they allow more general functions to be defined on the topology.

4.3 Example

Let $X = \{a, b, c, d\}$, and $\emptyset_N, X_N$ be of any type of neutrosophic empty and universal subsets as defined in Definition 2.3. For any two neutrosophic crisp subsets A, B, defined by:

$A = < \{a\}, \{b, d\}, \{c\} >$ and $B = < \{a\}, \{b\}, \{c\} >$.

The family $\tau = \{\emptyset_N, X_N, A, B\}$ is a neutrosophic crisp topologyin X.

4.4 Definition [8, 9]

Let $(X, \tau_1), (X, \tau_2)$ be two neutrosophic crisp topological spaces in X. Then τ_1 is contained in τ_2 (symbolized $\tau_1 \subseteq \tau_2$) if $G \in \tau_2$ for each $G \in \tau_1$. In this case, we say that τ_1 is coarser than τ_2 and τ_2 is finer than τ_1.

4.5 Definition [10, 11]

Consider (X, τ) to be neutrosophic crisp topological space and $A = \langle A_1, A_2, A_3 \rangle$ to be a neutrosophic crisp setin X. Then the neutrosophic crisp closure of A (*NCCl*(A), for short) and neutrosophic crisp interior (*NCInt*(A), for short) of A are defined by:

(a) $NCCl(A) = \cap \{ K : K$ is a neutrosophic closed set $in\ X$ and $A \subseteq K\}$;
(b) $NCInt(A) = \cup \{G : G$ is a neutrosophic crisp open set in X and $G \subseteq A\}$;

It can be also shown that NCCl(A) is aneutrosophic crisp closed set; and NCInt(A) is a neutrosophic crisp open set in X.

4.6 Remark [10, 11]

For any neutrosophic set A in X, we have that:

(a) A is a neutrosophic crisp open set in X, if and only $if\ A = NCInt(A)$.
(b) A is a neutrosophic crisp closed set in in X, if and only $if\ A = NCCl(A)$.

4.7 Proposition [10, 11]

For any neutrosophic crisp set A in (X, τ) , we have that:

(a) $NCCl(co\ A) = co(NCInt(A))$;
(b) $NCInt(co\ A) = co(NCCl(A))$.

4.8 Proposition [10, 11]

For any neutrosophic crisp topological space (X, τ), and any two neutrosophic crisp sets in X, A and$B$, the following properties hold:

(a) $NCInt(A) \subseteq A$, and $A \subseteq NCCl(A)$
(b) $A \subseteq B \Rightarrow NCInt(A) \subseteq NCInt(B)$
(c) $A \subseteq B \Rightarrow NCCl(A) \subseteq NCCl(B)$
(d) $NCInt(A \cap B) = NCInt(A) \cap NCInt(B)$
(e) $NCCl(A \cup B) = NCCl(A) \cup NCCl(B)$
(f) $NCInt(X_N) = X_N$, and$NCInt(\emptyset_N) = \emptyset_N$
(g) $NCCl(X_N) = X_N$, and $NCCl(\emptyset_N) = \emptyset_N$.

5 Neutrosophic Crisp Points

In [2], the nature of neutrosophic crisp set corresponding to an element$x$ in X,called the neutrosophic crisp point was defined as follows:

5.1 Definition [5]

For neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$ of the non-empty set X, the neutrosophic crisp point is defined to be the following triple structure $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$where $p_1 \neq p_2 \neq p_3 \in X$.

5.2 Remark[5]

Aneutrosophic crisp point $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$belongsto neutrosophic crisp set $A = \langle A_1$, A2 , A3 of X , denoted by p∈A if the following is true:

$\{p_1\} \subseteq A_1$, $\{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$.

5.3 Definition [6]

Consider the non-empty set X and the two neutrosophic crisp sets $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$. Then $A \subseteq B$ if and only if for each $p \in A$ we have that $p \in B$, where p is any neutrosophic crisp point in X.

5.4 Definition [5]

Consider $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp subset of X, then $A = \cup \{p : p \in A\}$
Since $\cup \{p : p \in A\}$ are given by $\langle \cup \{p_1 : p_1 \in A_1\}, \cup \{p_2 : p_2 \in A_2\}, \cup \{p_3 : p_3 \in A_3\} \rangle$.

