ENTANGLED ANTIPODAL POINTS ON BLACK HOLE SURFACES: THE BORSUK-ULAM THEOREM COMES INTO PLAY

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The entangled antipodal points on black hole surfaces, recently described by t'Hooft, display an unnoticed relationship with the Borsuk-Ulam theorem. Taking into account this observation and other recent claims, suggesting that quantum entanglement takes place on the antipodal points of a S^3 hypersphere, a novel framework can be developed, based on algebraic topological issues: a feature encompassed in an S^2 unentangled state gives rise, when projected one dimension higher, to two entangled particles. This allows us to achieve a mathematical description of the holographic principle occurring in S^2. Furthermore, our observations let us to hypothesize that a) quantum entanglement might occur in a four-dimensional spacetime, while disentanglement might be achieved on a motionless, three-dimensional manifold; b) a negative mass might exist on the surface of a black hole.

KEYWORDS: Borsuk-Ulam theorem; antipodal points; quantum entanglement; holographic principle

The space-time topology of a black hole has been recently described in four dimensions (t'Hooft 2108). The impenetrable, continuous curtain surrounding the black hole, termed firewall, displays antipodal quantum states with matching description. This also means that particles emerging at opposite sides of the 4-dimensional hypersphere are strongly entangled. In turn, recent claims suggest that quantum entanglement can be described in terms of opposite features on a 4D hypersphere. Indeed, Peters and Tozzi (2016) showed that a separable state can be achieved for each of the entangled particles lying in S^2, just by embedding them in a higher dimensional S^3 space. Indeed, the Authors view quantum entanglement as the simultaneous activation of signals in a 3D space mapped into a S^3 hypersphere. Because the particles are entangled at the S^2 level and un-entangled at the S^3 hypersphere level, therefore a composite system is achieved, in which each local constituent is equipped with a pure state.

It is noteworthy that both the issues, i.e., the black hole's antipodal points and the entanglement on a hypersphere, are assessable through the framework described by the Borsuk-Ulam theorem (BUT), which states that e*very continuous map* $f: S^n \rightarrow R^n$ *must identify a pair of antipodal points* – diametrically opposite points on an n-sphere (Borsuk 1933; Henderson 2001; Matousek 2003). Points are *antipodal*, provided they are diametrically opposite (Borsuk, 1969; Borsuk and Gmurczyk, 1980). Examples of antipodal points are the endpoints of a line segment, or opposite points along the circumference of a circle, or poles of a sphere, or the opposite quantum states with matching description embedded in the t'Hooft's four-dimensional black hole surface (Krantz, 2009; Manetti, 2015; Moura and Henderson, 1996). In other words, the BUT states that two features with matching description are mapped to a single feature one dimension lower, provided the function under assessment is continuous. In the case of t'Hooft's account of black holes, the continuity is preserved, because the firewalls of their surfaces are continuous. In the sequel, we will show how the BUT is correlated with the holographic principle and will draw other unexpected consequences.

HOLOGRAPHIC PRINCIPLE AND TOPOLOGICAL MAPPINGS

The possibility dictated by the Borsuk-Ulam theorem to proceed from higher to lower dimensions and vice versa leads us to the realm of the holographic principle (HP). It states that the description of a volume of space can be thought of as encoded on a lower-dimensional boundary to the region (T'Hooft 1993; Susskind 1995). The theory suggests that the entire universe can be seen as two-dimensional information on the cosmological horizon. In HP, information (albeit quantum states evolving in spacetime) can be represented as a hologram, explainable via the theory of topological deformation retracts (Ahmed, Rafat, 2018). A deformation retract is a mapping of the boundary of a shape (surface) to its skeleton (Hatcher, 2002). In the context of black holes, we have a deformation of quantum states in their neighborhood. Here we show how, starting from the t'Hooft black hole equipped with quantum strates entangled on its horizon, an algebraic topological description of the HP can be provided.

The derivation of the holographic principle is represented concisely as a fibre bundle. Briefly, a ***fibre bundle*** is a triple $\left(E,\pi,B\right)$, where $\pi : E \to B$ is a projection mapping from a bundle space E to a base space B (Husemoller 1994). Fibre bundles are on the threshold of an operational view of a complex collection of mappings that includes a projection mapping. This is the case, since it is a straightforward task to extract from a fibre bundle the steps of an algorithm (aka, precise prescription leading to implementations in different settings, such as a black hole's horizon equipped with antipodal points). A fibre bundle representation of the holographic principle L is given in **Figure 1**, that illustrates how the holographic manifold L(M, X) can be extracted from a cross product of mappings:

