Coincidence and Non-coincidence Using Optical Circulators

M. W. Roberts

Abstract

An optical experiment is described in which pairs of quantum entangled photons are sent into separate optical circulators. Theoretical analysis is used to predict the number of coincident detections between these photons at the output from the circulators.

With proper control of non-local, two-photon interference, the photon pairs can be put in perfect coincidence or in perfect non-coincidence, as selected by the experimenter. These results contradict the predictions made using classical probability analysis.

1. Introduction

The experimental setup is shown in Figure 1. The setup is composed of a Source (Src) and two optical circulators (OC1 and OC2).

To simplify the description of this experiment, the effects of optical filters, detector quantum efficiency and dark counts, and other potential losses are not included in the following discussion.

2a. Source

The Source (Src) contains a single-mode, continuous wave (cw) pump laser (LSR), a periodically-poled lithium niobate crystal (PPLN), a long-pass dichroic mirror (DM), a polarizing beam splitter (PBSA), and a beam stop (Stp).

The PPLN is temperature-controlled, and is set to allow collinear, degenerate, type II spontaneous parametric down-conversion (SPDC) in which a photon from pump laser LSR is annihilated and a signal and idler pair of photons is created. The signal photon is horizontally (H) polarized, and the idler photon is vertically (V) polarized.

The photons from pump laser LSR have a very long coherence length. The short wavelength pump photons from LSR that are not down-converted in the PPLN are reflected at long-pass dichroic mirror DM and are incident on beam stop Stp.

On average, one of every 10^6 of the pump photons is annihilated in a SPDC event within the PPLN that creates a signal/idler pair of photons. These photons are energy/time entangled.

The long wavelength signal and idler photons exit from the PPLN and are transmitted through the DM to PBSA. The H polarized signal photons are transmitted through PBSA and travel to optical circulator OC1. The V polarized idler photons are reflected by PBSA and travel to optical circulator OC2.

2b. Optical Circulators

Optical circulator OC1 consists of amplitude beam splitter ABS1, two Pockels cells (PC1 and PC2), two polarizing beam splitters (PBS1 and PBS2), and four mirrors (m1 – m4).

Optical circulator OC2 consists of amplitude beam splitter ABS2 and three mirrors (m5 – m7). Amplitude beam splitters ABS1 and ABS2 may be partially-silvered plate beam splitters.

Detectors D1 and D2 and coincidence counter CC are located at the output from the two optical circulators. The detectors are capable of photon counting.

An idler photon that reaches OC2 may immediately reflect from ABS2 and travel to detector D2, or it may be transmitted through ABS2 into OC2. If it enters OC2, the idler photon makes one or more cycles around through OC2 before exiting from ABS2 and travelling to detector D2.

The time required for an idler photon to make one cycle through OC2 is equal to X. Fixed time X should be of sufficient duration to be discernable by coincidence counter CC. Time X should be much longer than the coherence time of an idler photon but should also be much shorter than the coherence time of a photon from pump laser LSR in the Source.

The optical path length for one cycle through OC2 is equal to L. Optical path length L should be an integer multiple of the wavelength of a photon from pump laser LSR [1].

A signal photon that reaches OC1 may immediately reflect from ABS1 and travel to detector D1, or it may be transmitted through ABS1 into OC1. If it enters OC1, the signal photon makes one or more cycles around through OC1 before exiting from ABS1 and travelling to detector D1.

Pockels cells PC1 and PC2 and polarizing beam splitters PBS1 and PBS2 are located at the far end of OC1, well away from entrance beam splitter ABS1.

The Pockels cells in OC1 may be used to rotate the polarization direction of a signal photon. The two Pockels cells are either both turned off or are both turned on. The Pockels cells

are set so that, when they are turned on, the "fast" axis of PC2 is orthogonal to the "fast" axis of PC1.