5.5 Proposition[5]

For any arbitrary family, $\{A_j : j \in J\}$ of neutrosophic crisp subsets of X, a generalization for the neutrosophic intersection and the neutrosophic union given in **Definition 5.4**, is defined as follows:

1) $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in \cup_{j \in J} A_j$ if $p \in A_j$ for any $j \in J$,
2) $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in \cap_{j \in J} A_j$ if $p \in A_j$ for each $j \in J$.

6 Star Neutrosophic Crisp Sets

As a retraction of neutrosophic crisp set, the star neutrosophic crisp set was introduced in [6].The new defined set was chosen to be a NCS-Class1 as given in Definition 2.2,where its three components are disjoint.

6.1 Definition [1]

For a neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$ of the non-empty fixed set X,the star neutrosophic crisp set A^*, is defined to be the following triple structure: $A^* = \langle A_1^*, A_2^*, A_3^* \rangle$; where $A_1^* = A_1 \cap co(A_2 \cup A_3)$, $A_2^* = A_2 \cap co(A_1 \cup A_3)$ and $A_3^* = A_3 \cap co(A_1 \cup A_2)$.

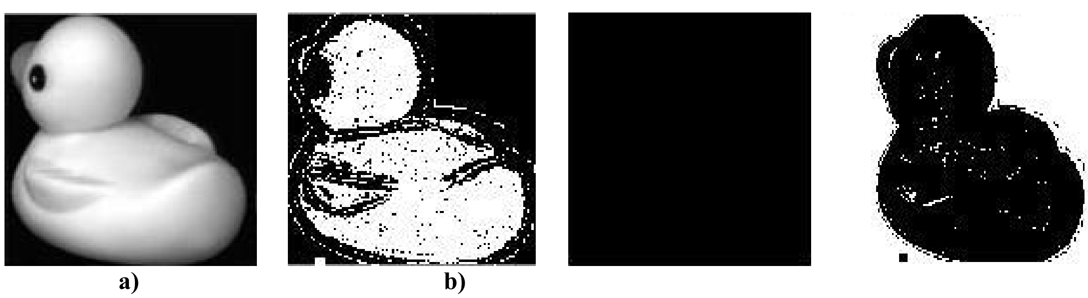


Fig. 2. a) Gray scale image b)Neutrosophic Star Crisp Components $\langle A_1, A_2, A_3 \rangle$ respectively

6.2 Corollary [1]

For any non-empty set X, the star null and the star universe sets ($\emptyset_N^*$ and X_N^*,respectively) are also neutrosophic crisp set.

6.3 Theorem [1]

Consider the non-empty set X, the two neutrosophic crisp sets $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$, and the two star neutrosophic crisp sets $A^* = \langle A_1{}^*, A_2{}^*, A_3{}^* \rangle$ and $B^* = \langle B_1{}^*, B_2{}^*, B_3{}^* \rangle$. If $A \subseteq B$ then $A^* \subseteq B^*$.

Proof

Given that $A \subseteq B$, it is easy to prove that $A_1{}^* \subseteq B_1{}^*$, $A_2{}^* \subseteq B_2{}^*$, $A_3{}^* \supseteq B_3{}^*$ and also that $A_1{}^* \subseteq B_1{}^*$, $A_2{}^* \supseteq B_2{}^*$, $A_3{}^* \supseteq B_3{}^*$.

Hence, $A^* \subseteq B^*$ according to the two types given in Definition 3.1.

6.4 Remark [1]

$A^* = B^*$ if and only if $A^* \subseteq B^*$ and $B^* \subseteq A^*$.

6.5 Remark [1]

1) All types of $\emptyset_N^*$ and $\emptyset_N$ are conceded.
2) All types of X_N^* and X_N are conceded.

That is from **Definition 2.3** we can see that:

n1: $\emptyset_N^* = \varphi_N = \langle \varphi, \varphi, X \rangle$, and **n2**: $\emptyset_N^* = \varphi_N = \langle \varphi, X, X \rangle$.
u1: $X_N^* = X_N = \langle X, X, \varphi \rangle$, and **u2**: $X_N^* = X_N = \langle X, \varphi, \varphi \rangle$.

