A note on some class of prime-generating quadratics

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Abstract

This note lists all the known prime-generating quadratics with at most two-digit positive coefficients that generate at least 20 primes in a row. The Euler polynomial is the best-known member of this class of six.

Prime-generating polynomials have been the subject of study for over two hundreds years now. These are polynomials f(n) with integer coefficients that generate a large number of primes for consecutive values of their argument n. By the convention used in [1], this number is greater than 10.

Among such polynomials, quadratics have been of special interest having been linked to the names of such well-known mathematicians as Euler or Legendre. The most famous of prime-generating quadratics is the Euler polynomial of the form n^2 + n + 41. It generates 40 distinct primes for consecutive values of n from n=0 to 39.

Among the prime-generating quadratics, one can distinguish a class of polynomials with at most two-digit coefficients. The Euler polynomial is the most recognizable member of this class, which also includes a polynomial attributed to Legendre, 2n^2 + 29, that generates 29 primes from n=0 to 28.

Rather surprisingly, it is not known how many unique quadratics belong to this class. We estimate that this number is not greater than 50 for quadratics with all positive coefficients. We are working on establishing it precisely, but this will be reported elsewhere.

However, it seems safe to say that we now know all the members of some subclass of this class of quadratics. Its members distinguish themselves by generating at least 20 primes in a row. Perhaps surprisingly as well, the last two polynomials in this class were discovered by this author only last year suggesting that the study of such polynomials has not been conducted in a systematic manner.

What follows is a complete list of unique quadratics that belong to this special class. By unique, we mean those prime-generating polynomials whose defining sequence of primes starting at n=0 is not contained in such a sequence of another prime-generating polynomial. For instance, n^2 + 7n + 53 generates 37 primes from n=0 to n=36, but all of these primes are also generated by the Euler polynomial, as n^2 + 7n + 53 = (n + 3)^2 + (n + 3) + 41. For this reason, n^2 + 7n + 53 is not considered unique.

The polynomials in this class generate only distinct positive primes - no duplicate or negative terms are allowed. Only polynomials with positive coefficients are considered.

The list below contains the polynomial, the length of the argument range for which it generates primes in a row, the most probable discoverer along with the year the discovery was communicated (in correspondence, print, or online), references to the OEIS, and references to the comments that follow.

1. **n^2 + n + 41**; 40, Euler (1772), http://oeis.org/A005846
2. **2n^2 + 29**; 29, Legendre (1798), http://oeis.org/A007641
3. **6n^2 + 6n + 31**; 29, Spencer (1982), http://oeis.org/A060844 (1)
4. **7n^2 + 49n + 41**; 24, Puszkarz (2018), http://oeis.org/A272077, (2)
5. **3n^2 + 3n + 23**; 22, Spencer (1982), http://oeis.org/A007637, (3)
6. **11n^2 + 55n + 43**; 20, Puszkarz (2017), http://oeis.org/A292578, (4)

Comments:
(1) This quadratic and 3n^ + 3n + 23 are mentioned on p. 119 of Computers in Number Theory by Donald D. Spencer published by Computer Science Press in 1982. It is likely that they had been known before, but this is the earliest source we know of.
(2) Primes generated by this quadratic can be found in sequence A272077 that is a tortuous version of the sequence we discovered in 2017.
(3) This quadratic is also known as 3n^2 - 3n + 23; it generates 22 consecutive primes starting at n=1. The sum of its coefficients is the smallest of all these quadratics.
(4) Let an^2+bn+c be a prime-generating quadratic. For this sequence and the other found by us, b=a+c+1 and b=qa, where q is prime. In general, b=ka for all quadratics above, where k is a non-negative integer, but only for these two k is prime.

The following list contains all the primes generated by the above polynomials within the range specified starting at n=0.

1.
{41,43,47,53,61,71,83,97,113,131,151,173,197,223,251,281,313,347,383,421,461,503,547,593,641,691,743,797,853,911,971,1033,1097,1163,1231,1301,1373,1447,1523,1601}

2.
{29,31,37,47,61,79,101,127,157,191,229,271,317,367,421,479,541,607,677,751,829,911,997,1087,1181,1279,1381,1487,1597}

3.
{31,43,67,103,151,211,283,367,463,571,691,823,967,1123,1291,1471,1663,1867,2083,2311,2551,2803,3067,3343,3631,3931,4243,4567,4903}

4.
{41,97,167,251,349,461,587,727,881,1049,1231,1427,1637,1861,2099,2351,2617,2897,3191,3499,3821,4157,4507,4871}

5.

{23,29,41,59,83,113,149,191,239,293,353,419,491,569,653,743,839,941,1049,1163,1283,1409}

6.
{43,109,197,307,439,593,769,967,1187,1429,1693,1979,2287,2617,2969,3343,3739,4157,4597,5059}

References

[1] http://mathworld.wolfram.com/Prime-GeneratingPolynomial.html