There are two different paths through OC1. In the "coincidence" path, a signal photon passes through PC1 and PBS1, reflects from mirror m1, and then passes through PBS2 and PC2. In the "non-coincidence" path, a signal photon passes through PC1, reflects at PBS1, mirror m2, and PBS2, and then passes through PC2.

The optical path length through a Pockels cell is slightly different when the cell is on as opposed to when it is off. Consequently, the lengths of the two different paths through OC1 are adjusted so that the time required for a signal photon to make one cycle through OC1 is always equal to X, regardless of which of the two paths is followed.

Fixed time X should be much longer than the coherence time of a signal photon. Time X is also the time required for an idler photon to make one cycle through OC2.

The distances external to the optical circulators are set equal. The distance from PBSA to ABS1 and then to D1 is set equal to the distance from PBSA to ABS2 and then to D2.

3. Notation

In the following discussion, both probability amplitude and probability will be calculated. As an example:

$$P[D1,D2;\Delta] = \left| pa[D1,D2;\Delta] \right|^2$$

In the above, $pa[D1,D2;\Delta]$ is the probability amplitude for the detection of a signal photon in detector D1 and the detection of an idler photon in detector D2 with time difference between detections equal to Δ. $P[D1,D2;\Delta]$ is the probability for the same detection events.

Both intensity and amplitude variables are used in the following. As an example, for amplitude beam splitter ABS1:

$$R_1 = \left| r_1 \right|^2 \ , \ T_1 = \left| t_1 \right|^2 \ \text{ and } \ R_1 + T_1 = 1$$

In the above, R_1 is the intensity reflectance, T_1 is the intensity transmittance, r_1 is the amplitude reflection coefficient, and t_1 is the amplitude transmission coefficient of ABS1.

4a. Perfect Coincidence [2][3]

To achieve perfect coincidence, the experimenter turns off both PC1 and PC2 in OC1.

The V polarized idler photon of a down-converted pair travels from the Source to optical circulator OC2. The idler photon then either reflects from entrance beam splitter ABS2, or it is transmitted through ABS2 and makes one or more cycles through OC2. The idler photon ultimately exits from OC2 and travels from ABS2 to detector D2.

The H polarized signal photon of the pair travels from the Source to optical circulator OC1. The signal photon then either reflects from entrance beam splitter ABS1, or it is transmitted through ABS1 and makes one or more cycles through OC1.

Each time the signal photon cycles through OC1, it passes through PC1 unchanged, is transmitted through PBS1, and reflects from mirror m1. The H polarized signal photon then passes through PBS2 and PC2 and returns to ABS1 via mirrors m3 and m4. The signal photon ultimately exits from OC1 and travels from ABS1 to detector D1.

Making explicit use of the non-local, two-photon interference between the photons, the probability amplitude and probability that the signal photon is incident on detector D1 and the idler photon of the pair is incident on detector D2 at the same time ($\Delta = 0$) in this perfect coincidence case are:

$$pa^{(C)}[D1,D2;\Delta=0] = [(ir_1)(ir_2)] + [(i^3 t_1^2)(i^3 t_2^2)]$$
$$+ [(i^7 t_1^2 r_1)(i^7 t_2^2 r_2)] + [(i^{11} t_1^2 r_1^2)(i^{11} t_2^2 r_2^2)] + ...$$

With: $r_2 = r_1$:
$$pa^{(C)}[D1,D2;\Delta=0] = [-r_1^2] + [(-t_1^4)/(1-r_1^2)] = -1$$
$$P^{(C)}[D1,D2;\Delta=0] = \left| pa^{(C)}[D1,D2;\Delta=0] \right|^2 = 1$$

The probability amplitude and probability that the signal photon is incident on detector D1 before the idler photon of the pair is incident on detector D2 with time difference $\Delta = X$ are:

$$pa^{(C)}[D1,D2;\Delta=X] = [(ir_1)(-it_2^2)] + [(-it_1^2)(-it_2^2 r_2)]]$$
$$+ [(-it_1^2 r_1)(-it_2^2 r_2^2)] + [(-it_1^2 r_1^2)(-it_2^2 r_2^3)] + ...$$

