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Multi-criteria Decision-making Approach based on Multi-valued Neutrosophic Geometric Weighted Choquet Integral Heronian Mean Operator

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Abstract. Multi-valued neutrosophic sets (MVNSs) have recently become a subject of great interest for researchers, and have been applied widely to multi-criteria decision-making (MCDM) problems. In this paper, the multi-valued neutrosophic geometric weighted Choquet integral Heronian mean (MVNGWCIHM) operator, which is based on the Heronian mean and Choquet integral, is proposed, and some special cases and the corresponding properties of the operator are discussed. Moreover, based on the proposed operator, an MCDM approach for handling multi-valued neutrosophic information where the weights are unknown is investigated. Furthermore, an illustrative example to demonstrate the applicability of the proposed decision-making approach is provided, together with a sensitivity analysis and comparison analysis, which proves that its results are feasible and credible.

Keywords: Multi-criteria decision-making, Multi-valued neutrosophic sets, Choquet integral, Heronian mean

1. Introduction

Multi-criteria decision-making (MCDM) is a human activity which helps in the decision-making process, mainly in terms of choosing, ranking or sorting the alternatives. In many cases, it is difficult for decision-makers to express precisely a preference regarding relevant alternatives under several criteria, especially when relying on inaccurate, uncertain, or incomplete information. Therefore, fuzzy sets (FSs) and their extensions are used to resolve MCDM or multi-criteria group decision-making (MCGDM) problems [1–7]. However, FSs cannot deal with indeterminate and inconsistent information. Consequently, Smarandache was the first person to introduce neutrosophic sets (NSs) [8–10], which are an extension of the standard interval $[0,1]$ of intuitionistic fuzzy sets (IFSs) [3].

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Due to the ambiguity and complexity of decision-making in the real world, it is difficult for decision-makers to express precisely their preferences by using NSs or any alternatives, such as single-valued neutrosophic sets (SNSs) and interval neutrosophic sets (INSs) [11–14]. It was for this reason that Wang and Li [15] and Ye [16] provided a definition of multi-valued neutrosophic sets (MVNSs) and single-valued neutrosophic hesitant fuzzy sets (SVNHFSs) respectively, which are both extensions of SNSs and the hesitant fuzzy sets (HFSs) introduced by Torra and Narukawa [17] and Torra [18]. Moreover, both MVNSs and SVNHFSs are represented by truth-membership, indeterminacy-membership and falsity-membership functions, which have a set of crisp values between zero and one. In fact there is no difference between MVNSs and SVNHFSs. Based on the definition of MVNSs, Peng et al. [19–21] further defined multi-valued neutrosophic preference, aggregation operators and outranking relations, and applied them to resolve MCDM problems. Ji et al. [22] developed a projection-based an acronym in Portuguese of interactive and multi-criteria decision-making (TODIM) method with multi-valued neutrosophic information. Moreover, based on the definition of MVNSs, Peng et al. [23] defined probability multi-valued neutrosophic numbers (MVNNs), which are also an extension of MVNSs. Wu and Wang [24] investigated some cross-entropy measures of MVNNs and applied them to the problem of selecting a suitable middle-level manager. Finally, Wang and Li [25] developed generalized single-valued neutrosophic hesitant fuzzy prioritized aggregation operators.

The Choquet integral [26] and Heronian mean (HM) [27] are powerful tools for solving MCDM problems with correlated information in the decision-making process. The Choquet integral can focus on changing the weight vector of the aggregation operator, while the HM can capture the interrelationships of individual data. Recently, the two methods have been applied widely to solve various MCDM problems [28–40]. For example, Yager [28] introduced the induced Choquet ordered averaging operator based on the Choquet integral. Additionally, Yu et al. [30] developed hesitant Choquet integral fuzzy operators and applied them to MCDM problems. Wang et al. [35] proposed some Choquet integral operators with interval 2-tuple linguistic information and also applied them to MCDM problems. Liu and Zhang [38] defined the neutrosophic hesitant fuzzy improved generalized weighted Heronian mean (NHFIGWHM) operator and the neutrosophic hesitant fuzzy improved generalized geometric weighted Heronian mean (NHFIGGWHM) operator.

Based on the aforementioned studies, some attempts have been made to define outranking relations, preference, aggregation operators and cross-entropy measures of MVNNs. However, these methods cannot reflect the interrelationships between the weights and individual data simultaneously. Moreover, for some actual decision-making problems, each criterion has a relationship with the other criteria. For example, if we want to select an investment project, we may consider the basic criteria to be risk and profit. As is well known, the higher the risk, the bigger the profit; therefore, the two criteria are correlated with each other. Clearly, the existing methods presented above cannot resolve this type of decision-making problem. In order to address this shortcoming, we extended the HM operator and Choquet integral to handle multi-valued neutrosophic information. Consequently, a new approach is established by combining the advantages of the HM operator and Choquet integral to deal with multi-valued neutrosophic MCDM problems where the weight information is completely unknown. An illustrative example is also provided to demonstrate the applicability of the proposed method.

The rest of paper is organized as follows. In Section 2, some basic concepts of NSs, MVNSs and the operations of MVNNs are briefly reviewed. Then in Section 3, the multi-valued neutrosophic geometric weighted Choquet integral Heronian mean (MVNGWCIHM) operator is proposed and some of the operator's special cases and corresponding properties are discussed. In Section 4, the way in which the extended method can solve MCDM problems using MVNNs is outlined. In Section 5, an illustrative example is presented to verify the proposed approach. Finally, conclusions are drawn in Section 6.

2. Preliminaries

This section introduces the fuzzy measure and Choquet integral, and reviews NSs and SNSs and MVNSs. Additionally, some operations of MVNNs, which will be utilized in the later analysis, are also included.

2.1. The fuzzy measure and Choquet integral

Assume $X = \{x_1, x_2, \ldots, x_n\}$ is the set of the criteria and $P(X)$ is the power set of X.

Definition 1 [41,42]. A fuzzy measure μ on the set X is a set function μ: $P(X) \to [0,1]$ and satisfies the following axioms:

(1) $\mu(\varphi) = 0$, $\mu(X) = 1$;

(2) if $B_1 \subseteq B_2 \subseteq X$, then $\mu(B_1) \leq \mu(B_2)$;

(3) $\mu(B_1 \cup B_2) = \mu(B_1) + \mu(B_2) + \rho\mu(B_1)\mu(B_2)$, for $\forall B_1, B_2 \subseteq X$, $B_1 \cap B_2 = \varphi$, where $\rho \in (-1, +\infty)$.

If $\rho = 0$, then (3) is reduced to the additive measure: $\forall B_1, B_2 \subseteq X$, and $B_1 \cap B_2 = \varphi$, $\mu(B_1 \cup B_2) = \mu(B_1) + \mu(B_2)$.

If the elements of B_i are independent, then

$$\mu(B_i) = \sum_{x_i \in B_i} \mu(x_i) \text{ for all } B_i \subseteq X. \quad (1)$$

If X is a finite set, then the ρ -fuzzy measure is represented as:

$$\mu\left(B_1\right)=\begin{cases}\frac{1}{\rho}\left(\prod\limits_{i\in B_1}\left[1+\rho\mu\left(i\right)\right]-1\right), & \rho\neq 0;\\ \sum\limits_{i\in B_1}\mu\left(i\right), & \rho=0.\end{cases} \quad (2)$$

Here ρ is determined from $\mu\left(X\right)=1$, i.e., $\rho+1=\prod_{i=1}^{n}\left(1+\rho\mu\left(i\right)\right)$.

