Fermat's last theorem. Proof of P. Fermat

In Memory of my MOTHER

Contradiction: Any prime factor *r* of the number R in the equality $A^n=A^n+B^n$ [...=(A+B)R] has a single ending 0...01 of infinite length; where $r \neq n$.

All calculations are done with numbers in base n, a prime number greater than *2*.

FLT is proved for the **base** case with AB not divisible by n.

1°) $C^n=A^n+B^n$ [...=(A+B)R or ...=(A+B)(nR)] (see http://vixra.org/abs/1707.0410)), where
2°) the numbers A, B, C, R and A+B are co-primes,
with the help of **the Theorem on power-power binomial.**

Every prime divisor r (different from the prime n>2) of the factor R binomial $A^{n^k}+B^{n^k}=(A^{n^{k-1}}+B^{n^{k-1}})R$, where the numbers A and B are co-prime and k>1, has the form: $m=dn^k+1$ (the proof is in the Appendix).

Proof of the FLT

Let *r* be a prime factor of the number R different from n.
Take the numbers xr+A and yr+B from the equations
4°) $xr+A=A^{n^k}$ and $yr+B=B^{n^k}$, where x and y have integer solutions (see Annex) and k are arbitrarily large, and consider a number
5°) $D=(xr+A)^n+(yr+B)^n=(xr+A+yr+B)T$, which is divisible by r [since the number $A^{n^k}+B^{n^k}$ has a factor A^n+B^n, equal to (A+B)R], and its factor xr+A+yr+B is not divisible by r (see 2°).
Thus, given a prime number r, we find an arbitrarily large number k, the number r is a factor of number T and, according to 3°, has the form:
$r=dn^{k+1}+1$.

Which implies the truth of the FLT.

I firmly believe that Pierre de Fermat had this proof of the great theorem in mind.
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ANNEX

Theorem on power-power binomial.
Every prime divisor (different from the prime n>2) of the factor R binomial $A^{n^k}+B^{n^k}=(A^{n^{k-1}}+B^{n^{k-1}})R$, where the numbers A and B are co-prime and k>1, has the form: $m=dn^k+1$.

Proof

Suppose that among the prime divisors of the number R there is a divisor in the form: $m=dn^{k-1}+1$, where d is not divisible by n. Then the numbers

1°) $A^{n^k}+B^{n^k}$ and, according to the Little Fermat's theorem for prime degree r,
2°) $A^{dn^{(k-1)}}-B^{dn^{(k-1)}}$ (where d is an even number) are divisible by r.

The theorem about GCD of two power binomials $A^{dn}+B^{dn}$ and $A^{dq}+B^{dq}$, where the natural A and B are co-prime , n [>2] and q [>2] are co-prime and d>0, says that the greatest common divisor (not counting n) of these binomials is equal to A^d+B^d .

In our case, the GCD multiple of r is the number $A^{n^{(k-1)}}+B^{n^{(k-1)}}$, which is co-prime with the number R. Hence, any factor r in the form $r=dn^{n^{(k-1)}}+1$ does not belong to the number R. From which follows the truth of the lemma.

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Integer solution of the equation $xr+A=A^{n^k}$ (and $yr+B=B^{n^k}$).
Designation: V // r – number V is divisible by r and r is a factor of number V.

From $xr+A=A^{n^k}$ ➔ $A(A^{n^k-1}-1)$ // r ➔
number n^k-1 // r-1 (requirement of Fermat's small theorem for the divisibility of $A^{r-1}-1$ by r), it means: $n^k-1=v(r-1)$, where $r-1=s_1s_2\ldots s_m$ and s_1, s_2, $\ldots s_m$ are prime factors of the number r-1. ➔
$k=M(s_1-1)(s_2-1)\ldots(s_m-1)$ – is a requirement of Fermat's small theorem for divisibility of n^k-1 by the numbers s_1, s_2, $\ldots s_m$. After that $A(A^{v(r-1)}-1)$ // r ➔ from where we find $x=A(A^{v(r-1)}-1)/r$.
Thus, for a given prime number r, we find such k that $xr+A=A^{n^k}$.
I think there is no need to comment virtuosity of P. Fermat.