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Multiple Attribute Decision-Making Method Using Correlation Coefficients of Normal Neutrosophic Sets

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Abstract: The normal distribution is a usual one of various distributions in the real world. A normal neutrosophic set (NNS) is composed of both a normal fuzzy number and a neutrosophic number, which a significant tool for describing the incompleteness, indeterminacy, and inconsistency of the decision-making information. In this paper, we propose two correlation coefficients between NNSs based on the score functions of normal neutrosophic numbers (NNNs) (basic elements in NNSs) and investigate their properties. Then, we develop a multiple attribute decision-making (MADM) method with NNSs under normal neutrosophic environments, where, by correlation coefficient values between each alternative (each evaluated NNS) and the ideal alternative (the ideal NNS), the ranking order of alternatives and the best one are given in the normal neutrosophic decision-making process. Finally, an illustrative example about the selection problem of investment alternatives is provided to demonstrate the application and feasibility of the developed decision-making method. Compared to the existing MADM approaches based on aggregation operators of NNNs, the proposed MADM method based on the correlation coefficients of NNSs shows the advantage of its simple decision-making process.

Keywords: multiple attribute decision-making; normal neutrosophic set; normal neutrosophic number; correlation coefficient

1. Introduction

In probability theory [1], the normal (or Gaussian) distribution is a very common continuous probability distribution. Normal distribution is an important distribution form in statistics and is very useful in the natural and social sciences to express real-valued random variables whose distributions are not known. Hence, it has been widely applied to various fields. Then, the fuzziness and uncertainty of the real decision-making information are a common phenomenon because some numerical values may be inadequate or insufficient to complex decision-making problems. In some occasions, it can be more reasonable to describe the attribute values by the fuzzy numbers in a fuzzy environment. Thus, Zadeh [2] firstly introduced the fuzzy set, which is described by the membership function. After that, Yang and Ko [3] defined a normal fuzzy number (NFN) to express the normal fuzzy information in random fuzzy situations. It is obvious that its main advantage is reasonable and realistic to normal distribution environments. As an extension of the fuzzy set, Atanassov [4] proposed the intuitionistic fuzzy set (IFS) by adding the non-membership function to the fuzzy set. However, because NFN only contains its normal fuzzy membership degree, Wang et al. [5] presented an intuitionistic normal fuzzy number (INFN) based on the combination of both an NFN and an intuitionistic fuzzy number (IFN) (a basic element in IFS), defined the score function and operational laws of INFNs, and presented some aggregation operators of INFNs, including an ordered intuitionistic normal ordered fuzzy weighted averaging operator, an INFN ordered weighted geometric averaging operator, two INFN-related ordered weighted arithmetic and geometric averaging operators, two induced INFN-related ordered weighted arithmetic and geometric averaging operators, and they then applied them to multiple criteria decision-making (MCDM) problems, where the criteria are interactive and the criteria values are the INFNs. Then, Wang and Li [6] proposed a score function of INFN based on relative entropy and an INFN weighted arithmetic averaging operator, and then applied them to normal intuitionistic fuzzy MCAD problems. Wang and Li [7] also introduced Euclidean distance between INFNs and an INFN weighted arithmetic averaging operator and an INFN weighted geometric averaging operator for MCDM problems with INFNs. Wang et al. [8] further introduced a normal intuitionistic fuzzy number (NIFN) weighted arithmetic averaging operator, an NIFN weighted geometric averaging operator, an NIFN-induced ordered weighted averaging operator, an NIFN-induced ordered weighted geometric averaging operator, and an NIFN-induced generalized ordered weighted averaging (NIFN-IGOWA) operator, and then applied the NIFN-IGOWA operator to MCDM problems with NIFN information. To express the truth, indeterminacy, and falsity information in real world, Smarandache [9] proposed a concept of a neutrosophic set from a philosophical point of view. As a subclass of the neutrosophic set, Smarandache [9] and Wang et al. [10] introduced the concept of a single-valued neutrosophic set (SVNS). Obviously, SVNS is a generalization of IFS and represents incomplete, indeterminate, and inconsistent information, which cannot be expressed by IFS. For example, assume that an investment company wants to invest a sum of money to some investment alternative. Then, there are 10 voters in the voting process of the investment alternative. Five vote "aye", four vote 'blackball', and one votes 'indeterminacy/neutrality'. From neutrosophic notation, it can be represented as (x, 0.5, 0.4, 0.1). It is obvious that this expression is beyond the scope of IFS. Hence, SVNS is suitable for the expression of indeterminate and inconsistent information. Recently, the neutrosophic sets have been applied in many decision-making problems [11–17]. Liu and Teng [18] presented a normal neutrosophic number (NNN) as an extension of NIFN and its generalized weighted power averaging operator, and then applied it to multiple attribute decision-making (MADM) problems with normal neutrosophic information. Liu and Li [19] further introduced some normal neutrosophic Bonferroni mean operators for decision-making problems with normal neutrosophic information. After that, Sahin [20] proposed some normal neutrosophic generalized prioritized aggregation operators for MADM problems under normal neutrosophic environments.

