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Shortest Path Problem on Single Valued Neutrosophic Graphs

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Abstract**—** A single valued neutrosophic graph is a generalized structure of fuzzy graph, intuitionistic fuzzy graph that gives more precision, flexibility and compatibility to a system when compared with systems that are designed using fuzzy graphs and intuitionistic fuzzy graphs. This paper addresses for the first time, the shortest path in an acyclic neutrosophic directed graph using ranking function. Here each edge length is assigned to single valued neutrosophic numbers instead of a real number. The neutrosophic number is able to represent the indeterminacy in the edge (arc) costs of neutrosophic graph. A proposed algorithm gives the shortest path and shortest path length from source node to destination node. Finally an illustrative example also included to demonstrate the proposed method in solving path problems with single valued neutrosophic arcs.

Keywords— Single valued neutrosophic sets; Single valued neutrosophic graph; Shortest path problem.

I. Introduction

The concept of neutrosophic set (NS for short) proposed by Smarandache [8, 9] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It is a generalization of the theory of fuzzy set [26], intuitionistic fuzzy sets [22, 23], interval-valued fuzzy sets [18] and interval-valued intuitionistic fuzzy sets [25], then the neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]^-0, 1^+[$. Therefore, if their range is restrained within the real standard unit interval [0, 1], Nevertheless, NSs are hard to be apply in practical problems since the values of the functions of truth, indeterminacy and falsity lie in$]^-0, 1^+[$.The single valued neutrosophic set was introduced for the first time by Smarandache in his 1998 book. The single valued neutrosophic sets as subclass of neutrosophic sets in which the value of truth-membership, indeterminacy-membership and falsity-membership degrees are intervals of numbers instead of the real numbers. Later on ,Wang et al.[12] studied some properties related to single valued neutrosophic sets. The concept of neutrosophic sets and its extensions such as single valued neutrosophic sets, interval neutrosophic sets, simplified neutrosophic sets and so on have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine and economic [1,4-11, 15-17, 20-21, 25, 27-31,32-38, 40]. The shortest path problem (SPP) is one of the most fundamental and well-known combinatorial problems that appear in various fields of science and engineering, e.g.,road networks application, transportation, routing in communication channels and scheduling problems. The shortest path problems concentrate on finding the path of minimum length between any pair of vertices. The arc (edge) length of the network may represent the real life quantities such as, time, cost, etc. In a classical shortest path problem, the distance of the arc between different nodes of a network are assumed to be certain. In some uncertain situation, the distance will be computed as a fuzzy number depending on the number of parameters is considered.

In the recent past, There are many shortest path problems that have been studied with different types of input data, including fuzzy set, intuitionistic fuzzy sets , vague sets [2, 3, 30,39]. many new algorithm have been developed so far. To the best of our knowledge, determining the shortest path in the networks in terms of indeterminacy and inconsistency has been not studied yet.
The shortest path problem involves addition and comparison of the edge lengths. Since, the addition and comparison between two single valued neutrosophic numbers are not alike those between two precise real numbers, we have used the ranking method proposed by Ye [20].

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Therefore, in this study we extend the proposed method for solving fuzzy shortest path proposed by [2] to SVN-numbers for solving neutrosophic shortest path problems in which the arc lengths of a network are assigned by SVN-numbers

The remainder of this paper is organized as follows. In Section 2, we review some basic concepts about neutrosophic sets, single valued neutrosophic sets, single valued neutrosophic graph and complete single valued neutrosophic graph. In Section 3, an algorithm is proposed for finding the shortest path and shortest distance in single valued neutrosophic graph. In section 4 an illustrative example is provided to find the shortest path and shortest distance between the source node and destination node. Finally, Section 5outlines the conclusion of this paper and suggests several directions for future research.

II. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, single valued neutrosophic graphs, relevant to the present work. See especially [8, 12, 32, 37, 41] for further details and background.

Definition 2.1 [8]. Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ X}, where the functions T, I, F: X→]$^-$0,1$^+$[define respectively the truth-membership function, indeterminacy-membership function, and falsity-membership function of the element x ∈∈ X to the set A with the condition:

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]$^-$0,1$^+$[.