$$X(n) \otimes M\left(m\left(\varphi(X(n)), \varphi(-X(n))\right)\right) \to L(M,X)$$

$$X \otimes M \to L(M,X)$$

,

$$L(M,X),$$

where $X(n)$ is the Hamiltonian of a quantum signal impacting on the particle n and an antipodal gathering $M(m(\varphi(X(n)),\varphi(-X(n))))$ that includes input from black hole's antipodal evaluations of X(n) from a spherical view of the black hole's horizon, i.e., antipodal values $\varphi(X(n)), \varphi(-X(n))$ originated from a circle-shaped_region of the horizon. That is, the mapping

$$X \otimes M \to L(M,X)$$

models the mapping of the accumulation $\otimes$ accruing from the interaction of the results of the mappings X and M to the holographic manifold $L(M,X)$, which displays a dimension lower than the black hole's 4D surface.

Concerning **Figure 1**, each arrow $\longmapsto$ represents a mapping. The arrows represent both ordinary mappings that carry the derivation forward and a projection mapping from X(n) to the black hole's horizon, which results in a gathering of antipodal evaluations of X(n), namely, $m\left(\varphi(X(n)), \varphi(-X(n))\right)$. A particular value of a holographic manifold $L(M,X)$ results from a synthesis of two signals: $X(n)$ and $M\left(m\left(\varphi(X(n)), \varphi(-X(n))\right)\right)$.

