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# NEUTROSOPHIC (Q, L)-FUZZY SUBGROUP

K. THIRUSANGU*[1], S. POORNAVEL[2], AND R. VASUKI[1]

[1]Associate Professor of Mathematics,
SIVET College, Gowrivakkam, Chennai-73, India.

[2]Assistant Prof. in Mathematics,
Madha Engineering College, Kundrathur, Chennai-69, india.

[3]Associate Prof. of Mathematics,
SIVET College, Gowrivakkam, Chennai, India.

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***ABSTRACT***

*Goguen [1967] introduced L-fuzzy set. Murali [1991] analyzed lattice of fuzzy algebras and closure systems. Saibaba [2008] initiated fuzzy lattice ordered groups, and studied many properties on fuzzy lattice. Swamy and Raju [1991] gave a detailed findings on algebraic fuzzy system. In this paper, the notion of neutrosophic (Q, L)-fuzzy subgroup is introduced, and discussed some of its basic algebraic properties .Also the results on homomorphic image, pre image of neutrosophic (Q, L)-fuzzy subgroup are derived. Proved findings are that the direct product of any two neutrosophic (Q, L)-subgroups is a neutrosophic (Q, L)-fuzzy subgroup, and it is extended for finite number of neutrosophic groups.*

***Keywords:*** *neutrosophic Q-fuzzy set, neutrosophic Q-fuzzy subgroup (NQLFG), neutrosophic Q-fuzzy normal subgroup.*

## INTRODUCTION

Smarandache [2002] introduced the notion of Neutrosophy as a new branch of philosophy. Neutrosophic is a base of neutrosophic logic which is an extension of fuzzy logic in which indeterminacy is included .In neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, percentage of indeterminancy in a subset I, and the percentage of falsity in a subset. Maji *et al.* [2001] introduced the notion of fuzzy soft set. Afterwards, many researches were conducted on the notion of fuzzy sets. The theory of neutrosophic set have achieved great success in various fields like medical diagnosis, image processing decision making problem and so on. Arockiarani, and Martina Jency [2014] consider the neutrosophic set with value from the subset of [0, 1] and extended the research in fuzzy neutrosophic set. They [2016] initiated the concept of subgroupoid in fuzzy neutrosophic set.

## SECTION 2: BASIC DEFINITIONS

**Definition 2.1:** Let X be a non-empty set. A fuzzy set A is a map A: $X \rightarrow [0, 1]$. Let (L, ≤) be a lattice with the least element 0 and the greatest element 1. Any element a in L satisfies $0 \leq a \leq 1$. An L-fuzzy set on a non-empty set X is a map f: $X \rightarrow L$.

**Definition 2.2:** Let X, Q be two non-empty sets, and L be a lattice. A mapping T: $X \times Q \rightarrow L$ is a (Q, L)-fuzzy set in X.

**Definition 2.3:** A neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $T_A(x)$, an indeterminacy function $I_A(x)$ and a falsity membership function $F_A(x)$ is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in X\}$, where $T_A$ , $I_A$ ,$F_A$ : $X \rightarrow [0. 1]$ and $0 \leq T_A(x) \leq 1$; $0 \leq I_A(x) \leq 1$; $0 \leq F_A(x) \leq 1$, for all $x \in X$.

***Corresponding Author: K. Thirusangu*[1]***
***[1]Associate Professor of Mathematics, SIVET College, Gowrivakkam, Chennai-73, India.***

**Definition 2.4:** Let X, Y be two non-empty sets and $f: X \longrightarrow Y$ be a function.

(i) If $B = \{< y, T_B(y), I_B(y), F_B(y): y \in Y\}$ is a neutrosophic fuzzy set in Y then the preimage of B under $f$, denoted by $f^{-1}(B)$, is the neutrosophic fuzzy set in X defined by
$f^{-1}(B) = \{< x, f^{-1}(T_B(x)), f^{-1}(I_B(x)), f^{-1}(F_B(x)) >: x \in X\}$ where $f^{-1}(T_B(x)) = T_B(f(x))$.