Since it is difficult to apply NSs to practical problems, Smarandache [1998] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [12]. Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in$ [0, 1]. A SVNS A can be written as

$$A=\{< x: T_A(x), I_A(x), F_A(x) >, x \in X\} \quad (2)$$

Definition 2.3 [20]. Let $\tilde{A}_1 = (T_1, I_1, F_1)$ and $\tilde{A}_2 = (T_2, I_2, F_2)$ be two single valued neutrosophic number. Then, the operations for NNs are defined as below;

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2)$ (3)

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = (T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2)$ (4)

(iii) $\lambda\tilde{A} = (1-(1-T_1)^\lambda), I_1^\lambda, F_1^\lambda)$ (5)

(iv) $\tilde{A}_1^\lambda = (T_1^\lambda, 1-(1-I_1)^\lambda, 1-(1-F_1)^\lambda)$ where $\lambda > 0$ (6)

Definition 2.4 [20]. Let $\tilde{A}_1 = (T_1, I_1, F_1)$ be a single valued neutrosophic number. Then, the score function $s(\tilde{A}_1)$, accuracy function $a(\tilde{A}_1)$ and certainty function $c(\tilde{A}_1)$ of an SVNN are defined as follows:

(i) $s(\tilde{A}_1) = \dfrac{2 + T_1 - I_1 - F_1}{3}$ (7)

(ii) $a(\tilde{A}_1) = T_1 - F_1$ (8)

(iii) $a(\tilde{A}_1) = T_1$ (9)

Comparison of single valued neutrosophic numbers

Let $\tilde{A}_1 = (T_1, I_1, F_1)$ and $\tilde{A}_2 = (T_2, I_2, F_2)$ be two single valued neutrosophic numbers then

(i) $\tilde{A}_1 \prec \tilde{A}_2$ if $s(\tilde{A}_1) \prec s(\tilde{A}_2)$ (10)

(ii) $\tilde{A}_1 \succ \tilde{A}_2$ if $s(\tilde{A}_1) \succ s(\tilde{A}_2)$ (11)

(iii) $\tilde{A}_1 = \tilde{A}_2$ if $s(\tilde{A}_1) = s(\tilde{A}_2)$ (12)

Definition 2.5 [41]. 0_n may be defined as four types:

(0_1) Type 1. $0_n = \{< x, (0,0,1) >: x \in X\}$

(0_2) Type 2. $0_n = \{< x, (0,1,1) >: x \in X\}$

(0_3) Type 3. $0_n = \{< x, (0,1,0) >: x \in X\}$

(0_4) Type 4. $0_n = \{< x, (0,0,0) >: x \in X\}$

1_n may be defined as four types:

(1_1) Type 1. $1_n = \{< x, (1,0,0) >: x \in X\}$

(1_2) Type 2. $1_n = \{< x, (1,0,1) >: x \in X\}$

(1_3) Type 3. $1_n = \{< x, (1,1,0) >: x \in X\}$

(1_4) Type 4. $1_n = \{< x, (1,1,1) >: x \in X\}$

Definition 2.6 [32, 37].A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair G= (A, B) where

1.The functions T_A:V$\longrightarrow$[0, 1], I_A:V→$\longrightarrow$[0, 1] and F_A:V→$\longrightarrow$[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in$ V, respectively, and

$0 \le T_A(v_i) + I_A(v_i) + F_A(v_i) \le 3$ for all $v_i \in$ V. (13)

2. The Functions T_B : E ⊆ V x V $\longrightarrow$[0, 1], I_B :E ⊆ V x V $\longrightarrow$[0, 1] and I_B : E ⊆ V x V $\longrightarrow$[0, 1] are defined by

$T_B(v_i, v_j) \le \min [T_A(v_i), T_A(v_j)]$, (14)

$I_B(v_i, v_j) \ge \max [I_A(v_i), I_A(v_j)]$ and (15)

$F_B(v_i, v_j) \geq \max [F_A(v_i),] F_A(v_i)]$ (16)

denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in$ E respectively, where

$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + \leq F_B(v_i, v_j) \leq 3$ for all $(v_i, v_j) \in$ E (i, j = 1, 2,…, n) (17)

A is the single valued neutrosophic vertex set of V, B is the single valued neutrosophic edge set of E, respectively.