Again with: $r_2 = r_1$:
$$pa^{(C)}[D1,D2;\Delta=X] = [r_1 t_1^2] + [(-t_1^4 r_1)/(1-r_1^2)] = 0$$
$$P^{(C)}[D1,D2;\Delta=X] = \left| pa^{(C)}[D1,D2;\Delta=X] \right|^2 = 0$$

4

The probability amplitude and probability that the signal photon is incident on detector D1 before the idler photon of the pair is incident on detector D2 with time difference $\Delta = nX$ (integer n>0) are:

$$pa^{(C)}[D1,D2;\Delta=nX] = pa^{(C)}[D1,D2;\Delta=X] \cdot r_1^{(n-1)} = 0$$
$$P^{(C)}[D1,D2;\Delta=nX] = \left|pa^{(C)}[D1,D2;\Delta=nX]\right|^2 = 0$$

By symmetry, the probability amplitude and probability that the idler photon is incident on detector D2 before the signal photon of the pair is incident on detector D1 with time difference $\Delta = mX$ (integer m>0) are:

$$pa^{(C)}[D2,D1;\Delta=mX] = 0$$
$$P^{(C)}[D2,D1;\Delta=mX] = \left|pa^{(C)}[D2,D1;\Delta=mX]\right|^2 = 0$$

For this case with PC1 and PC2 both turned off, as long as R_2 is set equal to R_1, <u>all</u> pairs of signal and idler photons will be detected in coincidence.

Each term in the probability amplitude sum represents the contribution from a given number of cycles through the optical circulator (0 cycles, 1 cycle, 2 cycles, etc.).

As many terms should be included in the probability amplitude sum as possible. For two-photon interference to be effective, all of the terms that contribute a significant value to the probability amplitude sum must be enabled by ensuring that coherence is maintained between the signal and idler photons for at least this required number of cycles.

This is why the photons from the pump laser should have a very long coherence length. This maximizes the number of terms in the probability amplitude sum that are enabled for a given size (length) of the optical circulator.

4b. Perfect Non-coincidence [2][3]

To achieve perfect non-coincidence, the experimenter turns on both PC1 and PC2 in OC1.

The V polarized idler photon of a down-converted pair travels from the Source to optical circulator OC2. The idler photon then either reflects from entrance beam splitter ABS2, or it is transmitted through ABS2 and makes one or more cycles through OC2. The idler photon ultimately exits from OC2 and is incident on detector D2.

The H polarized signal photon of the pair travels from the Source to optical circulator OC1. The signal photon then either

reflects from entrance beam splitter ABS1, or it is transmitted through ABS1 and makes one or more cycles through OC1.

Each time the signal photon cycles through the circulator, it passes through PC1. The polarization direction of the signal photon is rotated, and it exits from PC1 V polarized. The now V polarized signal photon then reflects from PBS1, mirror m2, and PBS2. The polarization direction of the signal photon is again rotated when it passes through PC2. The signal photon exits from PC2 H polarized.

The H polarized signal photon returns to ABS1 via mirrors m3 and m4. The signal photon ultimately exits from OC1 and is incident on detector D1.

The optical path length for one cycle through OC1 is the same in the non-coincidence case as in the coincidence case. The only difference between the two cases is the two additional reflections per cycle (at PBS1 and PBS2) experienced by the signal photon in the non-coincidence case.