(ii) If $A = \{< x, T_A(x), I_A(x), F_A(x): x \in X\}$ is a Neutrosophic fuzzy set in X, then the image f(A) of A under f is the neutrosophic fuzzy set in Y defined by $f(A) = \{< y, f(T_A(y)), f(I_A(y)), f(F_A(y)) > : y \in Y\}$,

where

$$f(T_A(y)) = \begin{cases} \sup_{x \in f^{-1}(y)} T_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 0 & \text{otherwise} \end{cases}$$

$$f(I_A(y)) = \begin{cases} \sup_{x \in f^{-1}(y)} I_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 0 & \text{otherwise} \end{cases}$$

$$f_{\sim}(F_A(y)) = \begin{cases} \inf_{x \in f^{-1}(y)} F_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 1 & \text{otherwise} \end{cases}, \text{ where } f_{\sim}(F_A(y)) = (1 - f(1 - F_A))y$$

**Definition 2.5:** A neutrosophic (Q, L)-fuzzy set is an object having the form A = { < (x, q), $T_A(x, q)$, $I_A(x, q)$, $F_A(x, q)$ > : $x \in X$, $q \in Q$ }, where $T_A: X \times Q \to L$, $I_A: X \times Q \to L$, $F_A: X \times Q \to L$ denote the degree of truth membership function, degree of indeterminacy membership function and the degree of false membership function for each element (x, q) to the set A respectively, and $0 \leq T_A(x, q) \leq 1$; $0 \leq I_A(x, q) \leq 1$; $0 \leq F_A(x, q) \leq 1$, for all $x \in X$, and $q \in Q$.

**Definition 2.6:** A = < (x, q), $T_A(x, q)$, $I_A(x, q)$, $F_A(x, q)$ > and B = < ( x, q), $T_B(x, q)$, $I_B(x, q)$, $F_B(x, q)$ > are two neutrosophic (Q, L)-fuzzy sets on X. Then,

(i) $A \subseteq B$, if $T_A(x, q) \leq T_B(x, q)$, $I_A(x, q) \leq I_B(x, q)$, $F_A(x, q) \geq F_B(x, q)$, for all $x \in X$, and $q \in Q$.

(ii) $A \cup B = < (x, q), \max(T_A(x, q), T_B(x, q)), \max(I_A(x, q), I_B(x, q)), \min((F_A(x, q), F_B(x, q)) >$.

(iii) $A \cap B = < (x, q), \min(T_A(x, q), T_B(x, q)), \min(I_A(x, q), I_B(x, q)), \max((F_A(x, q), F_B(x, q)) >$.

**Definition 2.7:** The complement $A^c$ of a neutrosophic Q-fuzzy subset A on X is defined by
$A^c = \{< (x, q), T_{A^c}(x, q), I_{A^c}(x, q), F_{A^c}(x, q) > : x \in X, q \in Q$, where $T_{A^c}(x, q) = F_A(x, q)$,
$I_{A^c}(x, q) = 1 - I_A(x, q), F_{A^c}(x, q) = T_A(x, q)$, for all $x \in X$, and $q \in Q$.

Throughout the paper, (L, ≤) be a lattice.

## SECTION 3: PROPERTIES ON NEUTROSOPHIC (Q, L)-FUZZY SUBGROUP

**Definition 3.1:** Let (G, .) be a group and A be a neutrosophic (Q, L)-fuzzy subset in G. Then A is called a neutrosophic (Q, L)-fuzzy sub group of G (NQLFG) if it satisfies the conditions

(i) $T_A(xy, q) \geq T_A(x, q) \wedge (T_A(y, q)$, $I_A(xy, q) \geq I_A(x, q) \wedge (I_A(y, q)$, $F_A(xy, q) \leq F_A(x, q) \vee (F_A(y, q))$.

