DEMONSTRATING THE RELATIONSHIP BETWEEN QUANTUM MECHANICS AND RELATIVITY

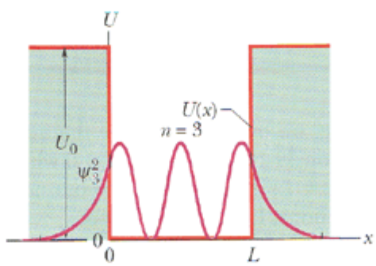
John Cipolla, January 9, 2019

Abstract

This paper postulates all massive objects from electrons to galaxies can be modeled as massless **superstrings** and the force of gravity is a vector component of string tension. Where, gravity, which is an emergent property of general relativity, is caused by tension in massless **supersymmetric strings** called **gravitons**. In addition, this paper attempts to unify Einstein's theory of general relativity and quantum mechanics or the wave nature of matter for a theory of quantum gravity. This paper also assumes all massive objects can be modeled as **heterotic** or closed strings and the force of gravity is a fundamental property of string theory. In this paper the relationship between string theory and general relativity are simplified by allowing the equations for general relativity to approach the Newtonian limit for small velocity. Background: The first particle string models were called **bosonic strings** because only **bosons** or force carriers like photons; gluons and the Higgs particles were modeled. Later, **superstring theory** was developed that predicted a connection or "super-symmetry" between bosons and **fermions** where fermions are elementary particles like electrons, protons and quarks that compose all ordinary matter.

Background

In this analysis we attempt to relate the gravitational forces predicted by string theory, Einstein's theory of general relativity and classical Newtonian gravity. Establishing the rudimentary relationship between general relativity and quantum mechanics requires a simple "model equation" that represents the basic physics but at a simplistic level. Using the concept of a "model equation" for this analysis is required because the full string theory equations are far too complex at the present time. This basic technique is used in computational fluid dynamics (CFD) where solution methods are tested using one-dimensional "model equations" and later extended to the full three dimensional nature of fluid flow.



For this analysis the one-dimensional string illustrated in Figure-2 represents the **wave equation**[1, 2] in Figure-3 and its solution. This is a notional string theory solution and not the "real" solution. But, the basic physics of any problem can sometimes be revealed using the "model equation" technique. The solution for the wave equation in Figure-3 reveals the traveling **wave velocity** of a disturbance on a string equals the square root of the **tension** on a string divided by the **linear mass density** of the string.

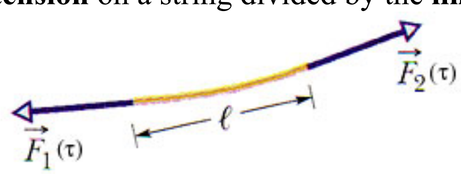


Figure-2, String segment

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad where \quad v = \sqrt{\tau/\mu}$$

Figure-3, String wave equation and solution

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad where \quad \psi(x) = Ae^{ikx} + Be^{-ikx}$$

Figure-4, Schrödinger's wave equation for a free particle[1] in a potential well and probability function, ψ

After acknowledging the **wave equation** in Figure-3 and its solution for wave velocity, v represents the physics of a vibrating string[3] it becomes clear the **Schrödinger's wave equation** for a free particle in Figure-4 has a clear analogy with the physics of a vibrating string. Therefore, its seems reasonable to approach the unification of general relativity and quantum mechanics by using physics of the string wave equation as a starting point to develop a model equation for a Theory of Everything (TOE).

GENERAL RELATIVITY IN THE NEWTONIAN LIMIT FOR $|v| \ll c$

Solutions using general relativity[4, 5] approach solutions using Newtonian mechanics when mass-energy generated gravitational fields produce velocities, v much less than the speed of light. $|\varphi| \ll 1$ and $|v| \ll c.$ For the low velocity approximation, general relativity must make the same predictions as Newtonian mechanics. In addition, it can be shown using tensor mathematics that the Einstein field equation described by Equation-1 for relativistic motion in strong gravity fields reduces to the Newtonian solution for $|v| \ll c$ in Equation-2 and Equation-3.