Definition 2 [26]. Let μ be a fuzzy measure on a finite set $X=\{x_1,x_2,\ldots,x_n\}$, $f:X\to[0,+\infty)$, then the discrete Choquet integral on f with respect to μ can be defined as:

$$\int_X fd\mu=\sum_{i=1}^{n}f\left(x_{\sigma(i)}\right)\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right), \quad (3)$$

where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $0\leq f\left(x_{\sigma(1)}\right)\leq f\left(x_{\sigma(2)}\right)\leq\cdots\leq f\left(x_{\sigma(n)}\right)$, $f\left(x_{\sigma(0)}\right)=0$, $B_{\sigma(i)}=\{x_{\sigma(i)},x_{\sigma(i+1)},\ldots,x_{\sigma(n)}\}$ and $\mu(B_{\sigma(n+1)})=0$.

2.2. MVNS

In this section, the definitions and operations of MVNNs, which will be utilized in the latter analysis, are introduced.

Definition 3 [15, 16]. Let X be a space of points (objects), with a generic element in X denoted by x. An MVNS ψ in X is characterized by

$$\psi=\left\{\left\langle x,\tilde{T}_\psi\left(x\right),\tilde{I}_\psi\left(x\right),\tilde{F}_\psi\left(x\right)\right\rangle|x\in X\right\}, \quad (4)$$

where $\tilde{T}_\psi\left(x\right)$, $\tilde{I}_\psi\left(x\right)$, and $\tilde{F}_\psi\left(x\right)$ are three sets of precise values in $[0,1]$ and are in the form of HFSs, denoting the truth-membership degree, indeterminacy-membership function and falsity-membership degree respectively, and satisfying $0\leq\gamma,\eta,\xi\leq 1, 0\leq\gamma^{+}+\eta^{+}+\xi^{+}\leq 3$, where $\gamma\in\tilde{T}_\psi\left(x\right),\eta\in\tilde{I}_\psi\left(x\right),\xi\in\tilde{F}_\psi\left(x\right)$, $\gamma^{+}=\sup\tilde{T}_\psi\left(x\right)$, $\eta^{+}=\sup\tilde{I}_\psi\left(x\right)$ and $\xi^{+}=\sup\tilde{F}_\psi\left(x\right)$.

If X has only one element, then ψ is called an MVNN, denoted by $\psi=\langle\tilde{T}_\psi\left(x\right),\tilde{I}_\psi\left(x\right),\tilde{F}_\psi\left(x\right)\rangle$. For convenience, an MVNN can be denoted by $\psi=\langle\tilde{T}_\psi,\tilde{I}_\psi,\tilde{F}_\psi\rangle$. The set of MVNNs are MVNNS. Obviously, MVNSs are generally considered as an extension of NSs. If each of $\tilde{T}_\psi\left(x\right),\tilde{I}_\psi\left(x\right)$ and $\tilde{F}_\psi\left(x\right)$ for any x has only one value, i.e., γ,η and ξ, and $0\leq\gamma+\eta+\xi\leq 3$, then MVNSs are reduced to SNSs; if $\tilde{I}_\psi\left(x\right)=\varnothing$ for any x, then MVNSs are reduced to DHFSs; and if $\tilde{I}_\psi\left(x\right)=\tilde{F}_\psi\left(x\right)=\varnothing$ for any x, then MVNSs are reduced to HFSs. Therefore, MVNSs are extensions of SNSs, DHFSs and HFSs.

Definition 4 [16, 19]. Let $\psi,\psi_1,\psi_2\in MVNNs$ and $\lambda>0$, then the following operations can be obtained:

(1) $\lambda\psi=\langle\cup_{\gamma_\psi\in\tilde{T}_\psi}\{1-(1-\gamma_\psi)^{\lambda}\}$,
$\cup_{\eta_\psi\in\tilde{I}_\psi}\{\{\eta_\psi\}^{\lambda}\},\cup_{\xi_\psi\in\tilde{F}_\psi}\{\{\xi_\psi\}^{\lambda}\}\rangle$;

(2) $\psi^{\lambda}=\langle\cup_{\gamma_\psi\in\tilde{T}_\psi}\{(\gamma_\psi)^{\lambda}\},\cup_{\eta_\psi\in\tilde{I}_\psi}\{1-(1-\eta_\psi)^{\lambda}\}$,
$\cup_{\xi_\psi\in\tilde{F}_\psi}\{1-(1-\xi_\psi)^{\lambda}\}\rangle$;

(3) $\psi_1\oplus\psi_2=$
$\langle\cup_{\gamma_{\psi_1}\in\tilde{T}_{\psi_1},\gamma_{\psi_2}\in\tilde{T}_{\psi_2}}\{\gamma_{\psi_1}+\gamma_{\psi_2}-\gamma_{\psi_1}\cdot\gamma_{\psi_2}\}$,
$\cup_{\eta_{\psi_1}\in\tilde{I}_{\psi_1},\eta_{\psi_2}\in\tilde{I}_{\psi_2}}\{\eta_{\psi_1}\cdot\eta_{\psi_2}\}$,
$\cup_{\xi_{\psi_1}\in\tilde{F}_{\psi_1},\xi_{\psi_2}\in\tilde{F}_{\psi_2}}\{\xi_{\psi_1}\cdot\xi_{\psi_2}\}\rangle$;

(4) $\psi_1\otimes\psi_2=$
$\langle\cup_{\gamma_{\psi_1}\in\tilde{T}_{\psi_1},\gamma_{\psi_2}\in\tilde{T}_{\psi_2}}\{\gamma_{\psi_1}\cdot\gamma_{\psi_2}\}$,
$\cup_{\eta_{\psi_1}\in\tilde{I}_{\psi_1},\eta_{\psi_2}\in\tilde{I}_{\psi_2}}\{\eta_{\psi_1}+\eta_{\psi_2}-\eta_{\psi_1}\cdot\eta_{\psi_2}\}$,
$\cup_{\xi_{\psi_1}\in\tilde{F}_{\psi_1},\xi_{\psi_2}\in\tilde{F}_{\psi_2}}\{\xi_{\psi_1}+\xi_{\psi_2}-\xi_{\psi_1}\cdot\xi_{\psi_2}\}\rangle$.

Definition 5 [19]. Let $\psi\in MVNNs$, then the complement of an MVNN can be denoted by ψ^{C}, which can be defined as follows:

$$\psi^{C}=\left\{\tilde{T}_\psi^{C},\tilde{I}_\psi^{C},\tilde{F}_\psi^{C}\right\}. \quad (5)$$

Here $\tilde{T}_\psi^{C}=\cup_{\xi\in\tilde{F}_\psi}\{\xi\}$, $\tilde{I}_\psi^{C}=\cup_{\eta\in\tilde{I}_\psi}\{1-\eta\}$ and $\tilde{F}_\psi^{C}=\cup_{\gamma\in\tilde{T}_\psi}\{\gamma\}$.

Definition 6 [19]. Let ψ_1 and ψ_2 be two MVNNs. The comparision method can be defined as follows:

(1) If $s\left(\psi_1\right)>s\left(\psi_2\right)$ or $s\left(\psi_1\right)=s\left(\psi_2\right)$ and $a\left(\psi_1\right)>a\left(\psi_2\right)$, then ψ_1 is superior to ψ_2, denoted by $\psi_1\succ\psi_2$;

(2) If $s\left(\psi_1\right)=s\left(\psi_2\right)$ and $a\left(\psi_1\right)=a\left(\psi_2\right)$, then ψ_1 is indifferent to ψ_2, denoted by $\psi_1\sim\psi_2$;

(3) If $s\left(\psi_1\right)=s\left(\psi_2\right)$ and $a\left(\psi_1\right)<a\left(\psi_2\right)$ or $s\left(\psi_1\right)<s\left(\psi_2\right)$, then ψ_1 is inferior to ψ_2, denoted by $\psi_1\prec\psi_2$;

