Dealing with optical fibers in General relativity

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Abstract

An example of how optical fibers can clarify the influence of a gravitational field on the propagation of light.

1 Introduction

Classical kinematics assumes that a free point particle moves with respect to a Galilean frame of reference as follows:

$$x^i = x_0^i + v^i(t - t_0) \tag{1}$$

where x_0^i is its position at time t_0 and v_0^i is its constant velocity.

Different frames of reference are related by Galilean transformations:

$$x^{i'} = R_j^{i'} x^j - A^{i'}, \quad t' = t - U \tag{2}$$

where R is a three dimensional rotation matrix. Special relativity introduces the concept of event with coordinates (x^i, t), or x^α with $\alpha = 1, 2, 3, 4$, and uses Lorentz transformations:

$$x^{\alpha'} = L_\beta^{\alpha'}(x^\beta - x_0^\beta) \tag{3}$$

instead of Galilean transformations to define the covariance of inertial systems of reference. $L_\beta^{\alpha'}$ are functions of v^i and a universal constant c that has been identified since then as the speed of light in vacuum.

While the identification of c with the speed of light is a deep insight in all branches of field theories, iit raises serious epistemology and metrology problems and may lead to paradoxes as rulers that contract or clocks that change

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their pace as they move. Calling c the "Universal space-time constant" or maybe, for short, the Roemer constant, would be safer and less controversial.

The problem is that from an epistemological point of view c is not measurable by separate measures of space intervals with rulers, and time intervals with clocks. Assuming rigidity or,what cant is the same, free mobility, we can prove that two rulers at different locations have the same length if moving one of them we can superpose it to the other. But we can not compare two time intervals at different times and different locations without a detailed discussion of how clocks behave when they are moved.

Let us consider an ideal optical fiber, i.e. a thin crystal thread with refractive index 1, supposed to have a fixed length whatever its shape and let us call L its finite length when stretched at its maximum ([4]), Appendix A,and B)). The concept of space has no meaning without this definition of length, and the main postulates of Euclidean geometry may be translated to this new terminology. The main point being the implementation of the Helmholtz free mobility postulate,

Space Geometry, the science describing the properties and relationships of figures build with stretched fibers like triangles , tetrahedrons, etc, or a variety circles, spheres, etc, would have no meaning without this postulate..

The dual second postulate to be considered is the existence of a large classes of clocks that when brought at a same location they beat at the same frequency. And when brought at a same location, after having been dispersed, they might not be synchronized but they still beat at the same frequency. Without them the concept of time in physics would have no meaning.

Classical mechanics uses the concept of Euclidean distance, as measured by stretched optical fibers called rods, and Euclidean geometry. And essential is also the assumption that any figure built with rods can be displaced from one location to another conserving any property they could have in the first.

What follows is inspired by these simple considerations and is all about measuring the time interval that light takes when circulating along circles of optical fibers located on the surface of the Earth depending on its position and its orientation, hoping that the simplicity of the experimental setting might in in the near future improve our knowledge of how gravitation affects light propagating along optical fibers.

2 Circles of optical fibers on the surface of the Earth

The simplest model satisfying the free mobility postulate is Euclidean space:

$$d\tilde{s}^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad \text{or} \quad d\tilde{s}^2 = dx^2 + dy^2 + dz^2 \tag{4}$$

and the simplest of the relativistic models derived from the Schwarzschild model where it is legitimate to claim that the coordinates r, θ, φ have the same meaning that they have in Euclidean space, is the Brillouin model ([2]),([4]):

$$ds^2 = -\frac{r}{r+2m}dt^2 + \frac{r+2m}{r}dr^2 + \left(1 + \frac{2m}{r}\right)^2 r^2(d\theta^2 + \sin\theta^2 d\varphi^2) \tag{5}$$

where it has been assumed that $c = 1$, and has been further approximated to linear terms of the parameter m/r:

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dt^2 + \left(1 + \frac{2m}{r}\right)dr^2 + \left(1 + \frac{4m}{r}\right)r^2(d\theta^2 + \sin\theta^2 d\varphi^2) \tag{6}$$

with:

$$r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \arctan\frac{\sqrt{x^2+y^2}}{z}, \quad \phi = \arctan\frac{y}{x} \tag{7}$$

dr,$d\theta$, $d\varphi$, being the corresponding differentials, the coefficients of the following quadratic form

$$ds^2 = g_{44}dt^2 + g_{11}dx^2 + g_{22}dy^2 + g_{33}dz^2 + 2(g_{12}dxdy + g_{13}dxdz + g_{23}dydz) \tag{8}$$

dr, $d\theta$ and $d\varphi$ are:

$$g_{44} = -1 + \frac{2m}{r} \tag{9}$$

$$g_{11} = 1 + \frac{2m}{r}(2 - \cos^2\varphi\sin^2\varphi) \tag{10}$$

$$g_{22} = 1 + \frac{2m}{r}(2 - \sin^4\varphi) \tag{11}$$

$$g_{33} = 1 + \frac{2m}{r}(1 + \sin^2\varphi) \tag{12}$$

$$g_{12} == -\frac{2m}{r}\cos\varphi\sin\varphi\sin^2\theta \tag{13}$$

$$g_{13} = -\frac{2m}{r}\cos\varphi\sin\theta\cos\theta \tag{14}$$

$$g_{23} = -\frac{2m}{r}\sin\theta\sin(\phi)\cos\theta. \tag{15}$$

the remaining coefficients being 0.

δt being the positive solution of the equation $ds^2 = 0$, approximated to order 1 of m/r on the surface $r = const$ we get:

$$\delta t = \delta t_{11}dx^2 + \delta t_{22}dy^2 + \delta t_{33}dz^2 + 2(\delta t_{12}dxdy + \delta t_{13}dxdz + \delta t_{23}dydz) \tag{16}$$

we have:

$$\delta t_{11} = 1 + \frac{m}{r}(3 - \cos^2\varphi\sin^2\theta) \tag{17}$$

$$\delta t_{22} = 1 + \frac{m}{r}(3 - \sin^2\theta\cos^2\varphi) \tag{18}$$

$$\delta t_{33} = 1 + \frac{m}{r}(2 + \sin^2\theta) \tag{19}$$

$$\delta t_{12} = -\frac{m}{r}\cos\phi\sin\phi\sin\theta \tag{20}$$

$$\delta t_{23} = -\frac{m}{r}\sin(\theta)\sin\varphi\cos\theta \tag{21}$$

$$\delta t_{31} = -\frac{m}{r}\sin\theta\cos\varphi\cos\theta \tag{22}$$

Defining now:

$$\Delta t = \int_C \delta t\, du,\ u = 0..2\pi \tag{23}$$

along any circuit $C(u)$, we get in particular: if:

$$Cx:\ dx = 0,\ dy = n\cos(u)du,\ dz = n\sin(u)du, \tag{24}$$

then:

$$\Delta tx = 2\pi n\left(1 + \frac{m}{2r}(5 + \cos^2\varphi\sin^2\theta)\right) \tag{25}$$

If:

$$Cy: \;\; dx = n\cos(u)du, \;\; dy = 0, \;\; dz = n\sin(u)du, \tag{26}$$

then:

$$\Delta ty = 2\pi n\left(1 + \frac{m}{2r}(5 + \sin^2\theta\sin^2\phi)\right) \tag{27}$$

and if:

$$Cz: \;\; dx = n\cos(u)du, \;\; dy = n\sin(u)du, \;\; dz = 0, \tag{28}$$

then

$$\Delta tz = 2\pi n\left(1 + \frac{m}{2r}(5 + \cos^2\theta)\right) \tag{29}$$

Assuming, for example, that $\phi = 0, \;\; \theta = \pi/4$ on the surface of the Earth with:

$$r = 6.3781365\,10^6, \;\; m = 0.004435022103 \text{ meters} \tag{30}$$

and $n = 1$ we get:

$$\Delta tx - \Delta tz = 0 \tag{31}$$

and:

$$\Delta ty - \Delta tx = 3.643352175\,10^{-18} \text{ seconds} \tag{32}$$

per unit cycle of a circle of radius 1 meter.

Are long optical fiber coils able to test these results?.

References

[1] (a) k. Schwarzschild, (a) *Sitzungsber. Preuss. Akad. Wiss.*, Phys. Math. Kl., 189 (1916) (b) Reprinted as Golden Oldie *Gen. Rel. Grav.*, **35**, 951 (2003)

[2] M. Brillouin, *Le Journal de Physique et Le Radium*, **23**, 43 (1923). English translation by S. Antoci: arXiv:Physics/0002009 v1

[3] Ll. Bel, arXiv:1103.3767v2 [gr-qc] (2011)

[4] Ll. Bel, viXra.org 1707.0303 (2017)