Potential Errors in the Lorentz Force Equation for Moving Charged Particles in Magnetic Fields

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Abstract- All forces in the universe are created from potential changes in energy levels. Even when particles are constrained so that they cannot expend the energy giving rise to those forces they still experience them; for the force to exist the energy system creating it must exist. We examine the second part of the Lorentz Force Equation, which looks at the forces experienced by a charged particle travelling through a fixed magnetic field. Here there is a transverse force on the electron normal to the direction of travel, and the electron's path is deflected into a curve, with no expenditure of energy. However, the existence of this force requires an energy mechanism. This paper analyses the energy system behind this force and shows that there is a dependency on magnetic geometry that has been neglected.

Keywords- *Lorentz Force, Electromagnetism, Field Theory*

I. INTRODUCTION (HEADING 1)

We consider the second component of the Lorentz Force Equation, which describes a force from the induced electric field on a charge moving through a magnetic field [1]. This force affects the electrons' path, changing it from a straight-line course to a circular one. Lorentz states that **F**=q(**v** x **B**), that is, force is equal to the moving charge times the cross product of the magnetic field strength and the charge's velocity through the field. Although it tells us that the electron experiences forces when moving through a magnetic field in a gap inside a magnet, the motionally-induced electric field is constant everywhere in the gap. The electron can move through any part of the gap and the combined energy of the electron's field and the induced electric field is constant everywhere inside the gap because the magnetic field is a constant in the gap. Without energy differentials no forces can come directly from the interaction of the electron with the local induced field inside the gap. We must look elsewhere for the deflecting force. This force-generating mechanism must pass two tests.

A. The neutron is not deflected inside a magnetic field

First, we know that the neutron has an intense electric field even though it is bounded at a femtometer-scale radius [2]. Nevertheless, it is not deflected inside a magnetic field, so the magnetic component of the Lorentz Force Equation seems to work with electrons, but not with neutrons. Our theory needs to show why this anomalous behavior happens.

B. There is an apparent violation of the Conservation of Energy

Second, the Lorentz Force Equation suggests that if we replace the magnetic field with an electric field (in order that the electric field always pointed in the same direction rather than remaining normal to **B** x **v** and thus causing the electron to follow a curved path) an infinite number of electrons could traverse the field, increasing their kinetic energy inside the field, without any expenditure of energy anywhere in the system; when they left the field with their augmented kinetic energy they would have gained something for nothing; this violates the Principle of Conservation of Energy. Again, our theory must demonstrate there is no such violation. Whilst this is a thought experiment rather than a practical one it nevertheless highlights an important issue.

II. IDENTIFYING THE ENERGY SYSTEM

Let us start with a simple square magnetic field with the electron travelling in the field, parallel to one of the sides, as shown in Fig. 1 in plan. This shows a magnetic field ***B*** depicted in half-tone with the magnetic field vector pointing up out of the page. A test electron 'e-' is travelling through this field with a velocity v, and perceives an electric field **E** induced in its own frame of reference. The electron will perceive forces that direct it towards the left in this picture. These forces must derive from a potential (but unrealized) reduction in energy somewhere in the system that creates attractive forces to the left via the equation **F**=d**E**/dl where force **F** equals rate of change of Energy **E** over distance l, and/or repulsive forces from the right that will have the same effect. Here the electron's kinetic energy remains constant, and it is merely deflected into a curved path. The fact that energy is not consumed does not change this argument as

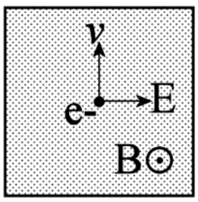

Figure 1. Electron moving through magnetic field in plan view.