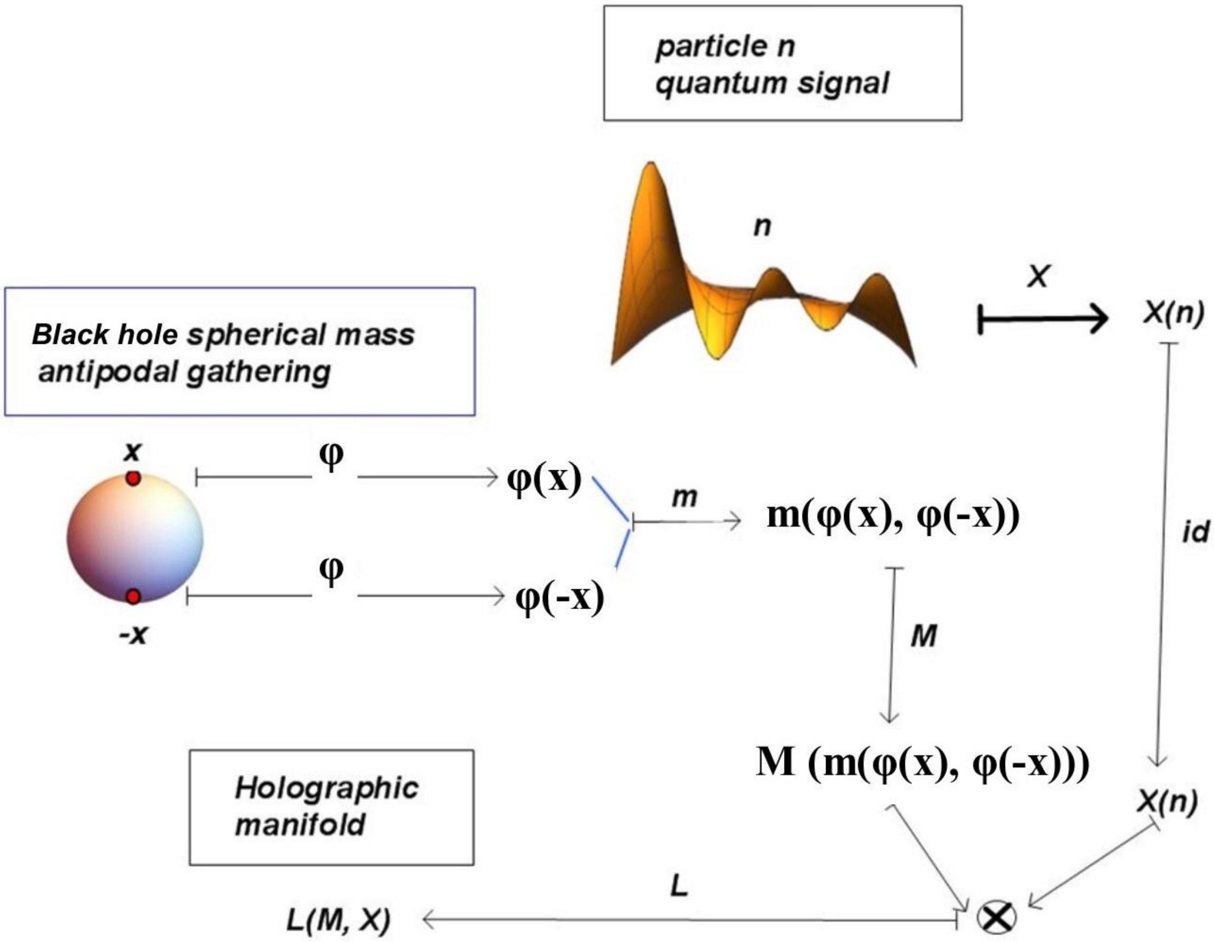

Figure 1. Fibre bundle representation the holographic principle and its relationships with black holes. The picture depicts the procedure to achieve a topological correlation between black hole's surface and the holographic principle. See text for further details.

INTRODUCING TIME IN THE BUT FRAMEWORK

The account of the cosmic holographic principle is generally provided by a framework which takes for granted that the event horizon is equipped with two spatial dimensions plus time. However, another possibility does exist, in order to describe a holographic manifold. A question arises: is it just a coincidence that the parameter time is not contemplated in two important formulas describing the Universe and the holographic principle? Indeed, both the Bekenstein-Hawking and Wheeler-De Witt equations take into account a static state of the related phenomena. Here the Moreva et al.'s results come into play. The latter Authors experimentally described how an observer located inside the Universe perceives the time flow, while a hypothetical external observer perceives the Universe as motionless (Moreva et al., 2013). According to their framework, entanglement discloses time as an emergent phenomenon. By running their experiment in two different modes ("observer" and "super-observer" mode) they showed how the same energy-entangled Hamiltonian eigenstate can be perceived as evolving by the internal observers that test the correlations between a clock subsystem and the rest, whereas it is static for the super-observer (Moreva et al., 2014). If we describe the Moreva et al.'s framework in terms of the BUT, we achieve the following topological result: an "observer" lies on a S^3 manifold, while a "superobserver" on a S^2 manifold. Indeed, the higher-dimensional manifold displays the coordinate of time, while the lower-dimensional does not. In physical terms, a manifold equipped with four dimensions (the three spatial dimensions plus time) encompasses two features with matching description. In turn, if we keep the dimension of time equal to zero (therefore removing it), we achieve a manifold, equipped with just three (spatial) dimensions, that encompasses just a single feature.

In sum, joining together the above-mentioned frameworks (by t'Hooft, Peters and Tozzi and Moreva et al.), it might be hypothesized that quantum entanglement occurs in spacetime, while disentanglement is achieved onto a motionless, three-dimensional manifold. Therefore, we may introduce the holographic principle in the following BUT terms: a motionless feature lying in a lower-dimensional stationary S^2 manifold gives rise to two moving features on a higher dimensional S^3 manifold, where time flow occurs. In other words, a feature encompassed in an unentangled state characterized by absence of time gives rise, when projected in one dimension higher (where time is not anymore zero), to two entangled particles. And vice versa.

ANTIPODAL MASSES: AN HYPOTHESIS

The above-mentioned frameworks allow to compare curved spacetime manifolds with structures equipped with antipodal symmetries. It is generally agreed that a black hole tends to deform the space around it, creating a vortex that captures nearby chunks of matter. The evolution of black holes can be represented by a Schwarzschild Spacetime Embedding Diagram (Zaslavskii 2011). In this approach, an embedding diagram for the vortex for a black hole can be visualized as a rubber sheet onto which a heavy mass is dropped. When an initial mass in increasing, the black hole's radius increases, burgeoning to a new mass with increasing gravitational pull. This observation allows us to tackle the issue in terms of antipodal points on black holes. Indeed, the different modes of the mass of a chunk of matter in the neighborhood of a black hole might reveal mass as an emergent phenomenon. The antipodal spacetime scenario for a chunk of matter being sucked into (of in the neighborhood of) a black hole is shown in **Figure 2** in a Penrose diagram, that is a conformal compactification of 2D Minkowski space. Using such a diagram to represent the evolution of soft particles populating spacetime was first suggested by Gerard t'Hooft (2017). Indeed, Penrose diagrams are able to capture the causal relations between antipodal points in spacetime. It is used to represent the infinities (timelike infinities vertically in two regions representing spacetime past and future, and spacelike infinities representing the evolution of the mass of a chunk of matter on the surface of a black hole). **Figure 2** extends t'Hooft's model, using the horizontal axis to represent the masses of soft matter (Parzygnat, 2014; Mitra 1999a; Mitra 1999b). The relationship between chunks of matter and a black hole in the neighborhood of surrounding ones, represented by the Penrose diagram in **Figure 2**, says to us that, as well as it is feasible to achieve antipodal points with matching features on a black hole horizon, we are allowed to hypothesize the simultaneous presence on the horizon of particles with positive and negative mass.

