A note on properties of a prime-generating quadratic polynomial 13n^2 + 53n + 41

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Abstract

This note presents some properties of a quadratic polynomial 13n^2 + 53n + 41. One of them is unique, while others are shared with other prime-generating quadratics. The main purpose of this note is to emphasize certain common features of such quadratics that may not have been noted before.

The quadratic 13n^2 + 53n + 41 has some interesting properties that we discuss in what follows. We label them A, B, and C. The truly unique is property A.

Property A. 1) it is a prime-generating polynomial (generates 11 primes in a row starting at n=0), see [1], 2) its coefficients are distinct positive primes, and 3) the sum of its coefficients is the smallest possible under conditions 1) and 2). This sum is 107, and 107 = 2*53 + 1 (see property B). The 11 consecutive primes mentioned are: 41, 107, 199, 317, 461, 631, 827, 1049, 1297, 1571, 1871.

To spell it out, this is a prime-generating quadratic polynomial with distinct positive prime coefficients whose sum is smaller than for any other such a polynomial.

If 2) is relaxed to allow non-distinct positive primes, then 1) and 3) are met by 2n^2 + 2n + 19 (same as [A007639] in the OEIS [2] for n running from 1) that generates 18 primes in a row. The prime-generating quadratic that generates at least 20 primes in a row and meets 2) and 3) is 43n^2 +151n + 1427; all 27 consecutive primes that it generates are in [A272285] as 43(n-8)^2 +151(n-8) + 1427= 43n^2 - 537n + 2971. If 2) is relaxed to allow non-distinct positive primes, the prime-generating quadratic that generates at least 20 primes in a row and meets 3) is 3n^2 + 3n + 23 (same as [A007637] for n running from 1); it generates 22 primes in a row.

Property B. If f(n) = an^2 + bn + c, then in this case b = a + c-1, implying a + b + c = 2b + 1, and ensuring that if b is prime, it is a Sophie Germain prime. This property is shared by a number of other prime-generating quadratics, including n^2 + 23n + 23 [A292509] and 2n^2 + 44n + 43 (17 primes in a row in both cases), as well as 3n^2 + 39n + 37 [A256585] and 47n^2 + 99n + 53 (18 primes in a row in both cases).

Let us note that there are also prime-generating quadratics with a property very similar to property B, namely, b = a + c + 1. Two of such quadratics are mentioned in [3] (7n^2 + 49n + 41 [A272077] and 11n^2 + 55n + 43 [A292578]) and yet another one, generating 18 primes in a row, is 10n^2 + 70n + 59.

Property C. Two of its coefficients are proper (of the form 6k + 5) distinct Sophie Germain primes: 41 and 53. 2 is the largest number of such primes that a prime-generating quadratic can have for its coefficients, whether they are distinct or not. If improper Sophie Germain primes are allowed, then one can have prime-generating quadratics whose all coefficients are Sophie Germain primes, e.g., 3n^2+3n+23. This property, properties A1, A2, and property B are shared by 41n^2 + 83n + 43 and 61n^2 + 113n + 53, generating 11 and 19 primes in a row, respectively.

The first of these quadratics also stands out in that its coefficients a and c form a prime pair and while 41 and 83 are Sophie Germain primes, we also have 83 = 2*41 + 1, which means that these primes are a part of a three-link Cunningham chain of the first kind (41, 83, 167) [4].

References

[1] http://mathworld.wolfram.com/Prime-GeneratingPolynomial.html

[2] http://oeis.org/

[3] W. Puszkarz, A note on some class of prime-generating quadratics, viXra preprint (2018)

[4] https://en.wikipedia.org/wiki/Cunningham_chain