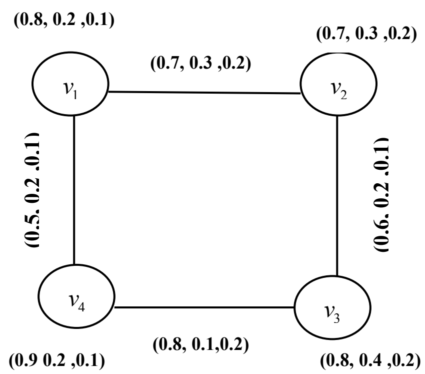

Fig.1. Single valued neutrosophic graph

III. An Algorithm for Neutrosophic Shortest Path in a Network

In this section an algorithm is proposed to find the shortest path and shortest distance of each node from source node. The algorithm is a labeling technique . Since the algorithm is direct extension of existing algorithm [30, 39, 41] with slightly modification. So it is very easy to understand and apply for solving shortest path problems occurring in real life problems.

Remark: In this paper, we are only interested in neutrosophic zero, given by:

$$0_n = (0,1,1)$$

Step 1 : Assume $\tilde{d}_1 = (0,1,1)$ and label the source node (say node1) as [(0,1,1) ,-].

Step 2: Find $\tilde{d}_j$ = minimum{ $\tilde{d}_i \oplus \tilde{d}_{ij}$ };j=2,3,…,n.

Step 3 : If minimum occurs corresponding to unique value of I i.e., i= r then label node j as [$\tilde{d}_j$,r]. If minimum occurs corresponding to more than one values of i then it represents that there are more than one neutrosophic path between source node and node j but neutrosophic distance along path is $\tilde{d}_j$, so choose any value of i.

Step 4 : Let the destination node (node n) be labeled as [$\tilde{d}_n$,l], then the neutrosophic shortest distance between source node is $\tilde{d}_n$.

Step 5 : Since destination node is labeled as [$\tilde{d}_n$,l], so, to find the neutrosophic shortest path between source node and destination node, check the label of node l. Let it be [$\tilde{d}_l$,p], now check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

Step 6 : Now the neutrosophic shortest path can be obtained by combining all the nodes obtained by the step 5.

Remark 1. Let $\tilde{A}_i$; i =1, 2,…, n be a set of neutrosophic numbers, if S($\tilde{A}_k$) < S($\tilde{A}_i$), for all i, the neutrosophic number is the minimum of $\tilde{A}_k$

Remark 2 : A node i is said to be predecessor node of node j if

(i) Node i is directly connected to node j.

(ii) The direction of path connecting node i and j from i to j.

In Fig 3, we present the flow diagram representing the neutrosophic shortest path algorithm

IV.ILLUSTRATIVE EXAMPLE

Let us consider a single valued neutrosophic graph given in figure 1, where the distance between a pair of vertices is a single valued neutrosophic number. The problem is to find the shortest distance and shortest path between source node and destination node on the network.

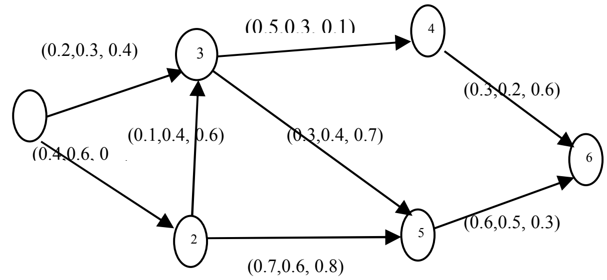

Fig.2 Network with neutrosophic shortest distance

Edges	Single valued Neutrosophic distance
1-2	(0.4, 0.6, 0.7)
1-3	(0.2, 0.3, 0.4)
2-3	(0.1, 0.4, 0.6)
2-5	(0.7, 0.6, 0.8)
3-4	(0.5, 0.3, 0.1)
3-5	(0.3, 0.4, 0.7)
4-6	(0.3, 0.2, 0.6)
5-6	(0.6, 0.5, 0.3)