However, the aforementioned decision-making methods depend on aggregation operators of NNNs in the normal neutrosophic decision-making process. Then, the correlation coefficient is an important mathematical tool in decision-making problems [11–13]. Compared with the decision-making methods using aggregation operators [18–20], the decision-making methods based on correlation coefficients imply relatively simple decision-making processes. However, there is no research on correlation coefficients of NNSs in existing normal neutrosophic decision-making methods. On the other hand, the applications of NNNs (basic elements in NNSs) in science and engineering fields are necessary and significant because the normal distribution is a typical and common distribution in the real world [18–20]. Additionally, NNN contain much more information than the general neutrosophic number because NNN is expressed by the combination information of both an NFN and a single-valued neutrosophic number (SVNN) (a basic element in SVNS). Hence, NNN used in decision-making can show its rationality and reality. Motivated by the decision-making methods [18–20], this study firstly proposes two correlation coefficients of normal neutrosophic sets (NNSs) based on the score functions of NNNs and then develops an MADM method using the correlation coefficients of NNSs to simplify the decision-making process under normal neutrosophic environments.

The rest of this paper is organized as follows. In Section 2, we review some basic concepts of NIFNs and NNSs. In Section 3, two correlation coefficients between NNSs are presented based on the score functions of NNNs. Section 4 develops an MADM method using the correlation coefficients of NNSs under normal neutrosophic environments. In Section 5, an illustrative example about the selection problem of investment alternatives is provided to demonstrate the applications and effectiveness of the proposed MADM method with normal neutrosophic information. Conclusions and future work are contained in Section 6.

2. Some Basic Concepts of NIFNs and NNSs

Yang and Ko [4] defined an NFN to express the normal fuzzy information in random fuzzy situations.

For a real number set X, if the membership function satisfies the form

$$N(x) = \mathrm{e}^{-\left(\frac{x-\mu}{\sigma}\right)^2} \quad (1)$$

then $N(x)$ is called NFN, where μ is the mean or expectation of the distribution (and its median and mode) and σ is standard deviation. Then, this NFN is symmetric around $x = \mu$, denoted by $N(\mu, \sigma)$.

Based on the combination of an IFN and an NFN, Wang et al. [8] defined an NIFN $A = \langle x \mid N(\mu, \sigma), t_A(x), v_A(x)\rangle$, where its membership function is expressed as

$$t_A(x) = t_A \mathrm{e}^{-\left(\frac{x-\mu}{\sigma}\right)^2}, x \in \mathrm{X}$$

and its non-membership function is expressed as

$$v_A(x) = 1 - (1 - v_A)\mathrm{e}^{-\left(\frac{x-\mu}{\sigma}\right)^2}, x \in \mathrm{X}$$

where t_A and v_A are a membership degree and a non-membership degree in an IFN and satisfy t_A, $v_A \in [0,1]$, and $0 \leq t_A + v_A \leq 1$.

To express indeterminate and inconsistent information in the real world, Smarandache [9] introduced a concept of a neutrosophic set from a philosophical point of view. A neutrosophic set B in a universe of discourse X can be described independently by its truth, indeterminacy, and falsity membership functions $t_B(x)$, $u_B(x)$, and $v_B(x)$ in real standard interval [0,1] or nonstandard interval $]^-0, 1^+[$, such that $t_B(x)$: $X \rightarrow\,]^-0, 1^+[$, $u_B(x)$: $X \rightarrow\,]^-0, 1^+[$, $v_B(x)$: $U \rightarrow\,]^-0, 1^+[$, and $^-0 \leq \sup t_B(x) + \sup u_B(x) + \sup v_B(x) \leq 3^+$ for $x \in X$.

However, when the three membership functions in the neutrosophic set lie in the nonstandard interval $]^-0, 1^+[$, the neutrosophic set shows the difficulty of its actual applications. Thus, Smarandache [9] and Wang et al. [10] introduced the concept of an SVNS as a subclass of the neutrosophic set when the three membership functions in the neutrosophic set are constrained in the real standard interval [0,1].

Definition 1. [9,10]. *Let X be a universe of discourse. An SVNS S in X is described independently by its truth, indeterminacy, and falsity membership functions $t_S(x)$, $u_S(x)$, and $v_S(x)$, where $t_S(x)$, $u_S(x)$, $v_S(x) \in [0,1]$, and $0 \leq t_S(x) + u_S(x) + v_S(x) \leq 3$ for $x \in X$. Then, the SVNS S can be denoted as. $S = \{\langle x, t_S(x), u_S(x), v_S(x)\rangle : x \in X\}$.*

For convenience, a basic element $\langle x, t_S(x), u_S(x), v_S(x)\rangle$ in S is denoted by $s = \langle t, u, v\rangle$ for short, which is called an SVNN.

As an extension of NIFN, Liu and Teng [11] and Liu and Li [12] presented a concept of NNS based on the combination of NFN and SVNN.