$s\left(\psi_i\right)=\frac{1}{l_{\tilde{T}_{\psi_i}}\cdot l_{\tilde{I}_{\psi_i}}\cdot l_{\tilde{F}_{\psi_i}}}\sum_{\gamma_{\psi_i}\in\tilde{T}_{\psi_i},\eta_{\psi_i}\in\tilde{I}_{\psi_i},\xi_{\psi_i}\in\tilde{F}_{\psi_i}}\frac{\left(\gamma_{\psi_i}-\eta_{\psi_i}-\xi_{\psi_i}\right)}{3}$ $(i=1,2)$ and $a\left(\psi_i\right)=\frac{1}{l_{\tilde{T}_{\psi_i}}\cdot l_{\tilde{I}_{\psi_i}}\cdot l_{\tilde{F}_{\psi_i}}}$

$\sum_{\gamma_{\psi_i}\in\tilde{T}_{\psi_i},\eta_{\psi_i}\in\tilde{I}_{\psi_i},\xi_{\psi_i}\in\tilde{F}_{\psi_i}} \frac{(\gamma_{\psi_i}+\eta_{\psi_i}+\xi_{\psi_i})}{3}$ $(i=1,2)$ represent the score function and accuracy function respectively. Here $\gamma_{\psi_i}\in\tilde{T}_{\psi_i}, \eta_{\psi_i}\in\tilde{I}_{\psi_i}$ and $\xi_{\psi_i}\in\tilde{F}_{\psi_i}$; $l_{\tilde{T}_{\psi_i}}, l_{\tilde{I}_{\psi_i}}$ and $l_{\tilde{F}_{\psi_i}}$ denote the number of elements in $\tilde{T}_{\psi_i}, \tilde{I}_{\psi_i}$ and $\tilde{F}_{\psi_i}$, respectively.

3. The MVNGWCIHM operator

Based on the HM and Choquet integral, the MVNG-WCHM operator is now proposed.

Definition 7 [43]. Let $Z_i\,(i=1,2,\ldots,m)$ be a set of nonnegative real numbers, then the HM can be defined as:

$$HM(Z_1,Z_2,\ldots,Z_m)=\frac{2}{m(m+1)}\sum_{i=1}^{m}\sum_{j=i}^{m}\sqrt{Z_iZ_j}. \tag{6}$$

Definition 8 [44,45]. Let $Z_i\,(i=1,2,\ldots,m)$ be a set of nonnegative numbers, and $p,q\geq 0$, p and q not be equal to zero simultaneously, then the geometric Heronian mean (GHM) can be defined as:

$$GHM_{p,q}(Z_1,Z_2,\ldots,Z_m) = \frac{1}{p+q}\left(\prod_{i=1}^{m}\prod_{j=i}^{m}(pZ_i+qZ_j)\right)^{\frac{2}{m(m+1)}}. \tag{7}$$

Definition 9 Let $\psi_i\,(i=1,2,\ldots,m)$ be a group of MVNNs, and μ be a fuzzy measure on X. Based on the fuzzy measure, the MVNGWCIHM can be defined as:

$$MVNGWCIHM(\psi_1,\psi_2,\ldots,\psi_m)= \frac{1}{p+q}\left(\underset{i=1}{\overset{m}{\otimes}}\underset{j=i}{\overset{m}{\otimes}}\left(pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)\psi_{\sigma(i)}\right)\oplus\left(qm\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)\psi_{\sigma(j)}\right)\right)^{\frac{2}{m(m+1)}} \tag{8}$$

Here $p,q\geq 0$, p and q are not equal to zero simultaneously. m is the balance parameter. $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $\psi_{\sigma(1)}\leq\psi_{\sigma(2)}\leq\cdots\leq\psi_{\sigma(n)}$, $B_{\sigma(i)}=(\sigma(i),\ldots,\sigma(n))$, and $B_{\sigma(n+1)}=\varnothing$.

Theorem 1 Let $\psi_i\,(i=1,2,\cdots,m)$ be a group of MVNNs, and μ be a fuzzy measure on X. Then the aggregated value utilizing the MVNGWCIHM operator is also an MVNN, and

$$MVNGWCIHM_{p,q}(\psi_1,\psi_2,\ldots,\psi_m)=\bigcup_{\substack{\gamma_{\psi_{\sigma(i)}}\in\tilde{T}_{\psi_{\sigma(i)}},\gamma_{\psi_{\sigma(j)}}\in\tilde{T}_{\psi_{\sigma(j)}}\\ \eta_{\psi_{\sigma(i)}}\in\tilde{I}_{\psi_{\sigma(i)}},\eta_{\psi_{\sigma(j)}}\in\tilde{I}_{\psi_{\sigma(j)}}\\ \xi_{\psi_{\sigma(i)}}\in\tilde{F}_{\psi_{\sigma(i)}},\xi_{\psi_{\sigma(j)}}\in\tilde{F}_{\psi_{\sigma(j)}}}}$$
$$\left\{\left\{1-\left(1-\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\left(1-\gamma_{\psi_{\sigma(i)}}\right)^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\left(1-\gamma_{\psi_{\sigma(j)}}\right)^{qm\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},\right.$$
$$\left\{\left(1-\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\eta_{\psi_{\sigma(i)}}^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\cdot\eta_{\psi_{\sigma(j)}}^{qm\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},$$
$$\left.\left\{\left(1-\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\xi_{\psi_{\sigma(i)}}^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\cdot\xi_{\psi_{\sigma(j)}}^{qm\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\}\right\}. \tag{9}$$

Here $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $\psi_{\sigma(1)}\leq\psi_{\sigma(2)}\leq\cdots\leq\psi_{\sigma(n)}$, $B_{\sigma(j)}=(\sigma(j),\ldots\sigma(n))$, and $B_{\sigma(n+1)}=\varnothing$.

The process of proof is omitted here.

Some special cases of the MVNGWCIHM operator are now discussed.

Case 1 (1) If Eq. (1) holds, then $\mu(\{6(i)\}) = \mu(6(i))-\mu(6(i+1)), i=1,2,\ldots,m$.

Thus, Thus, Eq. (9) is reduced to a multi-valued neutrosophic weighted geometric Heronian mean (MVN-WGHM) operator:

$$MVNWGHM_{p,q}\left(\psi_1,\psi_2,\ldots,\psi_m\right)$$
$$=\frac{1}{p+q}\left(\left(\underset{i=1}{\overset{m}{\otimes}}\underset{j=i}{\overset{m}{\otimes}}\left(pm\mu\left(\{X_i\}\right)\psi_{\sigma(i)}\oplus qm\mu\left(\{X_j\}\right)\psi_{\sigma(j)}\right)\right)^{\frac{2}{m(m+1)}}\right)$$
$$=\bigcup_{\substack{\gamma_{\psi_{\sigma(i)}}\in\tilde{T}_{\psi_{\sigma(i)}},\gamma_{\psi_{\sigma(j)}}\in\tilde{T}_{\psi_{\sigma(j)}}\\ \eta_{\psi_{\sigma(i)}}\in\tilde{I}_{\psi_{\sigma(i)}},\eta_{\psi_{\sigma(j)}}\in\tilde{I}_{\psi_{\sigma(j)}}\\ \xi_{\psi_{\sigma(i)}}\in\tilde{F}_{\psi_{\sigma(i)}},\xi_{\psi_{\sigma(j)}}\in\tilde{F}_{\psi_{\sigma(j)}}}}\left\{\left\{1-\left(1-\left(\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\left(1-\gamma_{\psi_{\sigma(i)}}\right)^{pm\mu(\{X_i\})}\left(1-\gamma_{\psi_{\sigma(j)}}\right)^{qm\mu(\{X_j\})}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},\right.$$
$$\left\{\left(1-\left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1-\eta_{\psi_{\sigma(i)}}^{pm\mu(\{X_i\})}\eta_{\psi_{\sigma(j)}}^{qm\mu(\{X_j\})}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},$$
$$\left.\left\{\left(1-\left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1-\xi_{\psi_{\sigma(i)}}^{pm\mu(\{X_i\})}\xi_{\psi_{\sigma(j)}}^{qm\mu(\{X_j\})}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\}\right\}. \tag{10}$$