(ii) $T_A(x^{-1}, q) = T_A(x, q), I_A(x^{-1}, q) = I_A(x, q), F_A(x^{-1}, q) = F_A(x, q)$ ,for all $x, y \in G$, $q \in Q$.

**Theorem 3.2:** A neutrosophic (Q, L)-fuzzy subset A of G is a neutrosophic (Q, L)-fuzzy subgroup of a group G if and only if $T_A(xy^{-1}, q) \geq (T_A(x, q) \wedge (T_A(y, q), I_A(xy^{-1}, q) \geq (I_A(x, q) \wedge (I_A(y, q)$
$F_A(xy^{-1}, q) \leq (F_A(x, q) \vee (F_A(y, q))$.

**Proof:** Let A be a neutrosophic (Q, L)-fuzzy subgroup of G.
$\Leftrightarrow T_A(xy, q) \geq T_A(x, q) \wedge T_A(y, q), I_A(xy, q) \geq I_A(x, q) \wedge I_A(y, q), F_A(xy, q) \leq F_A(x, q) \vee F_A(y, q))$, and
$T_A(x^{-1}, q) = T_A(x, q), I_A(x^{-1}, q) = I_A(x, q), F_A(x^{-1}, q) = F_A(x, q)$ for all $x, y \in G$, and $q \in Q$.
$\Leftrightarrow T_A(xy^{-1}, q) \geq T_A(x, q) \wedge T_A(y, q), I_A(xy^{-1}, q) \geq I_A(x, q) \wedge I_A(y, q), F_A(xy^{-1}, q) \leq F_A(x, q) \vee F_A(y, q)$.

**Definition 3.3**: A neutrosophic (Q, L)-fuzzy subgroup A of a group G is a neutrosophic (Q, L)-fuzzy normal subgroup of G (NQLFNG) if $T_A(xy, q) = T_A(yx, q), I_A(xy, q) = I_A(yx, q), F_A(xy, q) = F_A(yx, q)$ or
$T_A(xyx^{-1}, q) = T_A(y, q), I_A(xyx^{-1}, q) = I_A(y, q), F_A(xyx^{-1}, q) = F_A(y, q)$ for all $x, y \in G$, $q \in Q$.

**Definition 3.4**: Let A be a neutrosophic (Q, L)-fuzzy subset of X. Let α, β, γ be in L. Then [α, β, γ]-level subset of (Q, L)-of A is defined by $[A]_{(\alpha, \beta, \gamma)} = \{x \in X, q \in Q: T_A(x, q) \geq \alpha, I_A(x, q) \geq \beta, F_A(x, q) \leq \gamma\}$.

**Theorem 3.5:** If A is a NQLFG of G and $\alpha, \beta, \gamma \in [0,1]$, then [α, β, γ]-level subset $[A]_{(\alpha, \beta, \gamma)}$ of A is a subgroup of G where $T_A(e, q) \geq \alpha, I_A(e, q) \geq \beta, F_A(e, q) \leq \gamma$, where e is the identity element of G, and $q \in Q$.

**Proof:** Since, $T_A(e, q) \geq \alpha, I_A(e, q) \geq \beta, F_A(e, q) \leq \gamma$, $e \in [A]_{(\alpha, \beta, \gamma)}$.

Therefore $[A]_{(\alpha, \beta, \gamma)} \neq \{ \}$. Let $x, y \in [A]_{(\alpha, \beta, \gamma)}$ and $q \in Q$. Then it follows that
$T_A(x, q) \geq \alpha, I_A(x, q) \geq \beta, F_A(x, q) \leq \gamma$, $T_A(y, q) \geq \alpha, I_A(y, q) \geq \beta, F_A(y, q) \leq \gamma$.