Table 1.Weights of the graphs

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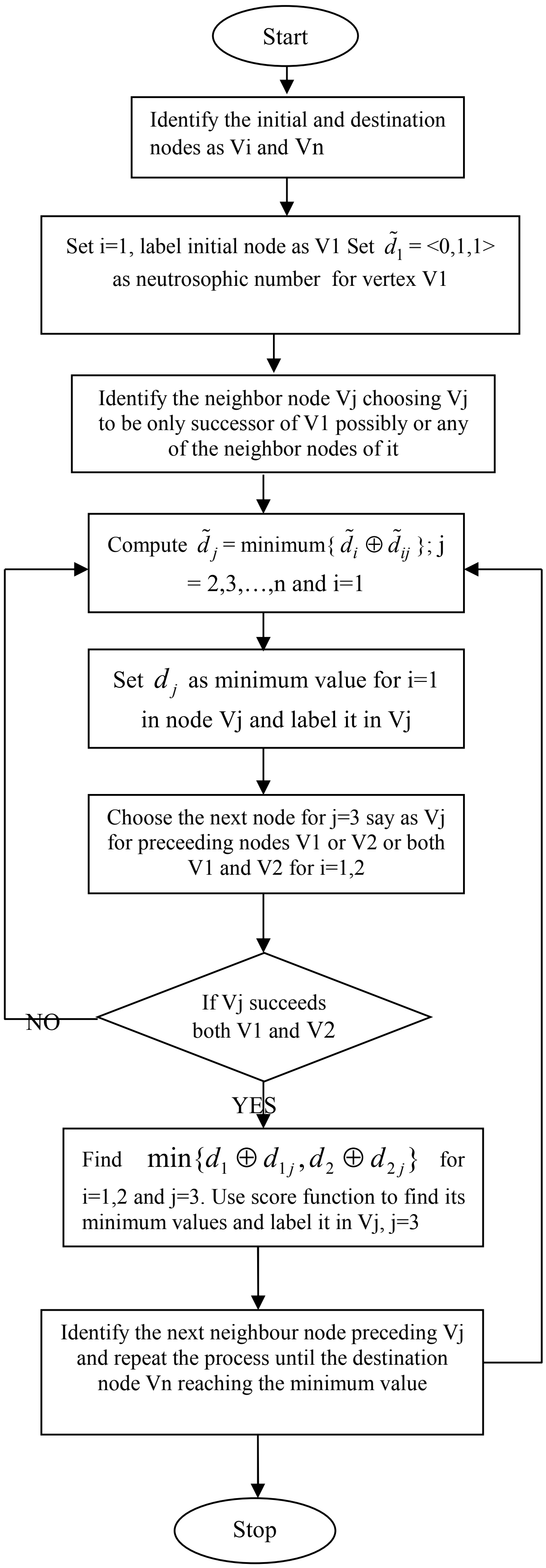

Fig 3. Flow diagram representing the neutrosophic shortest path algorithm.

Using the algorithm described in section 3, the following computational results are obtained
Since node 6 is the destination node, so n= 6.
assume $\tilde{d}_1 = (0,1,1)$ and label the source node (say node 1) as [$(0,1,1)$,-], the value of $\tilde{d}_j$; j= 2, 3, 4, 5 ,6 can be obtained as follows:

Iteration 1 : Since only node 1 is the predecessor node of node 2, so putting i=1 and j= 2 in step2 of the proposed algorithm, the value of $\tilde{d}_2$ is
$\tilde{d}_2$= minimum{ $\tilde{d}_1 \oplus \tilde{d}_{12}$ }=minimum{(0, 1, 1)$\oplus$ (0.4, 0.6, 0.7)= (0.4, 0.6, 0.7)
Since minimum occurs corresponding to i=1, so label node 2 as [$(0.4, 0.6, 0.7)$,1]