Definition 2. [11,12]. *Let X be a finite non-empty set and N(μ, σ) be a normal distribution function. An NNS is defined as*

$$P = \{\langle x | N(\mu_P, \sigma_P), (t_P(x), u_P(x), v_P(x))\rangle : x \in X\} \quad (2)$$

where the three functions $t_P(x)$, $u_P(x)$, *and* $v_P(x)$ *for* $x \in X$ *satisfy the following properties:*

$$t_P(x) = t_P e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$u_P(x) = 1 - (1 - u_P)e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$v_P(x) = 1 - (1 - v_P)e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$0 \leq t_P(x) + u_P(x) + v_P(x) \leq 1$$

and t_p, u_p, *and* v_p *are the truth, indeterminacy, and falsity degrees in the SVNN, respectively, and satisfy* t_p, u_p, *and* $v_p \in [0,1]$ *and* $0 \leq t_p + u_p + v_p \leq 3$.

Then, an NNN (a basic element) in the NNS P is denoted by $p = <N(\mu, \sigma), (t, u, v)>$ for convenience, where t, u, and v are the truth, indeterminacy, and falsity degrees, respectively, in the SVNN (t, u, v) and satisfy $t, u, v \in [0,1]$ and $0 \leq t + u + v \leq 3$.

Definition 3. [12]. *Let* $p = <N(\mu, \sigma), (t, u, v)>$ *be an NNN. Then, its score functions are defined as*

$$\begin{aligned} S_1(p) &= \mu(2 + t - u - v), \\ S_2(p) &= \sigma(2 + t - u - v). \end{aligned} \quad (3)$$

3. Correlation Coefficients between NNSs

Based on the score functions of NNNs in Definition 3, we can give the definitions of the correlation and correlation coefficients between NNSs under normal neutrosophic environments.

Definition 4. *Let two NNSs be* $P = \{p_1, p_2, \ldots, p_n\}$ *and* $Q = \{q_1, q_2, \ldots, q_n\}$, *where* $p_j = <N(\mu_{pj}, \sigma_{pj}), (t_{pj}, u_{pj}, v_{pj})>$ *and* $q_j = <N(\mu_{qj}, \sigma_{qj}), (t_{qj}, u_{qj}, v_{qj})>$ *for* $j = 1, 2, \ldots, n$ *are NNNs in P and Q. The correlation between two NNSs P and Q is defined as*

$$C(P, Q) = \sum_{j=1}^{n} \left[(2 + t_{Pj} - u_{Pj} - v_{Pj})(2 + t_{qj} - u_{qj} - v_{qj})(\mu_{pj}\mu_{qj} + \sigma_{pj}\sigma_{qj}) \right] \quad (4)$$

Thus, based on the correlation between two NNSs P and Q, we can introduce the definition of the following correlation coefficients between two NNSs P and Q.

Definition 5. *Let two NNSs be* $P = \{p_1, p_2, \ldots, p_n\}$ *and* $Q = \{q_1, q_2, \ldots, q_n\}$, *where* $p_j = <N(\mu_{pj}, \sigma_{pj}), (t_{pj}, u_{pj}, v_{pj})>$ *and* $q_j = <N(\mu_{qj}, \sigma_{qj}), (t_{qj}, u_{qj}, v_{qj})>$ *for* $j = 1, 2, \ldots, n$ *are NNNs in P and Q. The correlation coefficients between two NNSs P and Q are defined as*

$$\begin{aligned} \rho_1(P, Q) &= \frac{C(P,Q)}{[C(P,P)C(Q,Q)]^{1/2}} \\ &= \frac{\sum_{j=1}^{n} \left[(2+t_{Pj}-u_{Pj}-v_{Pj})(2+t_{qj}-u_{qj}-v_{qj})(\mu_{pj}\mu_{qj}+\sigma_{pj}\sigma_{qj}) \right]}{\left\{ \begin{array}{c} \sqrt{\sum_{j=1}^{n} \left\{ \left[(2 + t_{Pj} - u_{Pj} - v_{pj})\mu_{pj} \right]^2 + \left[(2 + t_{pj} - u_{pj} - v_{pj})\sigma_{pj} \right]^2 \right\}} \\ \times \sqrt{\sum_{j=1}^{n} \left\{ \left[(2 + t_{qj} - u_{qj} - v_{qj})\mu_{qj} \right]^2 + \left[(2 + t_{qj} - u_{qj} - v_{qj})\sigma_{qj} \right]^2 \right\}} \end{array} \right\}} \end{aligned} \quad (5)$$

$$\rho_2(P,Q) = \frac{C(P,Q)}{\max[C(P,P),C(Q,Q)]} = \frac{\sum_{j=1}^{n}\left[(2+t_{Pj}-u_{Pj}-v_{Pj})(2+t_{qj}-u_{qj}-v_{qj})(\mu_{pj}\mu_{qj}+\sigma_{pj}\sigma_{qj})\right]}{\max\left\{\begin{array}{c}\sum_{j=1}^{n}\left\{\left[(2+t_{Pj}-u_{Pj}-v_{pj})\mu_{pj}\right]^2+\left[(2+t_{pj}-u_{pj}-v_{pj})\sigma_{pj}\right]^2\right\}, \\ \sum_{j=1}^{n}\left\{\left[(2+t_{qj}-u_{qj}-v_{qj})\mu_{qj}\right]^2+\left[(2+t_{qj}-u_{qj}-v_{qj})\sigma_{qj}\right]^2\right\}\end{array}\right\}} \quad (6)$$