6.6 Definition [1]

The complement of a star neutrosophic crisp set $A^*(\text{co } A^*, \text{for short})$ may be defined as one of the following two types:

c1: $co\ A^* = \langle coA_1{}^*, coA_2{}^*, coA_3{}^* \rangle$
c2: $co\ A^* = \langle A_3{}^*, coA_2{}^*, A_1{}^* \rangle$

To this point, we notice that we can divide the family of all-star neutrosophic crisp sets into two major categories as in the following proposition.

6.7 Proposition

For a non-empty X, the family of all-star neutrosophic crisp sets can be categorizing as follows:

Category I: If for the neutrosophic crisp set A, the three components A_1, A_2, A_3 satisfy the condition: $A_i \cap A_j \neq \emptyset$, $\forall i = 1,2,3$. Then A^* will be a retraction of A where $A_i^* \subseteq A_i$, $\forall i = 1,2,3$.

Category II: If the components $A_i, i = 1,2,3$ are disjoint, then $A^* = A$, in this case, all the neutrosophic crisp definitions and operations are applied.

6.8 Example

Consider the set $X = \{a, b, c, d\}$ and the two neutrosophic crisp sets $A = \langle \{a\}, \{b, d\}, \{c\} \rangle$ and $B = \langle \{a\}, \{b\}, \{c\} \rangle$. Then the starneutrosophic crisp sets A^* and B^* are :

$A^* = \langle\{a\},\{b,d\},\{c\}\rangle$, $B^* = \langle\{a\},\{b\},\{c\}\rangle$, and can use the definitions [2.3] and [3.3] to deduce the following:

1) The complements of A^* and B^*may be equal one of the following forms:

c1:$co\ A^* = \langle\{b,c,d\},\{a,c\},\{a,b,d\}\rangle$and$co\ B^* = \langle\{b,c,d\},\{a,c,d\},\{a,b,d\}\rangle$.
c2:$co\ A^* = \langle\{c\},\{a,c\},\{a\}\rangle$and$co\ B^* = \langle\{c\},\{a,c,d\},\{a\}\rangle$.

2) $A^* \cup B^*$may be equal one of the following forms:

Type I:$A^* \cup B^* = \langle\{a\},\{b,d\},\{c\}\rangle$and **Type II:** $A^* \cup B^* = \langle\{a\},\{b\},\{c\}\rangle$.

3) $A^* \cap B^*$may be equal one of the following forms:

TypeI: $A^* \cap B^* = \langle\{a\},\{b\},\{c\}\rangle$ and **Type II**: $A^* \cap B^* = \langle\{a\},\{b,d\},\{c\}\rangle$.

4) $A^* - B^*$ may be equal one of the following forms:

TypeI: c1:$A^* \cap co\ B^* = \langle\emptyset,\{d\},X\rangle$and **TypeII: c1:** $A^* \cap co\ B^* = \langle\emptyset,X,X\rangle$.
TypeI: c2:$A^* \cap co\ B^* = \langle\emptyset,\{d\},\{a,c\}\rangle$ and**TypeII: c2:** $A^* \cap co\ B^* = \langle\emptyset,X,\{a,c\}\rangle$.

7 Retract Neutrosophic Crisp Sets and Operations

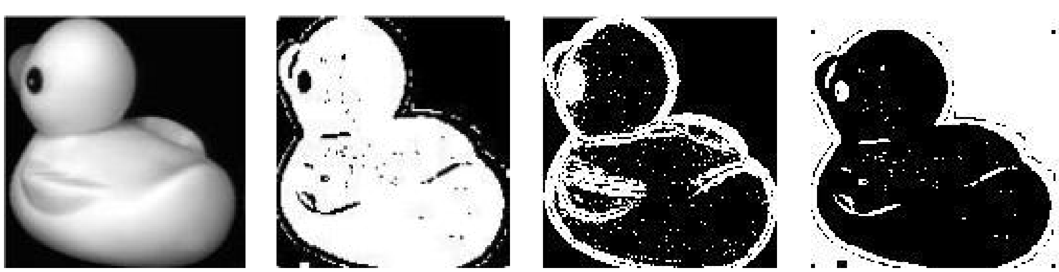
At this point, we have noticed that there exist some crisp sets which having the neutrosophic triple structure and are not classified in either categories of the neutrosophic crisp sets' classification. In this case, the three components of those sets may overlap.