 

$\Leftrightarrow T_A(x,q) \wedge T_A(y,q) \geq \alpha, I_A(x,q) \wedge I_A(y,q) \geq \beta, F_A(x,q) \vee F_A(y,q) \leq \gamma$
$\Leftrightarrow T_A(xy^{-1},q) \geq \alpha, I_A(xy^{-1},q) \geq \beta, F_A(xy^{-1},q) \leq \gamma$
$\Leftrightarrow xy^{-1} \in [A]_{(\alpha, \beta, \gamma)}$.
$\Leftrightarrow [A]_{(\alpha, \beta, \gamma)}$ is a subgroup of G.

**Theorem 3.6:** If A is a neutrosophic Q-fuzzy subset of a group G, then A is a NQLFG of G if and only if $[A]_{(\alpha, \beta, \gamma)}$ is a subgroup of G for $\alpha, \beta, \gamma \in L$.

**Proof:** Let x, y $\in [A]_{(\alpha, \beta, \gamma)}$ and $q \in Q$.

Let A is a neutrosophic Q-fuzzy subgroup of G.
$\Leftrightarrow T_A(xy^{-1},q) \geq T_A(x,q) \wedge T_A(y,q), I_A(xy^{-1},q) \geq I_A(x,q) \wedge I_A(y,q),$
$F_A(xy^{-1},q) \leq F_A(x,q) \vee F_A(y,q))$
$\Leftrightarrow T_A(xy^{-1},q) \geq \alpha, , I_A(x,q) \geq \beta, F_A(xy^{-1},q) \leq \gamma$
$\Leftrightarrow xy^{-1} \in [A]_{(\alpha, \beta, \gamma)}$.
$\Leftrightarrow [A]_{(\alpha, \beta, \gamma)}$ is a subgroup of G.

**Theorem 3.7:** A is a neutrosophic Q-fuzzy subset of a group G. Then A is a neutrosophic Q-fuzzy normal subgroup of G if and only if $[A]_{(\alpha, \beta, \gamma)}$ is a normal subgroup of G.

**Proof:** Let A be a neutrosophic Q-fuzzy normal subgroup of G. Then, $T_A(xyx^{-1},q) = T_A(y,q) \geq \alpha, I_A(xyx^{-1},q) = I_A(y,q) \geq \beta, F_A(xyx^{-1},q) = F_A(y,q) \leq \gamma$, for all $x, y \in G$, $q \in Q$. Hence $[A]_{(\alpha, \beta, \gamma)}$ is a normal subgroup of G.

**Theorem 3.8:** If $A_1, A_2, \ldots, A_n$ be neutrosophic Q-fuzzy subgroups of G. Then A = $\cup_{i=1}^{n} A_i$ is a neutrosophic Q-fuzzy subgroup of G.

**Proof:** Let $A_1, A_2, \ldots, A_n$ be neutrosophic Q-fuzzy subgroups of G.

Let A = $\cup_{i=1}^{n} A_i, x, y \in G, q \in Q$.

Then , $A(xy^{-1},q) = \cup_{i=1}^{n} A_i(xy^{-1},q)$

$$= < (x,q), \left(T_{\cup_{i=1}^{n} A_i}(xy^{-1},q)\right), \left(I_{\cup_{i=1}^{n} A_i}(xy^{-1},q)\right), (F_{\cap_{i=1}^{n} A_i}(xy^{-1},q) >$$

$$\begin{aligned} T_{\cup_{i=1}^{n} A_i}(xy^{-1},q) &= \vee T_{A_i}(xy^{-1},q) \geq \vee (T_{A_i}(x,q) \wedge T_{A_i}(y,q)) \\ &= (\vee T_{A_i}(x,q)) \wedge (\vee T_{A_i}(y,q)) \\ &= T_{\cup_{i=1}^{n} A_i}(x,q) \wedge T_{\cup_{i=1}^{n} A_i}(y,q) \end{aligned}$$