(2) If $\mu(\{X_i\}) = \frac{1}{m}$ for all $i = 1, 2, \ldots, m$, then Eq. (9) is reduced to a multi-valued neutrosophic averaging geometric Heronian mean (MVNAGHM) operator:

$$MVNAGHM_{p,q}\left(\psi_1,\psi_2,\ldots,\psi_m\right)=\frac{1}{p+q}\left(\left(\underset{i=1}{\overset{m}{\otimes}}\underset{j=i}{\overset{m}{\otimes}}\left(\frac{1}{m}pm\psi_{\sigma(i)}\oplus\frac{1}{m}qm\psi_{\sigma(j)}\right)\right)^{\frac{2}{m(m+1)}}\right)$$
$$=\bigcup_{\substack{\gamma_{\psi_{\sigma(i)}}\in\tilde{T}_{\psi_{\sigma(i)}},\gamma_{\psi_{\sigma(j)}}\in\tilde{T}_{\psi_{\sigma(j)}}\\ \eta_{\psi_{\sigma(i)}}\in\tilde{I}_{\psi_{\sigma(i)}},\eta_{\psi_{\sigma(j)}}\in\tilde{I}_{\psi_{\sigma(j)}}\\ \xi_{\psi_{\sigma(i)}}\in\tilde{F}_{\psi_{\sigma(i)}},\xi_{\psi_{\sigma(j)}}\in\tilde{F}_{\psi_{\sigma(j)}}}}\left\{\left\{1-\left(1-\left(\prod_{i=1}^{m}\prod_{j=i}^{m}\left(\left(1-\left(1-\gamma_{\psi_{\sigma(i)}}\right)^{p}\left(1-\gamma_{\psi_{\sigma(j)}}\right)^{q}\right)\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},\right. \tag{11}$$
$$\left.\left\{\left(1-\left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1-\eta_{\psi_{\sigma(i)}}^{p}\eta_{\psi_{\sigma(j)}}^{q}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},\left\{\left(1-\left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1-\xi_{\psi_{\sigma(i)}}^{p}\xi_{\psi_{\sigma(j)}}^{q}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\}\right\}.$$

(3) If $\mu(B) = \sum_{i=1}^{|B|} i$ for all $B \subseteq X$, (here $|B|$ represents the number of the elements in the set B), then $i = \mu(6(i)) - \mu(6(i+1)), i = 1, 2, \ldots, m$. Here $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ and $i \geq 0 (i = 1, 2, \ldots, m)$. In this case, Eq. (9) is reduced to an MVNAGHM operator:

$$MVNAGHM_{p,q}\left(\psi_1,\psi_2,\ldots,\psi_m\right)=\frac{1}{p+q}\left(\left(\underset{i=1}{\overset{m}{\otimes}}\underset{j=i}{\overset{m}{\otimes}}\left(pm\omega_i\psi_{\sigma(i)}\oplus qm\omega_j\psi_{\sigma(j)}\right)\right)^{\frac{2}{m(m+1)}}\right)$$
$$=\bigcup_{\substack{\gamma_{\psi_{\sigma(i)}}\in\tilde{T}_{\psi_{\sigma(i)}},\gamma_{\psi_{\sigma(j)}}\in\tilde{T}_{\psi_{\sigma(j)}}\\ \eta_{\psi_{\sigma(i)}}\in\tilde{I}_{\psi_{\sigma(i)}},\eta_{\psi_{\sigma(j)}}\in\tilde{I}_{\psi_{\sigma(j)}}\\ \xi_{\psi_{\sigma(i)}}\in\tilde{F}_{\psi_{\sigma(i)}},\xi_{\psi_{\sigma(j)}}\in\tilde{F}_{\psi_{\sigma(j)}}}}\left\{\left\{1-\left(1-\left(\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\left(1-\gamma_{\psi_{\sigma(i)}}\right)^{pm\omega_i}\left(1-\gamma_{\psi_{\sigma(j)}}\right)^{qm\omega_j}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},\right. \tag{12}$$

$$\left\{\left(1-\left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1-\eta_{\psi_{\sigma(i)}}^{pm\omega_i}\eta_{\psi_{\sigma(j)}}^{qm\omega_j}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},\left\{\left(1-\left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1-\xi_{\psi_{\sigma(i)}}^{pm\omega_i}\xi_{\psi_{\sigma(j)}}^{qm\omega_j}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\}\Bigg\}.$$

(4) In particular, if $\mu(B)=\frac{|B|}{m}$ for all $B\subseteq X$, then both Eq. (9) and Eq. (11) are reduced to an MVNAGHM operator.

Case 2 (1) If $p=q=\frac{1}{2}$, then the MVNGWCIHM operator, i.e. Eq. (9), is reduced to:

$$MVNGWCIHM_{\frac{1}{2},\frac{1}{2}}(\psi_1,\psi_2,\ldots,\psi_m)=\bigcup_{\substack{\gamma_{\psi_{\sigma(i)}}\in\tilde{T}_{\psi_{\sigma(i)}},\gamma_{\psi_{\sigma(j)}}\in\tilde{T}_{\psi_{\sigma(j)}}\\ \eta_{\psi_{\sigma(i)}}\in\tilde{I}_{\psi_{\sigma(i)}},\eta_{\psi_{\sigma(j)}}\in\tilde{I}_{\psi_{\sigma(j)}}\\ \xi_{\psi_{\sigma(i)}}\in\tilde{F}_{\psi_{\sigma(i)}},\xi_{\psi_{\sigma(j)}}\in\tilde{F}_{\psi_{\sigma(j)}}}}$$

$$\left\{\left\{1-\left(1-\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\sqrt{\left(1-\gamma_{\psi_{\sigma(i)}}\right)^{m\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\left(1-\gamma_{\psi_{\sigma(j)}}\right)^{m\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}}\right)^{\frac{2}{m(m+1)}}\right)\right\},\right.$$

$$\left\{1-\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\sqrt{\eta_{\psi_{\sigma(i)}}^{m\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\eta_{\psi_{\sigma(j)}}^{m\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}}\right)^{\frac{2}{m(m+1)}}\right\},$$

$$\left.\left\{1-\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\sqrt{\xi_{\psi_{\sigma(i)}}^{m\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\xi_{\psi_{\sigma(j)}}^{m\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}}\right)^{\frac{2}{m(m+1)}}\right\}\right\}. \tag{13}$$

(2) If $p=q=1$, then the MVNGWCIHM operator, i.e. Eq. (9), is reduced to:

$$MVNGWCIHM_{1,1}(\psi_1,\psi_2,\ldots,\psi_m)=\bigcup_{\substack{\gamma_{\psi_{\sigma(i)}}\in\tilde{T}_{\psi_{\sigma(i)}},\gamma_{\psi_{\sigma(j)}}\in\tilde{T}_{\psi_{\sigma(j)}}\\ \eta_{\psi_{\sigma(i)}}\in\tilde{I}_{\psi_{\sigma(i)}},\eta_{\psi_{\sigma(j)}}\in\tilde{I}_{\psi_{\sigma(j)}}\\ \xi_{\psi_{\sigma(i)}}\in\tilde{F}_{\psi_{\sigma(i)}},\xi_{\psi_{\sigma(j)}}\in\tilde{F}_{\psi_{\sigma(j)}}}}$$

$$\left\{\left\{1-\sqrt{1-\left(\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1-\left(1-\gamma_{\psi_{\sigma(i)}}\right)^{m\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\left(1-\gamma_{\psi_{\sigma(j)}}\right)^{m\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)\right)^{\frac{2}{m(m+1)}}}\right\},\right.$$

$$\left\{\sqrt{1-\left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1-\eta_{\psi_{\sigma(i)}}^{m\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\eta_{\psi_{\sigma(j)}}^{m\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)\right)^{\frac{2}{m(m+1)}}}\right\},$$

$$\left.\left\{\sqrt{1-\left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1-\xi_{\psi_{\sigma(i)}}^{m\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\xi_{\psi_{\sigma(j)}}^{m\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)\right)^{\frac{2}{m(m+1)}}}\right\}\right\}. \tag{14}$$