Iteration 2 : The predecessor node of node 3 are node 1 and node 2, so putting i= 1, 2 and j= 3 in step 2 of the proposed algorithm, the value of $\tilde{d}_3$ is $\tilde{d}_3$=minimum{ $\tilde{d}_1 \oplus \tilde{d}_{13}, \tilde{d}_2 \oplus \tilde{d}_{23}$ }=minimum{(0, 1, 1)$\oplus$ (0.2, 0.3, 0.4), (0.4, 0.6, 0.7)$\oplus$ (0.1, 0.4, 0.6)}= minimum{(0.2, 0.3, 0.4), (0.46, 0.24, 0.42)}

S (0.2, 0.3, 0.4) = $\dfrac{2+T-I-F}{3} = \dfrac{2+0.2-0.3-0.4}{3}$ =1.5

S (0.46, 0.24, 0.42) = $\dfrac{2+T-I-F}{3} = \dfrac{2+0.46-0.24-0.42}{3}$
=1.8
Since S (0.2, 0.3, 0.4) $<$ S (0.46, 0.24, 0.42)
So minimum{(0.2, 0.3, 0.4)$\oplus$ (0.46, 0.24, 0.42)}= (0.2, 0.3, 0.4)

Since minimum occurs corresponding to i=1, so label node 3 as [$(0.2, 0.3, 0.4)$,1]

Iteration 3. The predecessor node of node 4 is node 3, so putting i= 3 and j= 4 in step 2 of the proposed algorithm, the value of $\tilde{d}_4$ is $\tilde{d}_4$=minimum{ $\tilde{d}_3 \oplus \tilde{d}_{34}$ }=minimum{(0.2, 0.3,0.4)$\oplus$ (0.5, 0.3, 0.1)}={0.6, 0.09, 0.04)}
Since minimum occurs corresponding to i=3, so label node 4 as [$(0.6, 0.09, 0.04)$,3]

Iteration 4 The predecessor node of node 5 are node 2 and node 3, so putting i= 2, 3and j= 5 in step 2 of the proposed algorithm, the value of $\tilde{d}_5$ is $\tilde{d}_5$ =minimum{ $\tilde{d}_2 \oplus \tilde{d}_{25}, \tilde{d}_3 \oplus \tilde{d}_{35}$ }=minimum{(0.4, 0.6, 0.7)$\oplus$ (0.7, 0.6, 0.8), (0.2, 0.3, 0.4)$\oplus$ (0.3, 0.4, 0.7)}= minimum{(0.82, 0.36, 0.56), (0.44, 0.12, 0.28)}

S (0.82, 0.36, 0.56) = $\dfrac{2+T-I-F}{3}$ =1.9

S (0.44, 0.12, 0.28) = $\frac{2+T-I-F}{3}$ =2.04

Since S (0.82, 0.36, 0.56) < S (0.44, 0.12, 0.28)

minimum{(0.82, 0.36, 0.56), (0.44, 0.12, 0.28)}= (0.82, 0.36, 0.5)

$\tilde{d}_5$ =(0.82, 0.36, 0.56)

Since minimum occurs corresponding to i=2, so label node 5 as [(0.82, 0.36, 0.56) ,2]

Iteration 5 The predecessor node of node 6 are node 4 and node 5, so putting i= 4, 5and j= 6 in step 2 of the proposed algorithm, the value of $\tilde{d}_6$ is $\tilde{d}_6$ =minimum{ $\tilde{d}_4 \oplus \tilde{d}_{46}, \tilde{d}_5 \oplus \tilde{d}_{56}$ }=minimum{(0.6, 0.09, 0.04) $\oplus$ (0.7, 0.6, 0.8), (0.82, 0.36, 0.56) $\oplus$ (0.6, 0.5, 0.3)}= minimum{(0.88, 0.054, 0.32), (0.93, 0.18, 0.17)}