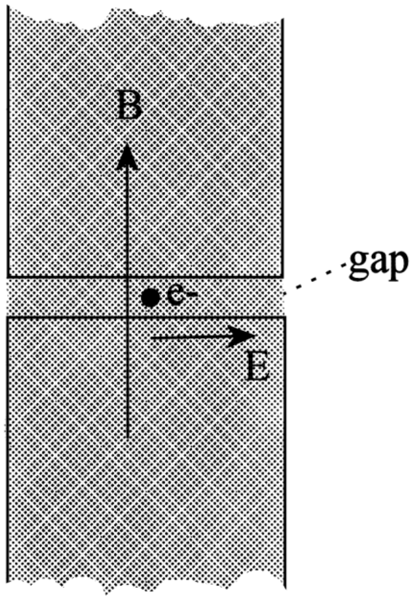

Figure 2. Side view of electron in magnet

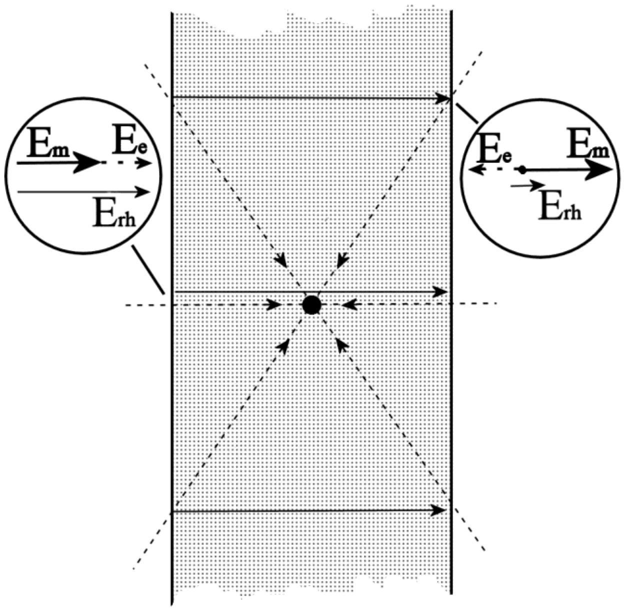

Figure 3. Electric field interaction at edges s of magnet

the expression of a force does not require an actual energy change in a system, it just requires the potential for one.

The obvious place to look is where the induced field changes its strength along the vector of the induced accelerative force that curves the path of the electron. The induced electric field has a sudden step in field strength at the left and right edges of the magnet, amounting to a massive localized gradient in the induced field, and this is therefore a candidate for the energy system. We now consider what happens at the left and right edges.

Although Fig. 1 refers to a simple slice through the magnetic field, it is important to realize we are not analyzing just one slice. Looking at the magnet side-on in Fig. 2 shows how the field extends in three dimensions. As before, the magnetic field is shown in halftone and its vector by **B**, and the induced electric field by **E**. The electron is shown travelling into the page inside the gap between the two poles of the magnet. As can be seen, the magnetic field, and therefore the induced electric field, continues through the poles of the magnet. The moving electron interacts with the whole field - the electron's electric field penetrates even into the atoms of the poles. Hence the computation must be over all interacting space, not just the gap between the poles.

In Fig. 3 we show the interaction between the electron's field and the induced electric field at the edges of the induced field. Two separate points are highlighted in inset pictures that show the electric vectors just within the edges of the magnet. The electron's field is shown in by dashed vectors $\mathbf{E}_e$ and the induced field is shown by solid vectors $\mathbf{E}_m$ describing the electric field induced by the magnetic field. In the right-hand inset the vertical component of the electron's field vector (a downward pointing vector in this inset) is not shown for clarity; it has no effect on the path of the electron, being orthogonal to the induced electric field and hence not interacting with it. The cutaways show only the effect on the horizontal component and the resultant horizontal electric field vector $\mathbf{E}_{rh}$ is shown as a solid vector.