Proposition 1. *The correlation coefficients of $\rho_k(P, Q)$ (k = 1, 2) satisfy the following properties:*

1. $0 \le \rho_k(P, Q) \le 1$;
2. $\rho_k(P, Q) = 1$ *if* $P = Q$, *i.e.,* $N(\mu_{pj}, \sigma_{pj}) = N(\mu_{qj}, \sigma_{qj})$ *and* $(t_{pj}, u_{pj}, v_{pj}) = (t_{qj}, u_{qj}, v_{qj})$;
3. $\rho_k(P, Q) = \rho_k(Q, P)$.

Proof.

Firstly, we prove that the correlation coefficient of $\rho_1(P, Q)$ satisfies the properties (1)–(3).

The inequality $\rho_1(P, Q) \ge 0$ is obvious. Then, we only prove $\rho_1(P, Q) \le 1$.

Based on the Cauchy–Schwarz inequality:

$$(x_1y_1 + x_2y_2 + \cdots + x_ny_n)^2 \le \left(x_1^2 + x_2^2 + \cdots x_n^2\right) \times \left(y_1^2 + y_2^2 + \cdots y_n^2\right)$$

where $(x_1, x_2, \ldots, x_n) \in R^n$ and $(y_1, y_2, \ldots, y_n) \in R^n$, we can yield the following inequality:

$$(x_1y_1 + x_2y_2 + \cdots + x_ny_n) \le \sqrt{\left(x_1^2 + x_2^2 + \cdots x_n^2\right)} \times \sqrt{\left(y_1^2 + y_2^2 + \cdots y_n^2\right)}$$

Corresponding to the above inequality and the definition of correlations coefficients in Definition 3, we have the following inequality:

$$\sum_{j=1}^{n}\left[(2+t_{pj}-u_{pj}-v_{pj})\mu_{pj}\times(2+t_{qj}-u_{qj}-v_{qj})\mu_{qj}\right]+\sum_{j=1}^{n}\left[(2+t_{pj}-u_{pj}-v_{pj})\sigma_{pj}\times(2+t_{qj}-u_{qj}-v_{qj})\sigma_{qj})\right] \le$$
$$\sqrt{\sum_{j=1}^{n}\left[(2+t_{pj}-u_{pj}-v_{pj})\mu_{pj}\right]^2+\sum_{j=1}^{n}\left[(2+t_{pj}-u_{pj}-v_{pj})\sigma_{pj}\right]^2} \times \sqrt{\sum_{j=1}^{n}\left[(2+t_{qj}-u_{qj}-v_{qj})\mu_{qj}\right]^2+\sum_{j=1}^{n}\left[(2+t_{qj}-u_{qj}-v_{qj})\sigma_{qj}\right]^2}$$

Hence, there is the following result:

$$\sum_{j=1}^{n}\left[(2+t_{pj}-u_{pj}-v_{pj})(2+t_{qj}-u_{qj}-v_{qj})(\mu_{pj}\mu_{qj}+\sigma_{pj}\sigma_{qj})\right] \le$$
$$\sqrt{\sum_{j=1}^{n}\left\{\left[(2+t_{pj}-u_{pj}-v_{pj})\mu_{pj}\right]^2+\left[(2+t_{pj}-u_{pj}-v_{pj})\sigma_{pj}\right]^2\right\}} \times \sqrt{\sum_{j=1}^{n}\left\{\left[(2+t_{qj}-u_{qj}-v_{qj})\mu_{qj}\right]^2+\left[(2+t_{qj}-u_{qj}-v_{qj})\sigma_{qj}\right]^2\right\}}$$

Based on Equation (5), we have $\rho_1(P, Q) \le 1$. Hence, $0 \le \rho_1(P, Q) \le 1$ holds.

(2) $P = Q \Rightarrow N(\mu_{pj}, \sigma_{pj}) = N(\mu_{qj}, \sigma_{qj})$ and $(t_{pj}, u_{pj}, v_{pj}) = (t_{qj}, u_{qj}, v_{qj}) \Rightarrow \mu_{pj} = \mu_{qj}$, $\sigma_{pj} = \sigma_{qj}$, $t_{pj} = t_{qj}$, $u_{pj} = u_{qj}$, and $v_{pj} = v_{qj}$ for $j = 1, 2, \ldots, n \Rightarrow \rho_1(P, Q) = 1$.

(3) It is straightforward.