In this section, we deduced a new triple structured set; where the three components are disjoint.

7.1 Definition

For any triple structured crisp set A, of the form $A = \langle A_1, A_2, A_3\rangle$;the retract neutrosophic crisp set A^{r} is the structure $A^{r} = \langle A_1^{r}, A_2^{r}, A_3^{r}\rangle$, where$A_1^{r} = A_1 \cap co(A_2 \cup A_3)$, $A_2^{r} = A_2 \cap co(A_1 \cup A_3)$ and$A_3^{r} = A_3 \cap co(A_1 \cup A_2)$.Furthermore, the three components A_1^{r} , A_2^{r} and A_3^{r} are disjoint and $A_i^{r} \subseteq A_i\ \forall\ i = 1,2,3$.

Fig. 3, shows a grayscale image in a 2D Cartesian domain, and its neutrosophic retract crisp components in the neutrosophic domain.



a) b)

Fig. 3. a) Gray scale image b) Neutrosophic Retract Components $\langle A_1, A_2, A_3\rangle$ respectively

7.2 Definition

The complement of a retract neutrosophic crisp set A^{r} (co A^{r}, for short) may be defined as: $r co\, A^{r} = \langle A_3^{r}, A_2^{r}, A_1^{r} \rangle$.

7.3 Definition

Consider a non-empty set X , and two retract neutrosophic crisp sets A^{r} , and B^{r}; then the retract intersection and retract union of any two retract neutrosophic sets are defined as follows:

1. The retract neutrosophic intersection of A^{r}, B^{r} is defined as :

   $A^{r} \breve{\cap} B^{r} = \langle A_1{}^{r} \cap B_1{}^{r}, A_2{}^{r} \cap B_2{}^{r}, A_3{}^{r} \cap B_3{}^{r} \rangle$.

2. The retract neutrosophic union of A^{r}, B^{r} is defined as :

   $A^{r} \breve{\cup} B^{r} = \langle A_1{}^{r} \cup B_1{}^{r}, A_2{}^{r} \cup B_2{}^{r}, A_3{}^{r} \cup B_3{}^{r} \rangle$.

3. The retract neutrosophic difference of A^{r}, B^{r} is defined as :

   $A^{r} \breve{-} B^{r} = (A_1{}^{r} \cap \mathrm{co}B_1{}^{r}, A_2{}^{r} \cap \mathrm{co}B_2{}^{r}, A_3{}^{r} \cap \mathrm{co}B_3{}^{r})$.

4. The retract neutrosophic symmetric difference of A^{r}, B^{r} is defined as :
5.
   $A^{r} \breve{\ominus} B^{r} = (A^{r} \breve{-} B^{r}) \breve{\cup} (B^{r} \breve{-} A^{r})$, or equivalently
   $A^{r} \breve{\ominus} B^{r} = (A^{r} \breve{\cup} B^{r}) \breve{-} (A^{r} \breve{\cap} B^{r})$.

7.4 Theorem

Let A^{r} and B^{r} two retract neutrosophic crisp sets, then:

(a) $A^{r} \cap B^{r} \breve{\subseteq} (A \cap B)^{r}$;
(b) $(A \cup B)^{r} \breve{\subseteq} A^{r} \cup B^{r}$.

7.5 Remark

As one can see that, when we deduced a retract neutrosophic crisp set, A; they both coincide. That is, if A is a neutrosophic crisp set, then $A^{r} = A^{*}$.

Moreover, if A is a neutrosophic crisp set of category II, then $A^{r} = A^{*} = A$.

7.6 Example

Consider the set, $X = \{a, b, c, d, e\}$ and $\emptyset_N^{r}$, X_N^{r} be any type of universal and empty subset,
If A, B are two neutrosophic crisp sets in X, defined as:

$A = \langle \{a, b\}, \{b, c, d\}, \{e, d\} \rangle, B = \langle \{a, c\}, \{b, d, e\}, \{b, e\} \rangle$ and $A \cap B = <\{a\}, \{b, d\}, , \{b, d, e\}>$, $A \cup B = <\{a, b, c\}, \{b, c, d, e\}, , \{e\}>$.