$$\begin{aligned} I_{\cup_{i=1}^{n} A_i}(xy^{-1},q) &= \vee I_{A_i}(xy^{-1},q) \geq \vee (I_{A_i}(x,q) \wedge I_{A_i}(y,q)) \\ &= (\vee I_{A_i}(x,q)) \wedge (\vee I_{A_i}(y,q)) \\ &= I_{\cup_{i=1}^{n} A_i}(x,q) \wedge I_{\cup_{i=1}^{n} A_i}(y,q) \end{aligned}$$

$$\begin{aligned} F_{\cup_{i=1}^{n} A_i}(xy^{-1},q) &= \wedge F_{A_i}(xy^{-1},q) \leq \wedge (F_{A_i}(x,q) \vee F_{A_i}(y,q)) \\ &= (\wedge (F_{A_i}(x,q)) \vee (\wedge (F_{A_i}(y,q)) \\ &= F_{\cup_{i=1}^{n} A_i}(x,q) \vee F_{\cup_{i=1}^{n} A_i}(y,q) \end{aligned}$$

Hence, A = $\cup_{i=1}^{n} A_i$ is a neutrosophic Q-fuzzy subgroup of G.

**Theorem 3.9:** If $A_1, A_2, \ldots, A_n$ be neutrosophic Q-fuzzy subgroups of G. Then A = $\cap_{i=1}^{n} A_i$ is a neutrosophic Q-fuzzy subgroup of G.

**Proof:** Let $A_1, A_2, \ldots, A_n$ be neutrosophic Q-fuzzy subgroups of G.
Let A = $\cap_{i=1}^{n} A_i, x, y \in G, q \in Q$.

Then $A(xy^{-1},q) = \cap_{i=1}^{n} A_i(xy^{-1},q)$

$$= < (x,q), \left(T_{\cap_{i=1}^{n} A_i}(xy^{-1},q)\right), \left(I_{\cap_{i=1}^{n} A_i}(xy^{-1},q)\right), (F_{\cup_{i=1}^{n} A_i}(xy^{-1},q) > .$$

$$\begin{aligned} T_{\cap_{i=1}^{n} A_i}(xy^{-1},q) &= \wedge T_{A_i}(xy^{-1},q) \geq \wedge (T_{A_i}(x,q) \wedge T_{A_i}(y,q)) \\ &= (\wedge T_{A_i}(x,q)) \wedge (\wedge T_{A_i}(y,q)) \\ &= T_{\cap_{i=1}^{n} A_i}(x,q) \wedge T_{\cap_{i=1}^{n} A_i}(y,q). \end{aligned}$$

$$\begin{aligned}I_{\cap_{i=1}^{n}A_i}(xy^{-1},q) &= \wedge\, I_{A_i}(xy^{-1},q) \geq \wedge\,(I_{A_i}(x,q)\wedge I_{A_i}(y,q))\\ &= (\wedge\, I_{A_i}(x,q))\wedge(\wedge\, I_{A_i}(y,q))\\ &= I_{\cap_{i=1}^{n}A_i}(x,q)\wedge I_{\cap_{i=1}^{n}A_i}(y,q).\end{aligned}$$

$$\begin{aligned}F_{\cap_{i=1}^{n}A_i}(xy^{-1},q) &= \vee\, F_{A_i}(xy^{-1},q) \leq \vee\,(F_{A_i}(x,q)\vee F_{A_i}(y,q))\\ &= (\vee\,(F_{A_i}(x,q))\vee(\vee\,(F_{A_i}(y,q))\\ &= F_{\cap_{i=1}^{n}A_i}(x,q)\vee F_{\cap_{i=1}^{n}A_i}(y,q).\end{aligned}$$

Hence, A = $\cap_{i=1}^{n} A_i$ is a neutrosophic Q-fuzzy subgroup of G.

## SECTION 4: HOMOMORPHISM OF NEUTROSOPHIC Q-FUZZY SUBGROUPS

**Definition 4.1:** Let G, G’ be any two groups .The function f: G × Q → G’ × Q is a group Q-homomorphism if (i). f: G → G’ is a group homomorphism, and (ii). f(xy, q) = f(x. q) f(y, q) for all x, y ∈ G and q ∈ Q.