Where two electric fields interact then the energy density is given by

$$\frac{dW}{dx\,dy\,dz} = \frac{\varepsilon\left(|\,E_1 + E_2\,|^2\right)}{2}$$

$$= \frac{\varepsilon E_1 \bullet E_1}{2} + \varepsilon E_1 \bullet E_2 + \frac{\varepsilon E_2 \bullet E_2}{2}$$

The first and third terms are simply the energy densities of the individual fields. The middle term is the energy density of interaction, or the *potential* energy density. If this term is positive there is an increase in energy when the fields interact and therefore forces will be generated that oppose the interaction. If it is negative there is a fall in energy and forces will be generated that encourage the interaction. It is positive where the originating field vectors are aligned within +90 to -90 degrees of each other, generating repulsive forces. It is negative where they are in opposition within +90 to +270 degrees of each other, generating attractive forces.

We now consider the interaction of the electron's fields with the edges of the magnet. If the electron moves a little to the left those parts of the fields of the electron that remain inside the magnetic field see no change in their energy density as the induced electric field has a zero gradient there, so those parts of the field produce no force. Equally all those parts of the fields that remain outside the magnet see no change in their energy density and produce no force. However, those horizontal components of the electron's field that enter at the right edge of the magnet as a result of this movement will go from their normal field to a partial cancellation with the magnet's induced electric field and hence to a reduced energy density and a leftwards force. Those horizontal components of the electron's

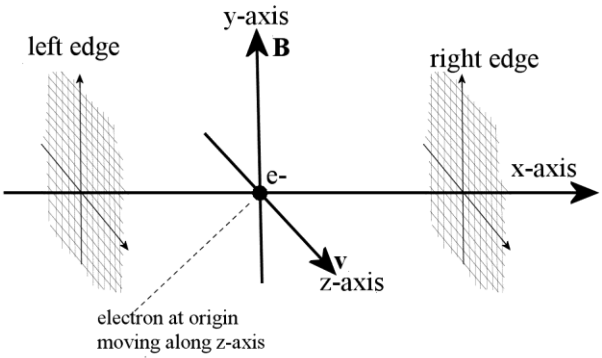

Figure 4. The interacting edges of the magnetic field

field leaving the magnet on the left will go from the increased energy density of interaction with the induced electric field to their normal independent energy density outside the magnet, leading to a drop in the energy density and a leftwards force. Hence there is a drop in the energy densities at both edges leading to forces that tend to force the electron to the left. There is clearly an energy system operating at the edges of the magnet.

III. Comparison with the Lorentz Force Equation

How does this compare with the Lorentz Force Equation? Let us use the following co-ordinate system for Fig. 4. The x-axis is parallel with the lines of the induced electric field, that is, left to right as in Fig. 2 and Fig. 3. The y-axis is parallel with the lines of the magnetic flux, that is, vertically as in Fig. 2. The z-axis is aligned with the electron's instantaneous velocity vector, that is, vertically in Fig. 3. In Fig. 4 the z-axis points out of the page and the left and right edges of the magnet are shown. The magnetic field B lies vertically along the y-axis and the motion v of the electron is instantaneously along the z-axis.

The whole of the induced electric field is normal to both the magnetic field and to the direction of motion. There is no component of the induced field in any other axis and the potential energy lies along only the x-axis component field of the electron in Fig. 4. Hence any forces must be along the x-axis. Next, if we consider that the induced electric field is some non-zero constant value within the magnet and zero elsewhere, then for any specific point in the electron's field the energy density of interaction with the induced field will be constant inside that field and zero outside. There are therefore changes in the energy density of interaction *only* on the boundaries of the induced field. We need consider only the potential energy associated with this edge-plane normal to the x-axis as shown in Fig. 4.