The two retract neutrosophic crisp subsets of X (A^{r}, B^{r}) can be defined as follows:

$A^{r} = \langle\{a\},\{c\},\{e\}\rangle$, $B^{r} = \langle\{a\},\{d\},\emptyset\rangle$. Hence, we may also deduce the following:

$A^{r} \breve{\cap} B^{r} = <\{a\},\emptyset,\emptyset>$, $(A \cap B)^{r} = <\{a\},\emptyset,\{e\}>$,
$A^{r} \breve{\cup} B^{r} = <\{a\},\{c,d\},\{e\}>$, $(A \cup B)^{r} = <\{a\},\{d\},\emptyset>$.

7.7 Lemma

For any two retract neutrosophic crisp sets, A^{r} and B^{r}, it is easy to show that the left side of each equality given in **Definition 7.2** is also a retract neutrosophic crisp sets.

7.8 Example

Consider the set, $X = \{a, b, c, d, e\}$ and $\emptyset_N^{r}$, X_N^{r} be any type of universal and empty subset,
If A, B are two neutrosophic crisp sets in X, defined as:

$A = \langle\{a,b\},\{b,c,d\},\{e,d\}\rangle$, $B = \langle\{a,c\},\{b,d,e\},\{b,e\}\rangle$.The two retract neutrosophic crisp subsets of X (A^{r}, B^{r}) can be defined as follows:

$A^{r} = \langle\{a\},\{c\},\{e\}\rangle$, $B^{r} = \langle\{a,c\},\{d\},\emptyset\rangle$. Hence, we may also deduce the following:

1. **c1:** $co\ A^{r} = \langle\{b,c,d,e\},\{a,b,d,e\},\{a,b,c,d\}\rangle$and $co\ B^{r} = \langle\{b,d,e\},\{a,b,c,e\},X\rangle$.

   c2:$co\ A^{r} = \langle\{e\},\{a,b,d,e\},\{a\}\rangle$ and $co\ B^{r} = \langle\emptyset,\{a,b,c,e\},\{a,c\}\rangle$.

   $r co\ A^{r} = \langle\{e\},\{c\},\{a\}\rangle$and$r co\ B^{r} = \langle\emptyset,\{d\},\{a,c\}\rangle$.

2. $A^{r} \breve{\cup} B^{r} = \langle\{a,c\},\{c,d\},\{e\}\rangle$.
3. $A^{r} \breve{\cap} B^{r} = \langle\{a\},\emptyset,\emptyset\rangle$.
4. **c1:**$A^{r} \breve{-} B^{r} = A^{r} \breve{\cap}\ co\ B^{r} = \langle\emptyset,\{c\},\{e\}\rangle$.

   c2:$A^{r} \breve{-} B^{r} = A^{r} \breve{\cap}\ co\ B^{r} = \langle\emptyset,\{c\},\emptyset\rangle$.

5. $A^{r} \breve{\ominus} B^{r} = (A^{r} \breve{-} B^{r}) \breve{\cup} (B^{r} \breve{-} A^{r}) = \langle\{c\},\{c,d\},\{e\}\rangle$or
   $A^{r} \breve{\ominus} B^{r} = (A^{r} \breve{\cup} B^{r}) \breve{-} (A^{r} \breve{\cap} B^{r}) = \langle\{c\},\{c,d\},\{e\}\rangle$.

7.9 Proposition

For any arbitrary family,$\{A_j^{r} : j \in J\}$, of retract neutrosophic crisp subsets of X, a generalization for the retract neutrosophic intersection and the retract neutrosophic union given in **Definition 6.2**, are defined as follows:

1) $\breve{\bigcap}_{j \in J} A_j^{r}$ is defined as:$\breve{\bigcap}_{j \in J} A_j^{r} = < \bigcap_{j \in J} A_{j1}^{r}\ ,\ \bigcap_{j \in J} A_{j2}^{r}\ ,\ \bigcap_{j \in J} A_{j3}^{r} >$.
2) $\breve{\bigcup}_{j \in J} A_j^{r}$ is defined as:$\breve{\bigcup}_{j \in J} A_j^{r} = < \bigcup_{j \in J} A_{j1}^{r}\ ,\ \bigcup_{j \in J} A_{j2}^{r}\ ,\ \bigcup_{j \in J} A_{j3}^{r} >$.