Using scoring function we have the values to be 2.5 and 2.58

Since S (0.88, 0.054, 0.32) < S (0.93, 0.18, 0.17)

So minimum{(0.88, 0.054, 0.32), (0.93, 0.18, 0.17)} = (0.88, 0.054, 0.32)

$\tilde{d}_6$ =(0.88, 0.054, 0.32)

Since minimum occurs corresponding to i=4, so label node 6 as [(0.88, 0.054, 0.32) , 4]

Since node 6 is the destination node of the given network, so the neutrosophic shortest distance between node 1 and node 6 is (0.88, 0.054, 0.32) .

Now the neutrosophic shortest path between node 1 and node 6 can be obtained by using the following procedure:

Since node 6 is labeled by [(0.88, 0.054, 0.32) , 4], which represents that we are coming from node 4. Node 4 is labeled by [(0.6, 0.09, 0.04) ,3]which represents that we are coming from node 3. Node 3 is labeled by [(0.2, 0.3, 0.4) ,1], which represents that we are coming from node 1. Now the neutrosophic shortest path between node 1 and node 6 is obtaining by joining all the obtained nodes. Hence the neutrosophic shortest path is $1 \to 3 \to 4 \to 6$ with the neutrosophic value (0.88, 0.054, 0.32) . In figure 4, the dashed lines indicate the shortest path from the source node to the destination node.

The neutrosophic shortest distance and the neutrosophic shortest path of all nodes from node 1 is shown in the table 2 and the labeling of each node is shown in figure 4

Node No.(j)	$\tilde{d}_i$	Neutrosophic shortest path between jth and 1st node
2	(0.4, 0.6, 0.7)	$1 \to 2$
3	(0.2, 0.3, 0.4)	$1 \to 3$
4	(0.6, 0.09, 0.04)	$1 \to 3 \to 4$
5	(0.82, 0.36, 0.56)	$1 \to 2 \to 5$
6	(0.88, 0.054, 0.32)	$1 \to 3 \to 4 \to 6$

Table 2. Tabular representation of different neutrosophic shortest paths

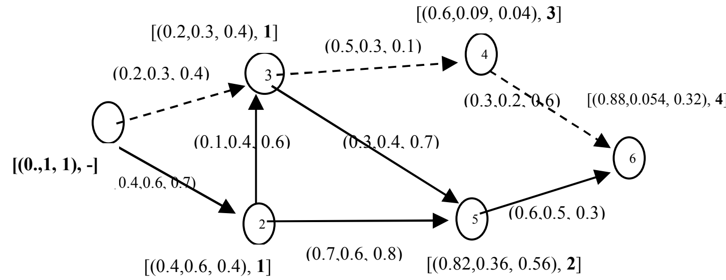

Fig.4. Network with neutrosophic shortest distance of each node from node 1

Since there is no other work on shortest path problem using single valued neutrosophic parameters for the edges (arcs), numerical comparison of this work with others work could not be done.

In this paper we find the shortest path from any source node to destination node using the Neutrosophic shortest path algorithm. The idea of this algorithm is to carry the distance function which works as a tool to identify the successor node from the source at the beginning till it reaches the destination node with a shortest path. Hence our neutrosophic shortest path algorithm is much efficient providing the fuzziness between the intervals classified with true, indeterministic and false membership values. This concept is ultimately differing with intuitionistic membership values as the case of intuitionistic considers only the true and the false membership values. Hence in neutrosophy all the cases of fuzziness is discussed and so the algorithm is effective in finding the shortest path.

V. Conclusion

In this paper we proposed an algorithm for finding shortest path and shortest path length from source node to destination node on a network where the edges weights are assigned by single valued neutrosophic number. The procedure of finding shortest path has been well explained and suitably discussed. Further, the implementation of the proposed algorithm is successfully illustrated with the help of an example. The algorithm is easy to understand and can be used for all types of shortest path problems with arc length as triangular neutrosophic, trapezoidal neutrosophic and interval neutrosophic numbers.

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