(3) If $q\to 0$, then the MVNGWCIHM operator, i.e. Eq. (9), is reduced to:

$$MVNGWCIHM_{p,0}(\psi_1,\psi_2,\ldots,\psi_m)$$

$$=\bigcup_{\substack{\gamma_{\psi_{\sigma(i)}}\in\tilde{T}_{\psi_{\sigma(i)}}\\ \eta_{\psi_{\sigma(i)}}\in\tilde{I}_{\psi_{\sigma(i)}}\\ \xi_{\psi_{\sigma(i)}}\in\tilde{F}_{\psi_{\sigma(i)}}}}\left\{\left\{1-\left(1-\left(\prod_{i=1}^{m}\left(1-\left(1-\gamma_{\psi_{\sigma(i)}}^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\right)\right)^{m+1-i}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p}}\right\},\right.$$

$$\left\{\left(1-\left(\prod_{i=1}^{m}\left(1-\eta_{\psi_{\sigma(i)}}^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\right)^{m+1-i}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p}}\right\},$$

$$\left\{\left(1-\left(\prod_{i=1}^{m}\left(1-\xi_{\psi_{\sigma(i)}}^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\right)^{m+1-i}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p}}\right\}\Bigg\}. \tag{15}$$

(4) If $p \to 0$, then the MVNGWCIHM operator, i.e. Eq. (9), is reduced to:

$$MVNGWCIHM_{0,q}\left(\psi_1,\psi_2,\ldots,\psi_m\right)$$

$$=\bigcup_{\substack{\gamma_{\psi_{\sigma(i)}}\in\tilde{T}_{\psi_{\sigma(i)}}\\ \eta_{\psi_{\sigma(i)}}\in\tilde{I}_{\psi_{\sigma(i)}}\\ \xi_{\psi_{\sigma(i)}}\in\tilde{F}_{\psi_{\sigma(i)}}}}\left\{\left\{1-\left(1-\left(\prod_{i=1}^{m}\left(1-\left(1-\gamma_{\psi_{\sigma(i)}}^{qm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\right)\right)^{i}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{q}}\right\},\right.$$

$$\left\{\left(1-\left(\prod_{i=1}^{m}\left(1-\eta_{\psi_{\sigma(i)}}^{qm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\right)^{i}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{q}}\right\},$$

$$\left\{\left(1-\left(\prod_{i=1}^{m}\left(1-\xi_{\psi_{\sigma(i)}}^{qm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\right)^{i}\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{q}}\right\}\Bigg\}. \tag{16}$$

Some desirable properties of the MVNGWCIHM operator can be obtained.

Property 1 (Idempotency). Let $\psi_i\,(i=1,2,\ldots,m)$ be a group of MVNNs, and $p,q \geq 0$, p and q be not equal to zero simultaneously. If $\psi_i = \psi = \{\langle\gamma,\eta,\xi\rangle\}\,(i=1,2,\ldots,m)$, then $MVNGWCIHM_{p,q}(\psi_1,\psi_2,\ldots,\psi_m) = \psi$ can be obtained.

Property 2 (IdemMonotonicity). Let $\psi_i(i = 1,2,\ldots,m)$ and $\pi_i(i=1,2,\ldots,m)$ be two groups of MVNNS, $p,q \geq 0$, p and q be not equal to zero simultaneously. If $\forall\,\gamma_{\psi_{\sigma(i)}} \in \tilde{T}_{\psi_{\sigma(i)}}, \eta_{\psi_{\sigma(i)}} \in \tilde{I}_{\psi_{\sigma(i)}}, \xi_{\psi_{\sigma(i)}} \in \tilde{F}_{\psi_{\sigma(i)}}$ and $\gamma_{\pi_{\sigma(i)}} \in \tilde{T}_{\pi_{\sigma(i)}}$, $\eta_{\pi_{\sigma(i)}} \in \tilde{I}_{\pi_{\sigma(i)}}$, $\xi_{\pi_{\sigma(i)}} \in \tilde{F}_{\pi_{\sigma(i)}}$ satisfy $\gamma_{\psi_{\sigma(i)}} \leq \gamma_{\pi_{\sigma(i)}}, \eta_{\psi_{\sigma(i)}} \geq \eta_{\pi_{\sigma(i)}}$ and $\xi_{\psi_{\sigma(i)}} \geq \xi_{\pi_{\sigma(i)}}$ for all $\psi_i\,(i=1,2,\ldots,m)$ and $\pi_i\,(i=1,2,\ldots,m)$, then $MVNGWCIHM_{p,q}(\psi_1,\psi_2,\ldots,\psi_m) \leq MVNGWCIHM_{p,q}(\pi_1,\pi_2,\ldots,\pi_m)$ can be obtained.

Property 3 (Boundedness). Let $\psi_i(i=1,2,\ldots,m)$ be a group of MVNNS and $p,q \geq 0$, p and q be not equal to zero simultaneously. If $\psi^- = \{\gamma^-,\eta^+,\xi^+\}$ and $\psi^+ = \{\gamma^+,\eta^-,\xi^-\}$, here $\gamma^- = \min_i \min_{\gamma_{\psi_i}\in T''}$, $\eta^- = \min_i \min_{\eta_{\psi_i}\in I''}$ and $\xi^- = \min_i \min_{\xi_{\psi_i}\in F''}$, $\gamma^+ = \max_i \max_{\gamma_{\psi_i}\in T''}$, $\eta^+ = \max_i \max_{\eta_{\psi_i}\in I''}$ and $\xi^+ = \max_i \max_{\xi_{\psi_i}\in F''}$, then $\psi^- \leq MVNGWCIHM_{p,q}\left(\psi_1,\psi_2,\ldots,\psi_m\right) \leq \psi^+$ can be obtained.

Property 4 (Permutation). Let $\psi_i(i=1,2,\ldots,m)$ be a group of MVNNs, and $p,q \geq 0$, p and q be not equal to zero simultaneously. If $\bar{\psi}_i(i=1,2,\ldots,m)$ be any permutation of $\psi_i(i=1,2,\ldots,m)$, then $MVNGWCIHM_{p,q}(\psi_1,\psi_2,\ldots,\psi_m) = MVNGWCIHM_{p,q}(\bar{\psi}_1,\bar{\psi}_2,\ldots,\bar{\psi}_m)$ can be obtained.

4. The MCDM approach based on a MVNGWCIHM operator with MVNNs

In this section, an approach is proposed to resolve the MCDM problems where the data are expressed by MVNNs.

Assume there are n alternatives denoted by $A = \{\alpha_1,\alpha_2,\ldots,\alpha_n\}$ and m criteria denoted by $C = \{c_1,c_2,\ldots,c_m\}$, and $w = (w_1,w_2,\ldots,w_m)^T$ is the weight vector of criterion $c_j(j=1,2,\ldots,m)$, where $w_j \geq 0(j=1,2,\ldots,m)$, and $\sum_{j=1}^{m} w_j = 1$. Let $R = (\alpha_{ij})_{n\times m}$ be the multi-valued neutrosophic decision matrix, and $\alpha_{ij} = \langle\tilde{T}_{\alpha_{ij}},\tilde{I}_{\alpha_{ij}},\tilde{F}_{\alpha_{ij}}\rangle$ be the evaluation value of α_i for criterion c_j being in the form of MVNNs. Where $\tilde{T}_{(\cdot)}$ indicates the truth-membership function, $\tilde{I}_{(\cdot)}$ indicates the indeterminacy-membership function and $\tilde{F}_{(\cdot)}$ indicates the falsity-membership function.