The probability amplitude and probability that the signal photon is incident on detector D1 and the idler photon of the pair is incident on detector D2 at the same time ($\Delta = 0$) in this non-coincidence case are:

$pa^{(N)}[D1,D2;\Delta=0] = [(ir_1)(ir_2)] + [(i^5 t_1^2)(i^3 t_2^2)]$
$\quad + [(i^{11} t_1^2\ r_1)(i^7 t_2^2\ r_2)] + [(i^{17} t_1^2\ r_1^2)(i^{11} t_2^2\ r_2^2)] + \ldots$

With: $r_2 = r_1$:
$pa^{(N)}[D1,D2;\Delta=0] = [-r_1^2]+[(t_1^4)/(1-r_1^4)]+[(-t_1^4\ r_1^2)/(1-r_1^4)]$
$\qquad = [-r_1^2] + [(t_1^4)/(1+r_1^2)] = [(1-3r_1^2)/(1+r_1^2)]$

With: $r_1 = \{1/\sqrt{(3)}\}$, $t_1 = \{[\sqrt{(2)}]/\sqrt{(3)}\}$:
$pa^{(N)}[D1,D2;\Delta=0] = 0$
$P^{(N)}[D1,D2;\Delta=0] = |pa^{(N)}[D1,D2;\Delta=0]|^2 = 0$

The probability amplitude and probability that the signal photon is incident on detector D1 before the idler photon of the pair is incident on detector D2 with time difference $\Delta = X$ are:

$pa^{(N)}[D1,D2;\Delta=X] = [(ir_1)(-it_2^2)] + [(+it_1^2)(-it_2^2\ r_2)]]$
$\quad + [(-it_1^2\ r_1)(-it_2^2\ r_2^2)] + [(+it_1^2\ r_1^2)(-it_2^2\ r_2^3)] + \ldots$

Again with: $r_2 = r_1$:
$pa^{(N)}[D1,D2;\Delta=X] = [+r_1 t_1^2]+[(+t_1^4 r_1)/(1-r_1^4)]+[(-t_1^4 r_1^3)/(1-r_1^4)]$
$\qquad = [+r_1 t_1^2]+[(+t_1^4 r_1)/(1+r_1^2)]$

With: $r_1 = \{1/\sqrt{(3)}\}$, $t_1 = \{[\sqrt{(2)}]/\sqrt{(3)}\}$:
$pa^{(N)}[D1,D2;\Delta=X] = [1/\sqrt{(3)}]$
$P^{(N)}[D1,D2;\Delta=X] = |pa^{(N)}[D1,D2;\Delta=X]|^2 = [1/3]$

The probability amplitude and probability that the signal photon is incident on detector D1 before the idler photon of the pair is incident on detector D2 with time difference $\Delta = nX$ (integer $n>0$) are:

$$pa^{(N)}[D1,D2;\Delta=nX] = pa^{(N)}[D1,D2;\Delta=X] \cdot r_1^{(n-1)} = [1/\sqrt{3}]^n$$
$$P^{(N)}[D1,D2;\Delta=nX] = \left|pa^{(N)}[D1,D2;\Delta=nX]\right|^2 = [1/3]^n$$
$$P^{(N)}[D1,D2;\Delta>0] = [1/3] + [1/9] + [1/27] + ... = [1/2]$$

The probability amplitude and probability that the idler photon is incident on detector D2 before the signal photon of the pair is incident on detector D1 with time difference $\Delta = X$ are:

$$pa^{(N)}[D2,D1;\Delta=X] = [(ir_2)(+it_1^2)] + [(-it_2^2)(-it_1^2 \, r_1)]]$$
$$+ [(-it_2^2 \, r_2)(+it_1^2 \, r_1^2)] + [(-it_2^2 \, r_2^2)(-it_1^2 \, r_1^3)] + ...$$

With: $r_2 = r_1$:
$$pa^{(N)}[D2,D1;\Delta=X] = [-r_1 t_1^2] + [(-t_1^4 r_1)/(1-r_1^4)] + [(+t_1^4 r_1^3)/(1-r_1^4)]$$
$$= [-r_1 t_1^2] + [(-t_1^4 r_1)/(1+r_1^2)]$$