7.10 Definition

The retract neutrosophic crisp set A^{r} is said to be a retract neutrosophic crisp subset of the retract neutrosophic crisp set B^{r} ($A^{r} \breve{\subseteq} B^{r}$), if the following is true:

$A^{r} \breve{\subseteq} B^{r} \iff A_1^{r} \subseteq B_1^{r}$, $A_2^{r} \subseteq B_2^{r}$,and$A_3^{r} \subseteq B_3^{r}$.

7.11 Corollary

Let $\{A_j\}_{j\in J}$ be a family of neutrosophic crisp setsin X, where J is an index set, and let $\{A_j^{r}\}$ be the corresponding retractneutrosophic crisp sets in X, then

(a) If $A_j^{r} \breve{\subseteq} B^{r}$ for each $j \in J$, then $\breve{\bigcup}_{j\in J} A_j^{r} \breve{\subseteq} B^{r}$,
(b) If $B^{r} \breve{\subseteq} A_i^{r}$ for each $j \in J$, then $B^{r} \breve{\subseteq} \breve{\bigcap}_{j\in J} A_j^{r}$,
(c) $co(\breve{\bigcup}_{j\in J} A_j^{r}) = \breve{\bigcap}_{j\in J} coA_j^{r}$, and $co(\breve{\bigcap}_{j\in J} A_j^{r}) = \breve{\bigcup}_{j\in J} coA_j^{r}$,
(d) If $A^{r} \breve{\subseteq} B^{r}$, then co $A^{r} \breve{\subseteq} coB^{r}$,
(e) $co(\text{co } A^{r}) = A^{r}$,
(f) $co\emptyset_N^{r} = X_N^{r}$, and $coX_N^{r} = \emptyset_N^{r}$.

8 Retract Neutrosophic Crisp Topological Spaces

In this section we introduce a new type of neutrosophic Topological Spaces, based on the retract neutrosophic crisp sets.

8.1 Definition

Let X be a non empty set, and τ^{r} be a family of retract neutrosophic crisp subsets of X, then τ^{r} is said to be a retract neutrosophic crisp topology if it satisfies the following axioms:

rO1: $\emptyset_N^{r}$, $X_N^{r} \in \tau^{r}$
rO2: $A^{r} \breve{\cap} B^{r} \in \tau^{r}$, $\forall A^{r}, B^{r} \in \tau^{r}$
rO3: $\breve{\bigcup}_{j\in J} A_j^{r} \in \tau^{r}$, $\forall A_j^{r} \in \tau^{r}$, $j \in J$.

8.2 Remarks

Remark1: The pair (X, τ^{r}) is called a retract neutrosophic crisp topological space in X.
Remark2: The elements ofτ^{r} are called retract neutrosophic crisp open sets in X.
Remark3: A retract neutrosophic crisp set F^{r} is said to be retract star neutrosophic crisp closed set if and only if its complement, co F^{r}, is a retract neutrosophic crisp open set

8.3 Corollary

Let X be a non empty set, and τ^{r} is retract neutrosophic topology crisp subsets of X and $r(x)$ is the set of all retract subsets of X, then $\tau^{r} \breve{\subseteq} r(x)$.

Then, we can apply **Proposition[6.7]** in a retract neutrosophic crisp topological space

8.4 Example

Consider the set, $X = \{a, b, c, d, e\}$ and $\emptyset_N^{r}$, X_N^{r} be any type of universal and empty subset,
If A, B are two neutrosophic crisp sets in X, defined as:

$A = \langle\{a, b\}, \{b, c, d\}, \{e, d\}\rangle$, $B = \langle\{a, c\}, \{b, d, e\}, \{b, e\}\rangle$.The retract neutrosophic crisp subsets of X (A^{r}, B^{r}, C^{r}, D^{r}) can be defined as follows:
$A^{r} = \langle\{a\}, \{c\}, \{e\}\rangle$, $B^{r} = \langle\{a\}, \{d\}, \emptyset\rangle$, $C^{r} = \langle\{a\}, \{c, d\}, \{e\}\rangle$, $D^{r} = \langle\{a\}, \emptyset, \emptyset\rangle$.

Then the family $\tau^{r} = \{\emptyset_N^{r}, X_N^{r}, A^{r}, B^{r}, C^{r}, D^{r}\}$ is a retract neutrosophic crisp topology in X.