Each criterion can be divided into two types, including maximizing which means the larger the better, and cost-type which means the smaller the better. For the benefit-type criteria, nothing is done; whereas for the minimizing criteria, the criterion values can be transformed into maximizing criteria as follows:

$$\beta_{ij} = \begin{cases} \alpha_{ij}, & \text{for maximizing criteria } c_j \\ (\alpha_{ij})^c, & \text{for minimizing criteria } c_j \end{cases}, \quad (i = 1, 2, \cdots, n; j = 1, 2, \cdots, m). \tag{17}$$

Here $(\alpha_{ij})^c$ is the complement of α_{ij} as defined in Definition 5.

In the following steps, a procedure to rank and select the most desirable alternative(s) is provided.

Step 1. Transform the decision matrix.

According to Eq. (17), the MVNN decision matrix $R = (\alpha_{ij})_{n\times m}$ can be transformed into a normalized MVNN decision matrix $\tilde{R} = (\beta_{ij})_{n\times m}$.

For the minimizing criteria, the normalization formula is

$$\beta_{ij} = (\alpha_{ij})^c = \left\langle \cup_{\xi\in\tilde{F}_{\alpha_{ij}}} \{\xi\}, \cup_{\eta\in\tilde{I}_{\alpha_{ij}}}\{1-\eta\}, \cup_{\gamma\in\tilde{T}_{\alpha_{ij}}} \{\gamma\} \right\rangle; \tag{18}$$

for the maximizing criteria, the normalization formula is

$$\beta_{ij} = \alpha_{ij} = \left\langle \cup_{\gamma\in\tilde{T}_{\alpha_{ij}}} \{\gamma\}, \cup_{\eta\in\tilde{I}_{\alpha_{ij}}} \{\eta\}, \cup_{\xi\in\tilde{F}_{\alpha_{ij}}} \{\xi\} \right\rangle. \tag{19}$$

Step 2. Confirm the fuzzy measures on criterion set C.

Based on the fuzzy measures and criteria set C, the weight of the criterion can be obtained as follows:

$$\tilde{w}_{\sigma(j)} = \mu\left(B_{\sigma(j)}\right) - \mu\left(B_{\sigma(j+1)}\right), j = 1, 2, \ldots, m.$$

Here, $(\sigma(1), \sigma(2), \ldots, \sigma(m))$ is a permutation of $(1, 2, \ldots, m)$. Then the corresponding weight of criteria can be obtained.

Step 3. Aggregate all the performance values of each alternative.

Based on Step 2, we can aggregate all the performance values β_{ij} of each alternative and obtain the overall values β_i corresponding to the alternative $\alpha_i (i = 1, 2, \ldots, m)$ by using the MVNGWCIHM operator as follows:

$$\begin{aligned} MVNWGCHM_{p,q}\left(\beta_{i1}, \beta_{i2}, \ldots, \beta_{im}\right) = & \bigcup_{\substack{\gamma_{\sigma(ii)}\in\tilde{\mathrm{T}}_{\sigma(ii)}, \gamma_{\sigma(ij)}\in\tilde{\mathrm{T}}_{\sigma(ij)} \\ \eta_{\sigma(ii)}\in\tilde{\mathrm{I}}_{\sigma(ii)}, \eta_{\sigma(ij)}\in\tilde{\mathrm{I}}_{\sigma(ij)} \\ \xi_{\sigma(ii)}\in\tilde{F}_{\sigma(ii)}, \xi_{\sigma(ij)}\in\tilde{F}_{\sigma(ij)}}} \\ & \left\{\left\{1 - \left(1 - \left(\prod_{i=1}^{m}\prod_{j=i}^{m}\left(1 - \left(1-\gamma_{\beta_{\sigma(ii)}}\right)^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\left(1-\gamma_{\beta_{\sigma(ij)}}\right)^{qm\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\},\right. \\ & \left\{\left(1 - \left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1 - \eta_{\beta_{\sigma(ii)}}^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\eta_{\beta_{\sigma(ij)}}^{qm\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\}, \\ & \left.\left\{\left(1 - \left(\prod_{i=1}^{m}\prod_{j=1}^{m}\left(1 - \xi_{\beta_{\sigma(ii)}}^{pm\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\xi_{\beta_{\sigma(ij)}}^{qm\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)\right)^{\frac{2}{m(m+1)}}\right)^{\frac{1}{p+q}}\right\}\right\}. \end{aligned} \tag{20}$$

Step 4. Rank all the alternative(s).

According to the score function and accuracy function in Def. (3), we can obtain the final ranking and select the best one(s).

5. An illustrative example

In this section, an example is adapted from Wang et al. [46] for further illustration. Hunan Nonferrous Metals Holding Group Co. Ltd. is a large state-owned company whose main business is producing and selling nonferrous metals. It is also the largest manufacturer of multi-species nonferrous metals in China, with the exception of aluminum. In order to expand its main business, the company is always engaged in overseas investment, and a department which consists of executive managers and several experts in the field has been established specifically to make decisions on global mineral investment. Recently, the overseas investment department has decided to select a pool of alternatives from several foreign countries based on preliminary surveys. Subsequently, the projects in those countries are to be investigated in detail. In this survey, the focus is on the first step of finding suitable candidate countries. Three countries (alternatives) are taken into consideration, which are denoted by $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$. During the assessment, four factors, namely: c_1: resources (such as the suitability of the minerals and their exploration potential); c_2: politics and policy (such as corruption and political risks); c_3: economy (such as development vitality and stability); and c_4: infrastructure (such as railway and highway facilities) are considered according to the department's previous investment projects. The evaluation of five candidates $\alpha_i\,(i = 1, 2, 3, 4, 5)$ is performed using MVNNs by the three decision-makers under the criterion $c_k\,(k = 1, 2, 3, 4)$. One decision-maker could give several evaluation values for three membership degrees. In particular, in the case where two decision-makers set the same value, it is counted only once. Then the multi-valued neutrosophic decision matrix $R = (\alpha_{ij})_{5\times 4}$ is constructed and shown as follows:

$$R = \begin{pmatrix} \langle\{0.5,0.7\},\{0.3\},\{0.3\}\rangle & \langle\{0.4\},\{0.2,0.3\},\{0.1\}\rangle & \langle\{0.5\},\{0.3,0.4\},\{0.4\}\rangle & \langle\{0.5,0.6\},\{0.2\},\{0.3\}\rangle \\ \langle\{0.3,0.4\},\{0.2\},\{0.2\}\rangle & \langle\{0.6\},\{0.2\},\{0.3\}\rangle & \langle\{0.7\},\{0.2\},\{0.3\}\rangle & \langle\{0.4\},\{0.2\},\{0.1\}\rangle \\ \langle\{0.3\},\{0.2,0.3,0.4\},\{0.4\}\rangle & \langle\{0.4,0.5\},\{0.2.0.3\},\{0.2\}\rangle & \langle\{0.6\},\{0.1,0.2\},\{0.3\}\rangle & \langle\{0.5\},\{0.1\},\{0.2\}\rangle \\ \langle\{0.5\},\{0.3\},\{0.3\}\rangle & \langle\{0.6\},\{0.1,0.2\},\{0.2\}\rangle & \langle\{0.7\},\{0.2\},\{0.2,0.3\}\rangle & \langle\{0.6\},\{0.2,0.3,0.4\},\{0.2\}\rangle \\ \langle\{0.6\},\{0.2\},\{0.4\}\rangle & \langle\{0.7\},\{0.2,0.3\},\{0.4\}\rangle & \langle\{0.7\},\{0.3\},\{0.2\}\rangle & \langle\{0.5\},\{0.1\},\{0.3\}\rangle \end{pmatrix}.$$

5.1. An illustration of the proposed approach

The procedures for obtaining the optimal alternative using the proposed method are shown in the following steps.