With: $r_1 = \{1/\sqrt{3}\}$, $t_1 = \{[\sqrt{2}]/\sqrt{3}\}$:
$$pa^{(N)}[D2,D1;\Delta=X] = -[1/\sqrt{3}]$$
$$P^{(N)}[D2,D1;\Delta=X] = \left|pa^{(N)}[D2,D1;\Delta=X]\right|^2 = [1/3]$$

The probability amplitude and probability that the idler photon is incident on detector D2 before the signal photon of the pair is incident on detector D1 with time difference $\Delta = mX$ (integer $m>0$) are:

$$pa^{(N)}[D2,D1;\Delta=mX] = pa^{(N)}[D2,D1;\Delta=X] \cdot (-r_1)^{(m-1)} = [-1/\sqrt{3}]^m$$
$$P^{(N)}[D2,D1;\Delta=mX] = \left|pa^{(N)}[D2,D1;\Delta=mX]\right|^2 = [1/3]^m$$
$$P^{(N)}[D2,D1;\Delta>0] = [1/2]$$

For this case with PC1 and PC2 both turned on, there will be <u>no</u> coincident detections of signal and idler photon pairs, as long as:

$$R_2 = R_1 = \left|r_1\right|^2 = [1/3] \ , \ T_2 = T_1 = \left|t_1\right|^2 = [2/3]$$

5. Discussion

In this experiment, quantum entanglement allows the signal and idler photon pairs to defy classical expectations. With $R_1 = R_2 = [1/3]$, the classical probabilities for coincidence and non-coincidence for this experimental setup are:

P_{CL}[coincidence] = $(1/3)^2$ + $(4/9)^2$ + $(4/27)^2$ + ... = $(1/3)$
P_{CL}[non-coincidence] = 1 - P_{CL}[coincidence] = $(2/3)$

Instead, all signal/idler photon pairs will either be in perfect coincidence or in perfect non-coincidence, as selected by the experimenter.
Note that this also includes the "immediately-reflected" photon pairs, in which one or both of the photons never enter an optical circulator.
In the perfect coincidence case, on average, for one-third of all of the photon pairs, the signal photon will travel from the Source to ABS1, immediately reflect from ABS1 without entering OC1, and will then travel to detector D1. The idler photon of the pair will travel from the Source to ABS2, immediately reflect from ABS2 without entering OC2, and will then travel to detector D2.
The coincident detection of these immediately-reflected signal/idler pairs occurs in spite of the fact that neither of these photons entered its optical circulator. This is an example of Einstein's "spukhafte Fernwirkung".
In the non-coincidence case, on average, one-third of all of the signal photons will travel from the Source to ABS1, immediately reflect from ABS1 without entering OC1, and will then travel to detector D1. Associated with these immediately-reflected signal photons, their paired idler photons will never immediately reflect from ABS2. Rather these idler photons will always enter OC2 and make at least one cycle through OC2 before ultimately being incident on detector D2.
Also, in the non-coincidence case, on average, one-third of all of the idler photons will travel from the Source to ABS2, immediately reflect from ABS2 without entering OC2, and will then travel to detector D2. Associated with these immediately-reflected idler photons, their paired signal photons will never immediately reflect from ABS1. Rather these signal photons will always enter OC1 and make at least one cycle through OC1 before ultimately being incident on detector D1.
The non-coincident detection of these signal/idler pairs occurs in spite of the fact that the first detected photon (signal or idler) did not enter its optical circulator.
Note that with this experimental setup, it is not possible to know when an "immediately-reflected" event has occurred. All that

is known is the time difference between the detections of the signal and idler photons.

6. Conclusion

The signal or idler photons obey classical statistics with respect to their detection. For example, with $R_1 = (1/3)$, the probability that a signal photon will be detected after making zero cycles through OC1 is equal to $(1/3)$. The probability that a signal photon will be detected after making one cycle through OC1 is equal to $(4/9)$.