Using Fig. 4 we can derive the equation for the interaction energy density at a point [x,y,z] on one of these planes as the x-component of the electric field strength from the electron at the boundary, times the induced electric field strength from the magnet [3]. This plane is conceptually just inside the magnetic field, on its edge.

$$\frac{dU}{dx\,dy\,dz} = \frac{\varepsilon q}{4\pi\varepsilon\left(x^2+y^2+z^2\right)}\frac{x(\boldsymbol{v}\times\boldsymbol{B})}{\sqrt{\left(x^2+y^2+z^2\right)}}$$

$$= \frac{\varepsilon q(\boldsymbol{v}\times\boldsymbol{B})}{4\pi\varepsilon}\frac{x}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}$$

We then consider the edge as a boundary lying in axes y-z, and analyze what force we get from the energy system described above. The interaction energy outside the magnetic field is zero as there is no induced field there, so the whole of the energy density step across the boundary is the above equation. We take the rate of change of energy over potential motion dx_{mot} along the x-axis to give the sideways force 'F' on the electron from this interaction.

$$F = \frac{dU}{dx_{mot}} = \frac{\varepsilon q(\boldsymbol{v}\times\boldsymbol{B})}{4\pi\varepsilon}\iint\frac{x}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}\,dy\,dz$$

We can simplify this integration by recognizing that the term $(y^2+z^2) = r^2$ is constant for a circle around the x-axis, where 'r' is the radius of the circle. Substitute semi-polar co-ordinates and multiply the function by 2πr, equivalent to one rotation, and integrating over the radius. This changes our plane from a rectangular one to a circular one but that is unimportant for this analysis.

$$F = \frac{\varepsilon q(\boldsymbol{v}\times\boldsymbol{B})}{4\pi\varepsilon}\int\frac{2\pi r x}{\left(x^2+r^2\right)^{\frac{3}{2}}}\,dr$$

$$= \frac{q(\boldsymbol{v}\times\boldsymbol{B})}{2}\int\frac{r x}{\left(x^2+r^2\right)^{\frac{3}{2}}}\,dr$$

$$= -\frac{q(\boldsymbol{v}\times\boldsymbol{B})x}{2\sqrt{x^2+r^2}}$$

Evaluating for a plane of infinite size, evaluating from r=zero to r=infinity gives

$$F = \frac{q(\boldsymbol{v}\times\boldsymbol{B})}{2}$$

The same result holds for the edge plane at the opposite side of the magnet. The force in both cases is in the same direction as there is repulsion on the electron from one edge and attraction from the other, so the total force is twice the above, namely q(**v**x**B**). This is simply the Lorentz result, and so it can be seen that this energy system produces the same result as the Lorentz equation predicts.

IV. Issues that Arise from this Model

However, the integration above is over edge-planes of infinite extent; the left and right edges of the induced electric field parallel to the motion of the electron are assumed to be an infinite plane; anything less and we do not get the Lorentz result. It can therefore be seen that the result agrees with Lorentz only where the areas of the left and right edge-planes of the magnet are infinite in extent and at a finite distance along the x-axis from the center of the electron, and the particle's electric field extends across the edges of the magnetic field (as it does for the electron, but not the neutron). Cutting the sides down in size to some finite area will reduce the integration sum and so one can expect significant discrepancies for short magnets between the actual magnetic field strength and that strength as measured by the deflecting force on a charged particle. Any measurement of a practical magnetic field made by looking at the deflective force on a moving charged particle will therefore be artificially low in value, although it will obviously be in full agreement with every experiment involving the deflection of moving charged particles, which will suffer matching errors.

The term in ‘x’ disappears for the infinite-area integration. It does not matter what finite width the magnet is in ‘x’ if the length of the sides is infinite. The Lorentz value will hold only when the whole of the left hemisphere of the electron’s solid angle of electric field lines interact with the left edge of the magnetic fields, and the whole of the right hemisphere interacts with the right edge.

Therefore, if we make both the magnet’s y-axis and z-axis very much larger than its x-axis, we should find that the resultant computed force on the electron approaches the value given by the Lorentz Force Equation. Conversely, as the y-axis and/or the z-axis dimension of the magnet reduces in size whilst keeping the x-axis width constant, the resultant force drops further and further below the Lorentz value; the deflection of an electron by the induced field is reduced. Since many measurements of magnetic field strength are made by measuring the effect of the field on the motion of an electron (as in the Hall effect), in this model they would report a lower magnetic field than actually exists.