Step 1. Normalize the decision matrix.

Because all the criteria are of a maximizing type and have the same measurement unit, there is no need for normalization; thus, $\tilde{R} = (\beta_{ij})_{4\times 4} = (\alpha_{ij})_{4\times 4}$.

Step 2. Confirm the fuzzy measures on criteria set C.

Assume that $\mu(c_1) = 0.40$, $\mu(c_2) = 0.27$, $\mu(c_3) = 0.35$ and $\mu(c_4) = 0.30$, then $\rho = -0.58$ can be obtained. According to Eq. (1), $\mu(c_1, c_2) = 0.55$, $\mu(c_1, c_3) = 0.67$, $\mu(c_1, c_4) = 0.63$, $\mu(c_2, c_3) = 0.57$, $\mu(c_2, c_4) = 0.52$, $\mu(c_3, c_4) = 0.59$, $\mu(c_1, c_2, c_3) = 0.83$, $\mu(c_1, c_2, c_4) = 0.80$, $\mu(c_2, c_3, c_4) = 0.78$, $\mu(c_1, c_3, c_4) = 0.85$ and $\mu(c_1, c_2, c_3, c_4) = 1$ can be determined.

Take $\beta_{5j} (j = 1, 2, 3, 4)$ for example,

$$s(\beta_{51}) = 0, s(\beta_{52}) = 0.0171,$$
$$s(\beta_{53}) = 0.0672, s(\beta_{54}) = 0.0331.$$

Obviously,

$$s(\beta_{51}) < s(\beta_{52}) < s(\beta_{54}) < s(\beta_{53}).$$

Which implies $\beta_{51} < \beta_{52} < \beta_{54} < \beta_{53}$ such that

$$\beta_{5\sigma(1)} = \beta_{51}, \beta_{5\sigma(2)} = \beta_{52},$$
$$\beta_{5\sigma(3)} = \beta_{54}, \beta_{5\sigma(4)} = \beta_{53}.$$

Then

$$\begin{aligned} \omega_{\sigma(1)} &= \mu\left(B_{\sigma(1)}\right) - \mu\left(B_{\sigma(2)}\right) \\ &= \mu(c_1, c_2, c_3, c_4) - \mu(c_2, c_3, c_4) \\ &= 1 - 0.78 = 0.22; \\ \omega_{\sigma(2)} &= \mu(c_2, c_4, c_3) - \mu(c_4, c_3) \\ &= 0.78 - 0.59 = 0.19; \\ \omega_{\sigma(3)} &= \mu(c_4, c_3) - \mu(c_3) = 0.59 - 0.35 = 0.24; \\ \omega_{\sigma(4)} &= \mu(c_3) = 0.35. \end{aligned}$$

So $\omega_5 = (0.22, 0.19, 0.35, 0.24)$.

Thus, the corresponding weight matrix can be obtained as:

$$\omega = \begin{pmatrix} 0.28 & 0.22 & 0.2 & 0.3 \\ 0.22 & 0.22 & 0.35 & 0.21 \\ 0.22 & 0.19 & 0.29 & 0.3 \\ 0.22 & 0.22 & 0.35 & 0.21 \\ 0.22 & 0.19 & 0.35 & 0.24 \end{pmatrix}.$$

Step 3. Aggregate all the performance values of each alternative.

Utilizing the operator i.e., Eq. (20), to aggregate the criterion values for each alternative and $p = q = 2$, then the comprehensive values can be obtained as follows:

$$
\begin{aligned}
&MVNGWCIHM_{2,2}\left(\beta_{11},\beta_{12},\beta_{13},\beta_{14}\right) = \bigcup_{\substack{\gamma_{1\sigma(i)}\in\tilde{T}_{1\sigma(i)},\gamma_{1\sigma(j)}\in\tilde{T}_{1\sigma(j)}\\ \eta_{1\sigma(i)}\in\tilde{I}_{1\sigma(i)},\eta_{1\sigma(j)}\in\tilde{I}_{1\sigma(j)}\\ \xi_{1\sigma(i)}\in\tilde{F}_{1\sigma(i)},\xi_{1\sigma(j)}\in\tilde{F}_{1\sigma(j)}}}\\
&\left\{\left\{1-\left(1-\prod_{i=1}^{4}\prod_{j=i}^{4}\left(1-\left(1-\gamma_{\beta_{1\sigma(i)}}\right)^{2\times4\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\left(1-\gamma_{\beta_{1\sigma(j)}}\right)^{2\times4\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)^{\frac{2}{4\times5}}\right)^{\frac{1}{2+2}}\right\},\right.\\
&\left\{\left(1-\prod_{i=1}^{4}\prod_{j=i}^{4}\left(1-\eta_{\beta_{1\sigma(i)}}^{2\times4\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\eta_{\beta_{1\sigma(j)}}^{2\times4\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)^{\frac{2}{4\times5}}\right)^{\frac{1}{2+2}}\right\},\\
&\left.\left\{\left(1-\prod_{i=1}^{4}\prod_{j=i}^{4}\left(1-\xi_{\beta_{1\sigma(i)}}^{2\times4\left(\mu\left(B_{\sigma(i)}\right)-\mu\left(B_{\sigma(i+1)}\right)\right)}\xi_{\beta_{1\sigma(j)}}^{2\times4\left(\mu\left(B_{\sigma(j)}\right)-\mu\left(B_{\sigma(j+1)}\right)\right)}\right)^{\frac{2}{4\times5}}\right)^{\frac{1}{2+2}}\right\}\right\}\\
&=\left\{\left\{0.8288,0.8447,0.8402,0.8578\right\},\left\{0.3038,0.29,0.2975,0.2833\right\},\left\{0.304\right\}\right\}.
\end{aligned}
$$

$$
\begin{aligned}
&MVNGWCIHM_{2,2}\left(\beta_{21},\beta_{22},\beta_{23},\beta_{24}\right)\\
&=\left\{\left\{0.8311,0.8494\right\},\left\{0.3364\right\},\left\{0.3214\right\}\right\}.\\
&MVNGWCIHM_{2,2}\left(\beta_{31},\beta_{32},\beta_{33},\beta_{34}\right)\\
&=\{\{0.8156,0.8221\},\{0.4074,0.3983,0.3929,\\
&0.4004,0.391,0.3857,0.3749,0.3647,0.3587,\\
&0.3671,0.3566,0.3506\},\{\{0.2935\}\}.\\
&MVNGWCIHM_{2,2}\left(\beta_{41},\beta_{42},\beta_{43},\beta_{44}\right)\\
&=\{\{0.8819\},\{0.3501,0.3244,0.3397,0.3129,\\
&0.3337,0.3063\},\{0.3244,0.2941\}\}.\\
&MVNGWCIHM_{2,2}\left(\beta_{51},\beta_{52},\beta_{53},\beta_{54}\right)\\
&=\left\{\left\{0.8835\right\},\left\{0.3404,0.3314\right\},\left\{0.2869\right\}\right\}.
\end{aligned}
$$

Step 4. Rank all the alternatives.

The score values of each alternative(s) can be obtained as:

$$
\begin{aligned}
&s\left(\alpha_1\right)=0.0817; s\left(\alpha_2\right)=0.0608;\\
&s\left(\alpha_3\right)=0.0488; s\left(\alpha_4\right)=0.0816;\\
&s\left(\alpha_5\right)=0.0869.
\end{aligned}
$$

Then $s(\alpha_5) > s(\alpha_1) > s(\alpha_4) > s(\alpha_2) > s(\alpha_3)$ can be obtained. Therefore, the final ranking is $\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$. The best alternative is α_5 while the worst is α_3.