**Theorem 4.2:** Let G, G’ be any two groups and f be a homomorphism of G onto G’. If A is a NQLFG into of G’, then $f^{-1}(A)$ is a NQLFG of G.

**Proof:** Let A be a neutrosophic fuzzy subgroup of $G'$.

By definition, $f^{-1}(A) = (f^{-1}(T_A), f^{-1}(I_A), f^{-1}(F_A))$

Now for x, y ∈ G and q ∈ q, it gets that

$$\begin{aligned}f^{-1}(T_A)(xy^{-1},q) &= T_A(f(xy^{-1},q))\\ &= T_A(f(x)f(y^{-1}),q) \text{ (since f is a homomorphism)}\\ &\geq T_A(f(x),q)\wedge T_A(f(y^{-1}),q)\\ &= f^{-1}(T_A)(x,q)\wedge f^{-1}(T_A)(y^{-1},q).\end{aligned}$$

$$\begin{aligned}f^{-1}(I_A)(xy^{-1},q) &= I_A(f(xy^{-1},q))\\ &= I_A(f(x)f(y^{-1}),q) \text{ (since, f is a homomorphism)}\\ &\geq I_A(f(x),q)\wedge I_A(f(y^{-1}),q)\\ &= f^{-1}(I_A)(x,q)\wedge f^{-1}(I_A)(y^{-1},q)\\ &= f^{-1}(I_A)(x,q)\wedge f^{-1}(I_A)(y,q).\end{aligned}$$

$$\begin{aligned}f^{-1}(F_A)(xy^{-1},q) &= F_A(f(xy^{-1},q))\\ &= F_A(f(x)f(y^{-1}),q) \text{ (since, f is a homomorphism)}\\ &\leq F_A(f(x),q)\vee F_A(f(y^{-1}),q)\\ &= f^{-1}(F_A)(x,q)\wedge f^{-1}(F_A)(y^{-1},q)\\ &= f^{-1}(F_A)(x,q)\wedge f^{-1}(F_A)(y,q).\end{aligned}$$

Hence $f^{-1}(A)$is a NQLFG of G.

**Theorem 4.3:** Let X, Y be two groups and f be a homomorphism of X onto Y. If A is a neutrosophic (Q, L)-fuzzy subgroup of X, then f(A) is a neutrosophic (Q, L)-fuzzy subgroup of Y.

**Proof:** Let A be a neutrosophic Q-fuzzy subgroup of $X$.

By definition, $f(A) = (f(T_A), f(I_A), f(F_A))$

Now for $x_1, x_2 \in X$, $y_1, y_2 \in Y$ and q ∈ Q,

$$\begin{aligned}f(T_A)(y_1y_2,q) &= \sup_{x_1x_2\in f^{-1}(Y)} T_A(x_1x_2,q) \geq \sup_{x_1,x_2\in f^{-1}(Y)}(T_A(x_1,q)\wedge T_A(x_2,q)) \text{ (since A is a NQFSG )}\\ &= \sup_{x_1\in f^{-1}(Y)} T_A(x_1,q)\wedge \sup_{x_2\in f^{-1}(Y)} T_A(x_2,q)\\ &= f(T_A)(y_1,q)\wedge f(T_A)(y_2,q).\end{aligned}$$

$$f(T_A)(y^{-1},q) = \sup_{x^{-1}\in f^{-1}(Y)} T_A(x^{-1},q) = \sup_{x\in f^{-1}(Y)} T_A(x,q) = f(T_A)(y,q).$$

$$\begin{aligned}f(I_A)(y_1y_2,q) &= \sup_{x_1x_2\in f^{-1}(Y)} I_A(x_1x_2,q) \geq \sup_{x_1,x_2\in f^{-1}(Y)}(I(x_1,q)\wedge I_A(x_2,q)) \text{ (since A is a NQFSG )}\\ &= \sup_{x_1\in f^{-1}(Y)} I_A(x_1,q)\wedge \sup_{x_2\in f^{-1}(Y)} I_A(x_2,q)\\ &= f(I_A)(y_1,q)\wedge f(I_A)(y_2,q).\end{aligned}$$