The treatment here has been simplified. The electron in Fig. 4 interacts with all of the magnetic field, not just that inside the magnet. In actuality the magnetic field diverges beyond the end of the magnetic poles and then loops around to meet up with the magnetic flux lines from the opposite pole in a return loop outside the magnet. The initial diverging flux lines near a pole acts to extend the effective length of the magnet, but we can ignore the region where they have looped back to meet the flux from the opposite pole as the electric field lines from the electron intersects both the inside and outside edges of the return flux and the effects at these edges therefore cancel out even where those return flux edges are diffuse as happens in magnets without a yoke.

V. The Solutions to Our Conundrums

The solutions to our two conundrums are then

A. The neutron is not deflected inside a magnetic field

A neutron will not be subject to any forces inside the magnetic field despite having a strong electric field, because its electric field is wholly contained within the magnet where the induced electric field gradient is zero, and its electric field does not reach the edges of the magnetic field. There is therefore no potential change in energy density as it moves through the magnetic field, and hence no forces exist.

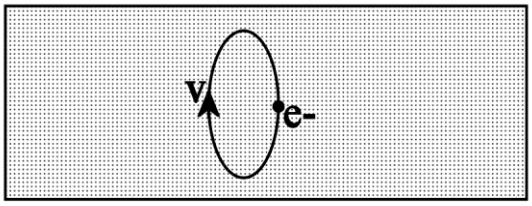

Figure 5. Electron circulating in magnet gap in plan view

B. There is an apparent violation of the Conservation of Energy

What if we replace the magnetic field with an electric field so that the electric field always pointed in the same direction rather than remaining normal to **B** x **v** and thus following a curved path? A theoretical electron falling from infinity is indeed accelerated across the field. But it is decelerated before entering the field by the interaction of the leading edge of the field with the electron’s field, its effect being reversed by the center of the electron being on the other side of the boundary, and likewise decelerated after exit from the field, the sum of the two decelerations matching the single acceleration. The overall effect is of no net change in the kinetic energy. The Principle of Conservation of Energy stands.

VI. Testing the Model

This model makes predictions at odds with the Lorentz Force Equation which can be tested. For example, consider an electron ‘**e-**’ circling inside a magnet, where the plan view of the magnetic field gap is rectangular, as shown in Fig. 5 (i.e. the magnetic field lines lie normal to the page). Here the Lorentz Force Equation predicts a perfectly circular path. However, this model predicts that the forces on the electron are greater when the electron is travelling parallel to the longer sides, causing the electron to follow an elliptical rather than a circular path. The major axis of the ellipse would therefore be parallel to the shorter sides.

Another prediction is that a short wide magnet and a long narrow magnet that have the same measurement of magnetic field strength when measured by electron deflection should have different measurements of magnetic field strength when measured by Paramagnetic Resonance.

VII. Conclusion

There is perhaps an unstated assumption in the magnetic component of the Lorentz Force Equations that a moving electron in a magnetic field is a point object sensitive only to the electric field strength at that point. Such a model cannot work because the induced field has a zero gradient in the region of the electron and no forces can result. The model presented here has no such defects, but indicates a sensitivity of the deflective force to the geometry of the magnetic field. Where the magnetic field strength is measured by the deflection of an electron inside the field in equipment such as Hall-effect devices and applied involving the deflection of electrons in the field, there is no conflict as the errors are identical in both cases. However, according to the model presented here, discrepancies should appear when a magnetic field is calibrated by electron deflection and then used to measure a magnetic dipole, as in paramagnetic resonance. For all realizable geometries electron deflection measurements should underestimate the strength of the magnetic field.

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