5.2. *A sensitivity analysis*

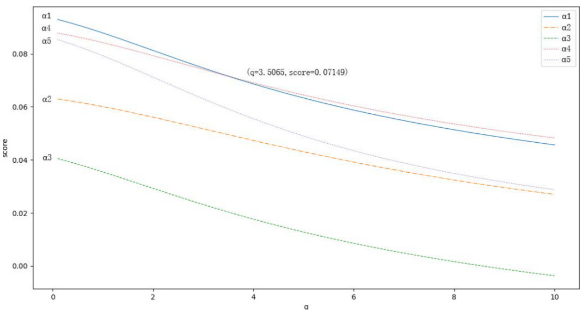

Fig. 1. The score values of alternatives and $p = 0$ and $q \in (0,\ 10]$.

In this subsection, the influence of different parameters on the ranking of alternatives is investigated by using the MVNGWCIHM operator. Since the evaluation values for three memberships in MVNNs are sets of precise values in [0, 1], the different values of $p, q \in (0, 10]$ are considered to conduct a sensitivity analysis. The results are represented in Table 1 and Fig.

Table 1

The results by using the different parameters

Parameter (p, q)	The final rankings	The best alternative(s)	The worst alternative(s)
$p = q = 0.5$	$\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_5	α_3
$p = 1, q = 0$	$\alpha_5 \succ \alpha_4 \succ \alpha_1 \succ \alpha_3 \succ \alpha_2$	α_5	α_2
$p = 0, q = 1$	$\alpha_1 \succ \alpha_4 \succ \alpha_5 \succ \alpha_2 \succ \alpha_3$	α_1	α_3
$p = q = 1$	$\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_5	α_3
$p = q = 2$	$\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_5	α_3
$p = q = 5$	$\alpha_4 \succ \alpha_1 \succ \alpha_5 \succ \alpha_2 \succ \alpha_3$	α_4	α_3
$p = q = 10$	$\alpha_4 \succ \alpha_1 \succ \alpha_5 \succ \alpha_2 \succ \alpha_3$	α_4	α_3

Table 2

The results obtained by utilizing the different methods

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
Ye [16]	$\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_5	α_3
Peng et al. [19]	$\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_5	α_3
Peng et al. [20]	$\alpha_1, \alpha_5 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_1, α_5	α_3
Peng et al. [21]	$\alpha_1 \succ \alpha_5 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_1	α_3
Wang and Li [25]	$\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_5	α_3
Liu and Zhang [38]	$\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_5	α_3
The proposed method	$\alpha_5 \succ \alpha_1 \succ \alpha_4 \succ \alpha_2 \succ \alpha_3$	α_5	α_3

1-Fig. 3. From the results, we can see that the final rankings may be different for the different parameters $p, q \in (0, 10]$, which can be considered as a reflection of the decision makers' preferences. The best alternative is always α_5, α_4 or α_1; while the worst alternative is always α_3 or α_2. Generally speaking, the greater the values of p and q, the more emphasized the interactions of criterion values. However, if the values of parameters are too big, then the difference between the scores of alternatives will not be so distinct. In other words, this will influence the accuracy of the final results. Therefore, we can determine the simple values for the ease of computation. Moreover, the MVNGW-CIHM operator can provide the decision-makers with more choices regarding the different values of the parameter, which are provided according to the decision-makers' preferences.

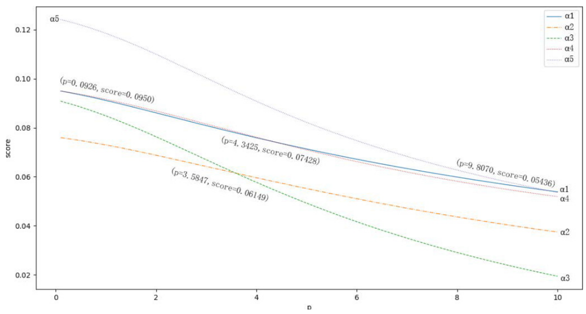

Fig. 2. The score values of alternatives and $p \in (0,\ 10]$ and $q = 0$.

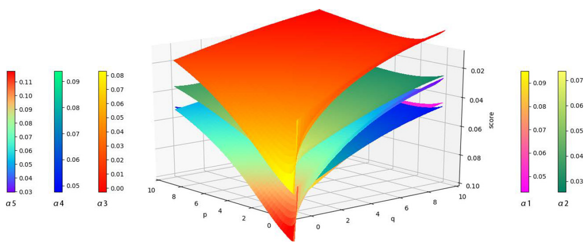

Fig. 3. The score values of alternatives and $p \in (0,\ 10]$ and $q \in (0\ ,\ 10]$.

5.3. A comparison analysis

In order to validate the feasibility of the proposed decision-making method, a comparative study was conducted with other methods; specifically those in Ye [16], Peng et al. [19–21], Wang and Li [25] and Liu and Zhang [38].

To facilitate a comparison analysis, the same example that was used in Section 5 is used here as well. Since the compared methods presented above cannot handle multi-valued neutrosophic information where the weight is completely unknown, we now use the same example but with the weights of criteria determined as $w = (0.25, 0.21, 0.35, 0.19)$. Subsequently, the proposed method is reduced to the methods used in Liu and Zhang [38]. For the proposed method, we can use the determined weight directly in Step 3 in Subsection 5.1 and $p = q = 2$. Then for the method in Ye [16] and Peng et al. [19], the weighted averaging operator and the weighted arithmetic power averaging operator are utilized respectively. For the method in Wang and Li [25], we assume that the criterion satisfy $C_1 \succ C_2 \succ C_3 \succ C_4$ and $\lambda = 1$. Moreover, the generalized single-valued neutrosophic hesitant fuzzy prioritized weighted geometric operator is used to deal with the same example. Consequently, the final results can be obtained as shown in Table 2.

Based on the results presented in Table 4, we can see that the results from the proposed approach are consistent with those that use the methods in Ye [16], Peng et al. [19], Wang and Li [25] and Liu and Zhang [38]; the best alternative is α_5 while the worst is α_3. For the other compared methods presented in Refs. [20, 21], although there is a slight difference in the final rankings of these methods, the alternative α_1 is always the best one(s).

Based on the results presented above, some conclusions can be drawn and these are now discussed. Firstly, although the result of the proposed approach is consistent with that using the method of Ye [16], Peng et al. [19], Wang and Li [25] and Liu and Zhang [38], these four methods are unable to consider the data interrelationships of the criterion. Secondly, the compared methods mentioned above cannot resolve multi-valued neutrosophic problems where the weight information is completely unknown. Thirdly, the methods developed by Peng et al. [20] can only resolve MCDM problems in which the number of criteria clearly exceeds the number of alternatives; while the method in Peng et al. [21] is better used in resolving problems with a large number of alternatives and few criteria. Otherwise, the final results cannot be obtained directly. However, the approach proposed in this paper is the optimal method for MCDM problems where the weight of criteria is completely unknown, and the relationships between the criteria and data should be considered. Therefore, the main advantages of the proposed approach are not only its ability to deal effectively with the preference information expressed by MVNNs, but also its consideration that the weights of the criteria and individual data are interrelated, which makes the final results correspond better with actual decision-making problems.

6. Conclusions

For real decision-making problems, the criteria and an individual's evaluation are always interrelated. MVNNs can be used widely to deal effectively with uncertain, imprecise and inconsistent information. Based on the HM and Choquet integral, the MVNGWCIHM operator has been proposed in this paper, and some special cases and the corresponding properties of the operator were discussed. Moreover, based on the MVNGWCIHM operator, a multi-valued neutrosophic approach was investigated to resolve MCDM problems where the data is in the form of MVNNs and the weights of criteria are completely unknown. Additionally, an illustrative example demonstrated the application of the proposed decision-making approach, and proved that its results are feasible and credible. The main advantages of the proposed approach are its ability to consider effectively the interrelationships of the criteria and data and that the weights of the criteria are completely unknown, which makes the final results better correspond with actual decision-making problems. In future research, the measures of MVNNs will be further investigated.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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