$f(I_A)(y^{-1},q) = \sup_{x^{-1}\in f^{-1}(Y)} I_A(x^{-1},q) = \sup_{x\in f^{-1}(Y)} I_A(x,q) = f(I_A)(y,q).$

$f(F_A)(y_1y_2,q) = \inf_{x_1x_2\in f^{-1}(Y)} F_A(x_1x_2,q) \le \inf_{x_1,x_2\in f^{-1}(Y)}(F_A(x_1,q) \vee F_A(x_2,q))$ (since A is a NQFSG )

$= \inf_{x_1\in f^{-1}(Y)} F_A(x_1,q) \vee \inf_{x_2\in f^{-1}(Y)} F_A(x_2,q)$

$= f(F_A)(y_1,q) \wedge f(F_A)(y_2,q).$

$f(F_A)(y^{-1},q) = \inf_{x^{-1}\in f^{-1}(Y)} F_A(x^{-1},q) = \inf_{x\in f^{-1}(Y)} F_A(x,q) = f(F_A)(y,q).$

Therefore f(A) is a neutrosophic (Q, L)-fuzzy subgroup of Y.

**Lemma 4.4:** For all $a, b \in I$ and $i$ is any positive integer, if $a \le b, then$
$(i)(a)^i \le (b)^i (ii)\ (a \wedge b)^i = (a)^i \wedge (b)^i (iii)\ (a \vee b)^i = (a)^i \vee (b)^i$

**Theorem 4.5:** Let A be a NQLFG of a group G. Then $A^i = \{< (x,\ q), (T_A(x,\ q))^i, (I_A(x,\ q))^i,\ (F_A(x,q))^i >: x \in G, q \in Q\}$ is a NQLFG of G, where i is a positive integer.

**Proof:** Let A be a NQLFG of a group G.

Now for x, y ∈ G and q ∈ Q,

$T_{A^i}(xy^{-1},q) = (T_A(xy^{-1},q))^i$

$\ge (T_A(x,q) \wedge T_A(y,q))^i$

$= (T_A(x,q))^i \wedge (T_A(y,q))^i$

$= T_{A^i}(x,q) \wedge T_{A^i}(y,q).$

$I_{A^i}(xy^{-1},q) = (I_A(xy^{-1},q))^i$

$\ge (I_A(x,q) \wedge I_A(y,q))^i$

$= (I_A(x,q))^i \wedge (I_A(y,q))^i$

$= I_{A^i}(x,q) \wedge I_{A^i}(y,q).$

$F_{A^i}(xy^{-1},q) = (F_A(xy^{-1},q))^i$

$\le (F_A(x,q) \vee F_A(y,q))^i$

$= (F_A(x,q))^i \vee (F_A(y,q))^i$

$= F_{A^i}(x,q) \vee F_{A^i}(y,q).$

Therefore $A^i$ is a NQFSG of G.

## SECTION 5: DIRECT PRODUCT OF NEUTROSOPHIC Q-FUZZY SUBGROUPS

**Definition 5.1:** Let A, B be neutrosophic (Q, L)–fuzzy subsets of X and Y respectively. Then the cartesian product $A \times B$ of A and B is defined by

$$A \times B = \{< ((x,y),q), T_{A\times B}((x,y),q), I_{A\times B}((x,y),q), F_{A\times B}((x,y),q) >: x \in X, y \in Y, q \in Q\}$$

Where

$T_{A\times B}((x,y),q) = T_A(x,q) \wedge T_B(y,q))$, $I_{A\times B}((x,y),q) = I_A(x,q) \wedge I_B(y,q))$, and
$F_{A\times B}((x,y),q) = F_A(x,q) \vee F_B(y,q)$.