Secondly, we prove that the correlation coefficient of $\rho_2(P, Q)$ satisfies the properties (1)–(3).

By the similar proof manner of the properties (1)–(3) of $\rho_1(P, Q)$, we can prove the properties (1)–(3) of $\rho_2(P, Q)$. It is not repeated here.

Therefore, we complete these proofs. □

When the weight of the elements p_j and q_j (j = 1, 2, . . . , n) is taken into account, $\boldsymbol{w}$ = {w_1, w_2, . . . , w_n} is given as the weight vector of the elements p_j and q_j (j = 1, 2, . . . , n) with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then, we have the following weighted correlation coefficients of NNSs:

$$\rho_{1w}(P,Q) = \frac{\sum_{j=1}^{n} w_j\left[(2+t_{pj}-u_{pj}-v_{pj})(2+t_{qj}-u_{qj}-v_{qj})(\mu_{pj}\mu_{qj}+\sigma_{pj}\sigma_{qj})\right]}{\left\{\begin{array}{c}\sqrt{\sum_{j=1}^{n} w_j\left\{\left[(2+t_{pj}-u_{Pj}-v_{pj})\mu_{pj}\right]^2+\left[(2+t_{pj}-u_{pj}-v_{pj})\sigma_{pj}\right]^2\right\}}\\ \times\sqrt{\sum_{j=1}^{n} w_j\left\{\left[(2+t_{qj}-u_{qj}-v_{qj})\mu_{qj}\right]^2+\left[(2+t_{qj}-u_{qj}-v_{qj})\sigma_{qj}\right]^2\right\}}\end{array}\right\}} \tag{7}$$

$$\rho_{2w}(P,Q) = \frac{\sum_{j=1}^{n} w_j\left[(2+t_{Pj}-u_{Pj}-v_{Pj})(2+t_{qj}-u_{qj}-v_{qj})(\mu_{pj}\mu_{qj}+\sigma_{pj}\sigma_{qj})\right]}{\max\left\{\begin{array}{c}\sum_{j=1}^{n} w_j\left\{\left[(2+t_{Pj}-u_{Pj}-v_{pj})\mu_{pj}\right]^2+\left[(2+t_{pj}-u_{pj}-v_{pj})\sigma_{pj}\right]^2\right\},\\ \sum_{j=1}^{n} w_j\left\{\left[(2+t_{qj}-u_{qj}-v_{qj})\mu_{qj}\right]^2+\left[(2+t_{qj}-u_{qj}-v_{qj})\sigma_{qj}\right]^2\right\}\end{array}\right\}} \tag{8}$$

Proposition 2. *The weighted correlation coefficients of $\rho_{kw}(P, Q)$ (k = 1, 2) also satisfy the following properties:*

1. $0 \le \rho_{kw}(P, Q) \le 1$;
2. $\rho_{kw}(P, Q) = 1$ *if and only if P = Q, i.e.,* $N(\mu_{pj}, \sigma_{pj}) = N(\mu_{qj}, \sigma_{qj})$ *and* $(t_{pj}, u_{pj}, v_{pj}) = (t_{qj}, u_{qj}, v_{qj})$;
3. $\rho_{kw}(P, Q) = \rho_{kw}(Q, P)$.

By the similar proofs of the properties in Proposition 1, we can prove the ones in Proposition 2. They are not repeated here.

Especially when w = {1/n, 1/n, . . . , 1/n}, Equations (7) and (8) are reduced to Equations (5) and (6).

4. The MADM Method Using the Correlation Coefficients of NNSs

In this section, we present a handling method for the MADM problems with normal neutrosophic information by means of the weighted correlation coefficients between NNSs.

In an MADM problem with normal neutrosophic information, assume that P = {P_1, P_2, . . . , P_m} is a set of m alternatives and R = {R_1, R_2, . . . , R_n} is a set of n attributes. The weight vector of the attributes is given as $\boldsymbol{w}$ = (w_1, w_2, . . . , w_n) satisfying $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then, the average value μ_{ij} and standard derivation σ_{ij} in the normal distribution $N(\mu_{ij}, \sigma_{ij})$ are obtained by the statistical analysis of data corresponding to the alternative P_i (i = 1, 2, . . . , m) over the attribute R_j (j = 1, 2, . . . , n), while the evaluation values of SVNNs corresponding to the alternative P_i (i = 1, 2, . . . , m) over the attribute R_j (j = 1, 2, . . . , n) are given by decision-makers. Based on the obtained NNNs p_{ij} = <$N(\mu_{ij}, \sigma_{ij})$, (t_{ij}, u_{ij}, v_{ij})> (i = 1, 2, . . . , m; j = 1, 2, . . . , n), we can yield the normal neutrosophic decision matrix $M(p_{ij})_{m\times n}$:

$$M(p_{ij})_{m\times n} = \begin{bmatrix} \langle N(\mu_{11},\sigma_{11}),(t_{11},u_{11},v_{11})\rangle & \langle N(\mu_{12},\sigma_{12}),(t_{12},u_{12},v_{12})\rangle & \cdots & \langle N(\mu_{1n},\sigma_{1n}),(t_{1n},u_{1n},v_{1n})\rangle \\ \langle N(\mu_{21},\sigma_{21}),(t_{21},u_{21},v_{21}\rangle & \langle N(\mu_{22},\sigma_{22}),(t_{22},u_{22},v_{22})\rangle & \cdots & \langle N(\mu_{2n},\sigma_{2n}),(t_{2n},u_{2n},v_{2n})\rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle N(\mu_{m1},\sigma_{m1}),(t_{m1},u_{m1},v_{m1})\rangle & \langle N(\mu_{m2},\sigma_{m2}),(t_{m2},u_{m2},v_{m2})\rangle & \cdots & \langle N(\mu_{mn},\sigma_{mn}),(t_{mn},u_{mn},v_{mn})\rangle \end{bmatrix}$$

In MADM problems, the concept of the ideal point has been used to help the identification of the best alternative in the decision set. It does provide a useful method to evaluate alternatives [13]. However, there are two types of attributes, i.e., benefit type and cost type, in decision-making problems. Hence, we firstly need to determinate an ideal solution/alternative (an ideal NNS) corresponding to the benefit type and cost type of attributes. Then, by correlation coefficient values between each alternative (each evaluated NNS) and the ideal alternative (the ideal NNS), the ranking order of alternatives and the best one are given in the normal neutrosophic decision-making process.

Thus, we use the developed method to deal with the MADM problem with normal neutrosophic information, which is described by the following procedures:

Step 1: Establish an ideal solution (an ideal alternative) $P^* = \{p_1^*, p_2^*, \ldots, p_n^*\}$ by the ideal NNN $p_j^* = \left\langle N\left(\max_i(m_{ij}), \min_i(\sigma_{ij})\right), \left(\max_i(t_{ij}), \min_i(u_{ij}), \min_i(v_{ij})\right)\right\rangle$ corresponding to the benefit type of attributes and $p_j^* = \left\langle N\left(\min_i(m_{ij}), \min_i(\sigma_{ij})\right), \left(\min_i(t_{ij}), \max_i(u_{ij}), \max_i(v_{ij})\right)\right\rangle$ corresponding to the cost type of attributes.

Step 2: Calculate the weighted correlation coefficients between an alternative P_i (i = 1, 2, ... , m) and the ideal solution P^* by using Equation (7) or Equation (8) and obtain the values of $\rho_{1w}(P_i, P^*)$ or $\rho_{2w}(P_i, P^*)$ (i = 1, 2, ... , m).

Step 3: Rank the alternatives in a descending order corresponding to the weighted correlation coefficient values and select the best one(s) according to the bigger value of $\rho_{1w}(P_i, P^*)$ or $\rho_{2w}(P_i, P^*)$.

Step 4: End.

5. Illustrative Example

For convenient comparison, an illustrative example about the selection problem of investment alternatives adopted from [18] is provided to demonstrate the applications and effectiveness of the proposed MADM method with normal neutrosophic information.

An investment company wants to invest a sum of money to the best industry. Then, four possible alternatives are considered as four potential industries: (1) P_1 is a car company; (2) P_2 is a food company; (3) P_3 is a computer company; (4) P_4 is an arms company. In the decision-making process, the four possible alternatives must satisfy the requirements of the three attributes: (1) R_1 is the risk; (2) R_2 is the growth; (3) R_3 is the environment, where the attributes R_1 and R_2 are benefit types and the attribute R_3 is a cost type. Assume that the weighting vector of the attributes is given by $\boldsymbol{w}$ = (0.35, 0.25, 0.4). By the statistical analysis and the evaluation of investment data regarding the four possible alternatives of P_i (i = 1, 2, 3, 4) over the three attributes of R_j (j = 1, 2, 3), we can establish the following NNN decision matrix [18]:

$$M(p_{ij})_{4\times 3} = \begin{bmatrix} \langle N(3,0.4),(0.4,0.2,0.3)\rangle & \langle N(7,0.6),(0.4,0.1,0.2)\rangle & \langle N(5,0.4),(0.7,0.2,0.4)\rangle \\ \langle N(4,0.2),(0.6,0.1,0.2)\rangle & \langle N(8,0.4),(0.6,0.1,0.2)\rangle & \langle N(6,0.7),(0.3,0.5,0.8)\rangle \\ \langle N(3.5,0.3),(0.3,0.2,0.3)\rangle & \langle N(6,0.2),(0.5,0.2,0.3)\rangle & \langle N(5.5,0.6),(0.4,0.2,0.7)\rangle \\ \langle N(5,0.5),(0.7,0.1,0.2)\rangle & \langle N(7,0.5),(0.6,0.1,0.1)\rangle & \langle N(4.5,0.5),(0.6,0.3,0.8)\rangle \end{bmatrix}$$

Then, we use Equation (7) to deal with the MADM problem with normal neutrosophic information, which is described by the following procedures:

Step 1: Establish an ideal solution (an ideal alternative) $P^* = \{p_1^*, p_2^*, \ldots, p_n^*\}$ expressed by the ideal NNS $P^* = \{\langle N(5,0.2),(0.7,0.1,0.2)\rangle, \langle N(8,0.2),(0.6,0.1,0.1)\rangle, \langle N(4.5,0.4),(0.3,0.5,0.8)\rangle\}$ corresponding to the benefit types and cost types of attributes.