To the contrary, the decidedly non-classical coincidence (or non-coincidence) of the signal and idler photon pairs is the result of non-local, two-photon interference that is allowed by the quantum entanglement between the two photons.

Appendix

With minor modification, the experimental setup (Figure 1) can be converted to act as a secure communications system.

The Source, optical circulator OC2, and detectors D1 and D2 are collocated to form the system's Source/Receiver (Src/Rx). Optical circulator OC1 is remotely located and acts as the system's Transmitter (Tx). The operator at the Tx can select to transmit a binary zero (coincidence condition) or a binary one (non-coincidence condition).

The external distance from PBSA to ABS2 and from ABS2 to detector D2 is now much shorter than the external distance from PBSA to ABS1 and from ABS1 to detector D1. The difference between these distances is known.

From creation to detection, the idler photon of a down-converted pair is always kept within the Src/Rx. Consequently, only the operator at the Src/Rx knows the time of detection of the idler photon.

The signal photon of the pair travels from the Src to the Tx (OC1). The signal photon then returns to and is detected in the Src/Rx.
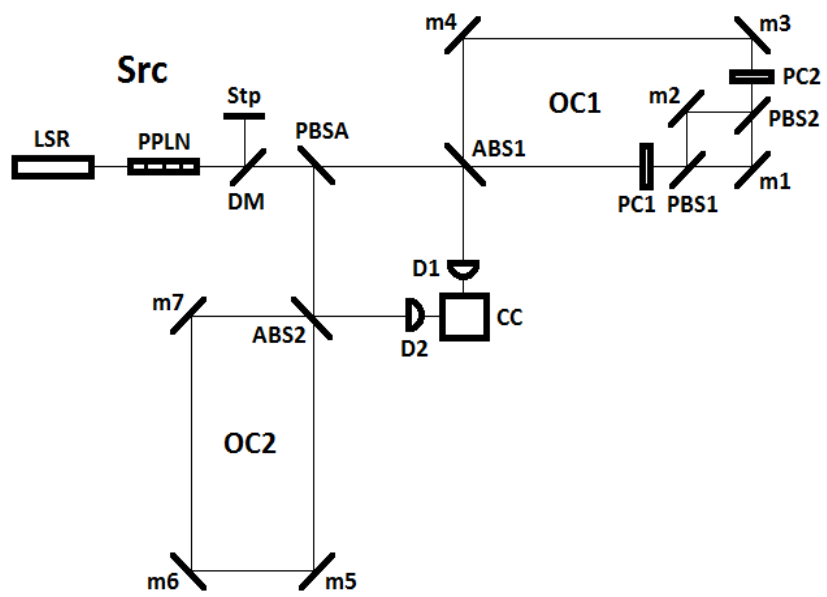
The operator at the Src/Rx knows the time of detection of both the idler photon and signal photon of each pair. By taking into account the known difference between the external distance from PBSA to detector D2 versus the external distance from PBSA to detector D1, the operator at the Src/Rx can determine whether a binary zero (coincidence condition) or a binary one (non-coincidence condition) has been transmitted.

An eavesdropper cannot know the time of detection of the idler photon, because the idler is never released from inside the Src/Rx. Consequently, an eavesdropper cannot determine whether a binary zero or a binary one has been transmitted.

This system provides a means for secure communication without the need for message encryption. Therefore, potentially, it allows secure communication to occur even in the presence of an eavesdropper using a quantum computer running Shor's algorithm.

References

[1] P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, <u>Physical Review A</u>, 47, 4, R2472 (1993).

[2] J. D. Franson, <u>Physical Review Letters</u>, 62, 2205 (1989).

[3] G. Weihs and A. Zeilinger, "Photon Statistics at Beam Splitters", <u>Coherence and Statistics of Photons and Atoms</u>, J. Perina (ed.), John Wiley & Sons, Inc. (2001).

Figure 1: Experimental Setup