**Theorem 5.2:** If A, B are neutrosophic (Q, L)-fuzzy subgroups of groups X and Y respectively, then A × B is a neutrosophic Q-fuzzy subgroup of X × Y.

**Proof:** Let A and B be neutrosophic Q-fuzzy subgroups of the group X and Y respectively.

Now for $(x_1,y_1), (x_2,y_2) \in A \times B$, $q \in Q$

$T_{A\times B}\left((x_1,y_1)(x_2,y_2),q\right) = T_{A\times B}\left((x_1x_2,y_1y_2),q\right)$

$= T_A\left((x_1x_2,q)\right) \wedge T_B\left((y_1y_2,q)\right)$

$\ge [T_A(x_1,q) \wedge T_A(x_2,q)] \wedge [T_B(y_1,q) \wedge T_B(y_2,q)]$

(Since A and B are NQFSG)

$= T_A(x_1,q) \wedge T_B(y_1,q) \wedge T_A(x_2,q) \wedge T_B(y_2,q)$

$= T_{A\times B}((x_1,y_1),q) \wedge T_{A\times B}((x_2,y_2),q).$

$$T_{A\times B}((x,y)^{-1},q)) = T_{A\times B}((x^{-1},y^{-1}),q) = T_A(x^{-1},q) \wedge T_B(y^{-1},q)$$
$$= T_A(x,q) \wedge T_B(y,q)$$
$$= T_{A\times B}((x,y),q).$$

$$I_{A\times B}\left(\left(x_1,y_1\right)\left(x_2,y_2\right),q\right) = I_{A\times B}\left(\left(x_1x_2,y_1y_2\right),q\right)$$
$$= I_A\left(\left(x_1x_2,q\right)\right) \wedge I_B\left(\left(y_1y_2,q\right)\right)$$
$$\geq [I_A(x_1,q) \wedge I_A(x_2,q)] \wedge [I_B(y_1,q) \wedge I_B(y_2,q)]$$
(Since A and B are NQFSG)
$$= I_A(x_1,q) \wedge I_B(y_1,q) \wedge I_A(x_2,q) \wedge I_B(y_2,q)$$
$$= I_{A\times B}((x_1,y_1),q) \wedge I_{A\times B}((x_2,y_2),q).$$

$$I_{A\times B}((x,y)^{-1},q)) = I_{A\times B}((x^{-1},y^{-1}),q) = I_A(x^{-1},q) \wedge I_B(y^{-1},q)$$
$$= I_A(x,q) \wedge I_B(y,q)$$
$$= I_{A\times B}((x,y),q).$$

$$F_{A\times B}\left(\left(x_1,y_1\right)\left(x_2,y_2\right),q\right) = F_{A\times B}\left(\left(x_1x_2,y_1y_2\right),q\right)$$
$$= F_A\left(\left(x_1x_2,q\right)\right) \vee F_B\left(\left(y_1y_2,q\right)\right)$$
$$\leq [F_A(x_1,q) \vee F_A(x_2,q)] \vee [F_B(y_1,q) \vee F_B(y_2,q)]$$
(Since A and B are NQFSG)
$$= F_A(x_1,q) \vee F_B(y_1,q) \vee F_A(x_2,q) \vee F_B(y_2,q)$$
$$= F_{A\times B}((x_1,y_1),q) \vee F_{A\times B}((x_2,y_2),q).$$

$$F_{A\times B}((x,y)^{-1},q)) = F_{A\times B}((x^{-1},y^{-1}),q) = F_A(x^{-1},q) \vee F_B(y^{-1},q)$$
$$= F_A(x,q) \vee F_B(y,q)$$
$$= F_{A\times B}((x,y),q).$$

Hence $A \times B$ is a neutrosophic (Q,, L)-fuzzy subgroup of X × Y.

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