Step 2: Calculate the weighted correlation coefficient between the alternative P_1 and the ideal solution P^* by using Equation (7) as follows:

$$\rho_{1w}(P_1,P^*) = \frac{\sum_{j=1}^{3} w_j\left[(2+t_{p_{1j}}-u_{p_{1j}}-v_{p_{1j}})(2+t_{p_j^*}-u_{p_j^*}-v_{p_j^*})(\mu_{p_{1j}}\mu_{p_j^*}+\sigma_{p_{1j}}\sigma_{p_j^*})\right]}{\left\{\begin{array}{c}\sqrt{\sum_{j=1}^{3} w_j\left\{\left[(2+t_{p_{1j}}-u_{P_{1j}}-v_{p_{1j}})\mu_{p_{1j}}\right]^2+\left[(2+t_{p_{1j}}-u_{p_{1j}}-v_{p_{1j}})\sigma_{p_{1j}}\right]^2\right\}}\\ \times\sqrt{\sum_{j=1}^{3} w_j\left\{\left[(2+t_{p_j^*}-u_{p_j^*}-v_{p_j^*})\mu_{p_j^*}\right]^2+\left[(2+t_{p_j^*}-u_{p_j^*}-v_{p_j^*})\sigma_{p_j^*}\right]^2\right\}}\end{array}\right\}}$$

$$= \frac{\left\{\begin{array}{c}0.35\times[(2+0.4-0.2-0.3)\times(2+0.7-0.1-0.2)\times(3\times5+0.4\times0.2)]\\ +0.25\times[(2+0.4-0.1-0.2)\times(2+0.6-0.1-0.1)\times(7\times8+0.6\times0.2)]\\ +0.4\times[(2+0.7-0.2-0.4)\times(2+0.3-0.5-0.8)\times(5\times4.5+0.4\times0.4)]\end{array}\right\}}{\left\{\begin{array}{c}\sqrt{\begin{array}{c}0.35\times\left\{[(2+0.4-0.2-0.3)\times3]^2+[(2+0.4-0.2-0.3)\times0.4]^2\right\}\\ +0.25\times\left\{[(2+0.4-0.1-0.2)\times7]^2+[(2+0.4-0.1-0.2)\times0.6]^2\right\}\\ +0.4\times\left\{[(2+0.7-0.2-0.4)\times5]^2+[(2+0.7-0.2-0.4)\times0.4]^2\right\}\end{array}}\\ \times\sqrt{\begin{array}{c}0.35\times\left\{[(2+0.7-0.1-0.2)\times5]^2+[(2+0.7-0.1-0.2)\times0.2]^2\right\}\\ +0.25\times\left\{[(2+0.6-0.1-0.1)\times8]^2+[(2+0.6-0.1-0.1)\times0.2]^2\right\}\\ +0.4\times\left\{[(2+0.3-0.5-0.8)\times4.5]^2+[(2+0.3-0.5-0.8)\times0.4]^2\right\}\end{array}}\end{array}\right\}}$$

$$= 0.8820.$$

By similar calculations, the weighted correlation coefficients between each alternative P_i (i = 2, 3, 4) and the ideal solution P^* can be given as the following values of $\rho_{1w}(P_i, P^*)$ (i = 2, 3, 4):

$\rho_{1w}(P_2, P^*)$ = 0.9891, $\rho_{1w}(P_3, P^*)$ = 0.9169, and $\rho_{1w}(P_4, P^*)$ = 0.9875.

Step 3: According to the values of $\rho_{1w}(P_i, P^*)$ (i = 1, 2, 3, 4), the ranking order of the alternatives is $P_2 > P_4 > P_3 > P_1$ and the best one is P_2. These results are the same as in [18].

We could also use Equation (8) to deal with the MADM problem with normal neutrosophic information, which is described by the following steps:

Step 1′: The same as Step 1.