8.5 Definition

Let (X, τ_1^{r}) and (X, τ_2^{r}) be two retract neutrosophic crisp topological spaces in X. Then τ_1^{r} is contained in τ_2^{r} (symbolize $\tau_1^{r} \subseteq \tau_2^{r}$) if $G \in \tau_2^{r}$ for each $G \in \tau_1^{r}$. In this case, we say that τ_1^{r} is coarser than τ_2^{r} and that τ_2^{r} is said to be finer than τ_1^{r}.

8.6 Definition

Let (X, τ^{r}) be a retract neutrosophic crisp topological space, and A^{r} be a retract neutrosophic crisp set in X, then the retract neutrosophic crisp interior ($r\text{int}(A^{r})$) of A^{r} and the retract neutrosophic crisp closure of A^{r} ($r\text{cl}(A^{r})$) are defined by:

(a) $rint(A^{r}) = \cup\{G^{r} : G^{r}$ is retract neutrosophic crisp open set in X and $G^{r} \subseteq A^{r}\}$;
(b) $rcl(A^{r}) = \cap\{F^{r} : F^{r}$ is retract neutrosophic crisp closed set in X and $A^{r} \subseteq F^{r}\}$;

Hence $r\text{int}(A^{r})$ is a retract neutrosophic crisp open set in X, that is $r\text{int}(A^{r}) \in \tau^{r}$, and $r\text{cl}(A^{r})$ is a retract neutrosophic crisp closed set in X, that is $r\text{cl}(A^{r}) \in \tau^{c^{r}}$.

8.7 Example

Consider the neutrosophic topological spaces (X, τ^{r}) defined in example [8.4], the retract neutrosophic crisp interior of the set $E^{r} =< \{a, c\}, \{b, e\}, \{d\} >$ is:

$$rint(E^{r}) = \langle\{a\}, \emptyset, \emptyset\rangle = D^{r}.$$

8.8 Definition

Let X be a non empty set, and f^{r} be a family of retract neutrosophic crisp subsets of X, then f^{r} is said to be a retract neutrosophic crisp co- topology if it satisfies the following axioms:

rC1: $\emptyset_N^{r}, X_N^{r} \in f^{r}$
rC2: $A^{r} \breve{\cup} B^{r} \in f^{r}, \quad \forall A^{r}, B^{r} \in f^{r}$
rC3: $\breve{\bigcap}_{j \in J} A_j^{r} \in f^{r}, \quad \forall A_j^{r} \in f^{r}, j \in J.$

8.9 Proposition

Let (X, τ) be a neutrosophic crisp topology spaces and A^{r}, B^{r} be two retract neutrosophic crisp sets in X holding the following properties:

a) A^{r} is retract neutrosophic crisp open if and only if $A^{r} = rint(A^{r})$;
b) $rint(A^{r}) \breve{\subseteq} A^{r}$, and $A^{r} \subseteq rcl(A^{r})$;
c) $A^{r} \breve{\subseteq} B^{r} \Rightarrow rint(A^{r}) \breve{\subseteq} rint(B^{r})$;
d) $rint(A^{r} \breve{\cap} B^{r}) = r\text{int}(A^{r}) \breve{\cap} r\text{rint}(B^{r})$;
e) $rcl(A^{r} \breve{\cup} B^{r}) = rcl(A^{r}) \breve{\cup} rcl(B^{r})$;
f) $rint(X_N^{r}) = X_N^{r}$, and $rint(\emptyset_N^{r}) = \emptyset_N^{r}$.
g) $rcl(X_N^{r}) = X_N^{r}$, and $rcl(\emptyset_N^{r}) = \emptyset_N^{r}$.

9 Conclusion

In this paper, a new generalization of the star intuitionistic topological space called the retract neutrosophic crisp topological space was introduced. The new topological space is constructed from some retraction of the neutrosophic crisp sets, or generally from any triple structured crisp set; called the retract neutrosophic crisp set. The properties and some basic operations relevant to the new type were investigated. The retract neutrosophic crisp set, has the triple structure as the neutrosophic crisp set. Whereas, it does not apply the property of the third component acts as the complement of the first component. In the view of this fact, we defined the retract neutrosophic operators to act similarly over each component.

Competing Interests

Authors have declared that no competing interests exist.

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