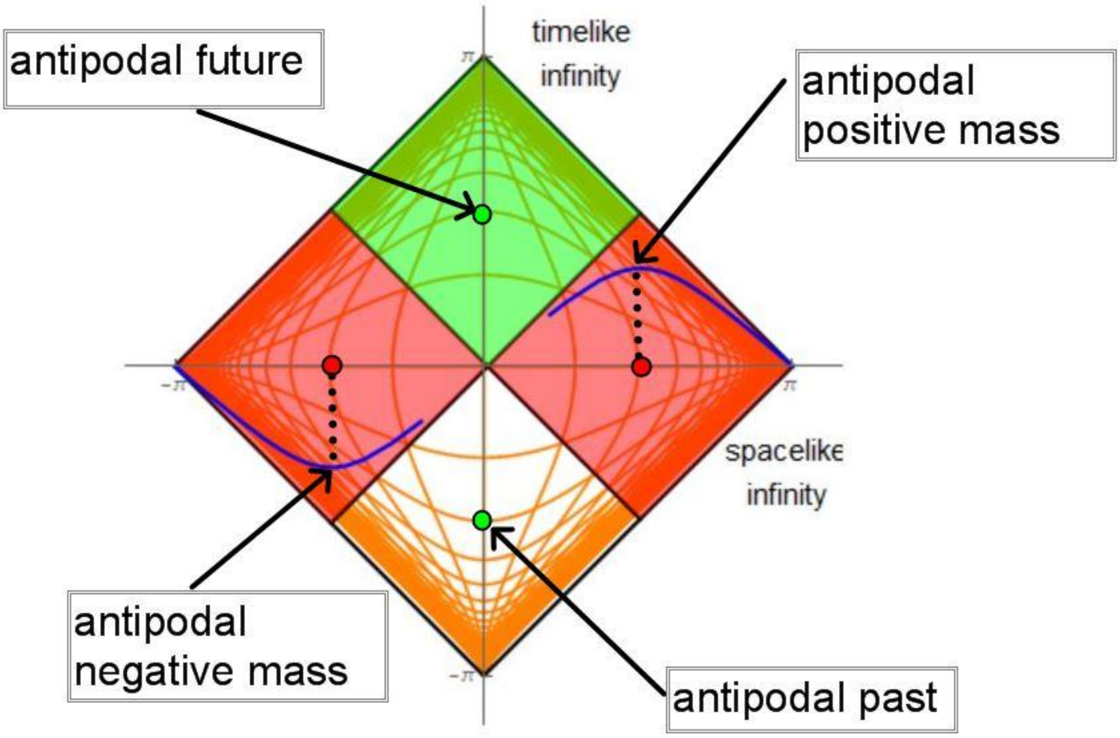

Figure 2. Evolution of chunks of soft matter in the neighborhood of a black hole. Antipodal spacetime is represented with a Penrose diagram, which also mimics the behavior of matter in the neighborhood of a black hole. Vertically, the green region represents future time and the orange region represents past time in the lifespan of a chuck of matter. Horizontally, the concave down blue geodesic line in the red region $+\pi$ represents the positive mass of a chuck matter on the S^4 surface of a black hole, whereas the concave up blue geodesic line in the red region $-\pi$ represents the negative mass of a chunk of matter on the opposite surface.

CONCLUSIONS

Here we showed how the black holes' antipodal points suggested by t'Hooft can be described in terms of the Borsuk-Ulam theorem. This led us to a algebraic topological description not just of black holes, but also of the holographic principle. The topological correlation between a black hole's surface and the holographic principle allows a fibre bundle representation, which stands for an algorithm that can be implemented in softwares.

The BUT approaches are potentially very fruitful, because they also allow to formulate intriguing theoretical claims. Indeed, they suggest the possible presence of antipodal positive and negative masses on black hole horizons. Furthermore, a feature encompassed in an unentangled state characterized by absence of time might give rise, when mapped to one dimension higher where time is introduced, to entangled particles. The last, but not the least, we need to remind that the proposal made by t'Hooft describes black holes in terms of pure quantum states. However, because he tackles antipodal entangled states in terms of opposite points on a S^3 hypersphere, his account might hold also for the 4-dimensional Minkowskian manifold of the general relativity, i.e., three spatial dimensions plus time. Therefore, we are in front of a potential unification of quantum mechanics and general relativity on a S^3 manifold.

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