Step 2′: Calculate the weighted correlation coefficient between the alternative P_1 and the ideal solution P^* by using Equation (8) as follows:

$$\rho_{2w}(P_1,P^*) = \frac{\sum_{j=1}^{3} w_j\left[(2+t_{p_{1j}}-u_{p_{1j}}-v_{p_{1j}})(2+t_{p_j^*}-u_{p_j^*}-v_{p_j^*})(\mu_{p_{1j}}\mu_{p_j^*}+\sigma_{p_{1j}}\sigma_{p_j^*})\right]}{\max\left\{\begin{array}{c}\sum_{j=1}^{3} w_j\left\{\left[(2+t_{p_{1j}}-u_{P_{1j}}-v_{p_{1j}})\mu_{p_{1j}}\right]^2+\left[(2+t_{p_{1j}}-u_{p_{1j}}-v_{p_{1j}})\sigma_{p_{1j}}\right]^2\right\},\\ \sum_{j=1}^{3} w_j\left\{\left[(2+t_{p_j^*}-u_{p_j^*}-v_{p_j^*})\mu_{p_j^*}\right]^2+\left[(2+t_{p_j^*}-u_{p_j^*}-v_{p_j^*})\sigma_{p_j^*}\right]^2\right\}\end{array}\right\}}$$

$$= \frac{\left\{\begin{array}{c}0.35\times[(2+0.4-0.2-0.3)\times(2+0.7-0.1-0.2)\times(3\times5+0.4\times0.2)]\\ +0.25\times[(2+0.4-0.1-0.2)\times(2+0.6-0.1-0.1)\times(7\times8+0.6\times0.2)]\\ +0.4\times[(2+0.7-0.2-0.4)\times(2+0.3-0.5-0.8)\times(5\times4.5+0.4\times0.4)]\end{array}\right\}}{\max\left\{\begin{array}{c}\left(\begin{array}{c}0.35\times\left\{[(2+0.4-0.2-0.3)\times3]^2+[(2+0.4-0.2-0.3)\times0.4]^2\right\}\\ +0.25\times\left\{[(2+0.4-0.1-0.2)\times7]^2+[(2+0.4-0.1-0.2)\times0.6]^2\right\}\\ +0.4\times\left\{[(2+0.7-0.2-0.4)\times5]^2+[(2+0.7-0.2-0.4)\times0.4]^2\right\}\end{array}\right),\\ \left(\begin{array}{c}0.35\times\left\{[(2+0.7-0.1-0.2)\times5]^2+[(2+0.7-0.1-0.2)\times0.2]^2\right\}\\ +0.25\times\left\{[(2+0.6-0.1-0.1)\times8]^2+[(2+0.6-0.1-0.1)\times0.2]^2\right\}\\ +0.4\times\left\{[(2+0.3-0.5-0.8)\times4.5]^2+[(2+0.3-0.5-0.8)\times0.4]^2\right\}\end{array}\right)\end{array}\right\}}$$

$$= 0.7544.$$

By similar calculations, the weighted correlation coefficients between each alternative P_i (i = 2, 3, 4) and the ideal solution P^* can be given as the following values of $\rho_{2w}(P_i, P^*)$ (i = 2, 3, 4):

$\rho_{2w}(P_2, P^*) = 0.9151$, $\rho_{2w}(P_3, P^*) = 0.6575$, and $\rho_{2w}(P_4, P^*) = 0.9522$.

Step 3′: According to the values of $\rho_{2w}(P_i, P^*)$ ($i = 1, 2, 3, 4$), the ranking order of the alternatives is $P_4 > P_2 > P_1 > P_3$, and the best one is P_4. These results also are the same as in [18].

Obviously, the above two ranking orders are different corresponding to different correlation coefficients for this decision-making problem; these results are thus in accordance with the ones in [18]. Hence, the proposed normal neutrosophic decision-making method based on the correlation coefficients illustrates its feasibility and effectiveness. Compared with existing decision-making methods based on aggregation operators of NNNs, the proposed decision-making method based on the correlation coefficients of NNSs shows that it is simpler to employ than existing normal neutrosophic decision-making methods in [18–20] under normal neutrosophic environments because the decision-making method proposed in this paper implies its simple algorithms and decision steps in the normal neutrosophic decision-making problems.

From the decision results of the illustrative example, we see that different correlation coefficients used in the decision-making problem can result in different ranking orders and selecting alternatives. Hence, the decision-maker can select one of both corresponding to his/her preference or actual requirements.

6. Conclusions

To simplify the complex decision-making process/steps and algorithms of existing normal neutrosophic decision-making methods in [18–20], this paper proposed two correlation coefficients between NNSs based on the score functions of NNNs under normal neutrosophic environments. Then, we developed an MADM method with normal neutrosophic information by using the correlation coefficients of NNSs under normal neutrosophic environments. An illustrative example about the selection problem of investment alternatives was provided to demonstrate the applications and effectiveness of the proposed MADM method under normal neutrosophic environments.

The main advantages of this study are (1) the evaluation information expressed by NNNs is relatively more reasonable and more realistic than the evaluation information expressed by general neutrosophic numbers in the decision-making process; (2) the proposed decision-making method based on the correlation coefficients of NNSs is simpler to employ than existing ones based on aggregation operators of NNNs in the normal neutrosophic decision-making algorithms; (3) the proposed decision-making method with NNNs contains much more information and shows its rationality and reality, while the existing decision-making methods with single neutrosophic information may lose some useful evaluation information of attributes in the decision-making process.

In future work, the study about new similarity measures of NNSs and applications in science and engineering fields are necessary and significant because the applications of the normal distribution widely exist in many domains.

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