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}. \tag{1}$$

$$\nabla^2 \Phi = 4\pi G \rho. \tag{2}$$

The solution of Equation-2 for a single gravitating body in Newtonian mechanics clearly shows the following potential field equation describes Newtonian gravity when $|v| \ll c$.

$$\Phi = -\frac{G\,M}{r}. \tag{3}$$

Finally, the acceleration of gravity in the Newtonian limit is easily derived and is used in this analysis to determine the gravitational force of attraction and acceleration of gravity for comparing string theory to Newtonian mechanics when $|v| \ll c$.

$$g = \frac{\mathrm{d}\Phi}{\mathrm{dr}} = \frac{G\,M}{r^2}. \tag{4}$$

$$F = mg. \tag{5}$$

GRAVITATIONAL FORCE OF ATTRACTION USING CLASSICAL THEORY

Derivation of the non-relativistic gravitational field using the equations of general relativity in the **Newtonian limit** follows. Please note the form of $\boldsymbol{F} = \boldsymbol{ma}$[1, 2] reduce to Newtonian mechanics for $v \ll c$ in the equations of general relativity[4, 5] as previously discussed. The Newtonian limited equations of general relativity are defined when three requirements are achieved. First, particles are moving slowly relative to the speed of light. Second, gravitational fields are weak meaning space-time can be considered flat. Third, gravitational fields are static and unchanging. These derivations will illustrate the equivalence between gravitational fields around massive objects for classical mechanics and the gravitational fields generated by **heterotic** strings when $v \ll c$. The Newtonian limited equations of general relativity for the classical gravitational force of attraction between planet Earth and the Sun is known to be.

$$F_{Newtonian} = M_e \frac{G\,M_{sun}}{{R_{sun}}^2}. \tag{6}$$

Where, the classical Newtonian acceleration of gravity at radius, $R_{sun}$is.

$$g_{Newtonian} = \frac{G\,M_{sun}}{{R_{sun}}^2}. \tag{7}$$

GRAVITATIONAL FORCE OF ATTRACTION FOR A STRING

This analysis assumes a **heterotic** or closed superstring[3] circles the Sun at a distance equal to the orbital radius of the Earth around the Sun. The total mass of the circular superstring is equal to the mass of the Earth having uniform mass-energy density along the string. Motion of the Earth particle is notionally like that of matter-wave trapped in a potential well as depicted in Figure-1. In this case the probability of finding the Earth in any particular location around the Sun is described by Schrödinger's wave equation of a free particle[1] in a potential well where the probability function and its solution is described in Figure-4. In this case the potential well where the Earth is "trapped" is the gravitational potential well generated by the mass of the Sun which curves space-time as described by the Einstein field equation in Equation-1. Also, the energy required for the Earth to escape the potential well where it is "trapped" is equal to the escape velocity from the Earth's location, R_{sun} orbiting the Sun. The velocity, V_{escape} is critical to describe the exact solution required for this string solution to match classical theories of gravitation. The following derivation describes in detail how the string solution produces forces of gravitational attraction from tension in the superstring. First, the length of the matter-wave representing the Earth is derived from the geometrized mass of the Earth. The remaining steps develop the proposed string theory force of gravitational attraction and string theory gravitational acceleration toward the Sun. The equations for force of gravitational attraction and acceleration between each planet in the solar system and the Sun indicate excellent agreement between string theory and classical gravitational theory.

Length of string using **geometrized mass** length units for the Earth.

$$L_s = \frac{G\,M_e}{c^2}\,. \tag{8}$$

Linear mass density of string at distance, R_{sun} from the Sun is.

$$\mu = \frac{M_e}{L_s}. \tag{9}$$

String **wave velocity** set equal to the **escape velocity** from the solar system at R_{sun}.

$$v = V_{escape}. \tag{10}$$

Where, the escape velocity at distance R_{sun} is the following.

$$V_{escape} = \sqrt{\frac{2\,G\,M_{sun}}{R_{sun}}}\,. \tag{11}$$

Then, the well-known wave velocity[1] (Equation-1) of a heterotic string follows.

$$v = \sqrt{\frac{\tau}{\mu}}\,. \tag{12}$$

Solving, tension in a heterotic string due to gravitational force at radius, R_{sun} becomes.

$$\tau = \mu\,v^2. \tag{13}$$

Compute string force of attraction to the Sun by first determining string end-point slope.

$$tan(\alpha) = \frac{x}{\sqrt{{R_{sun}}^2 - x^2}}\,. \text{ Where, } x = \frac{L_s}{2}. \tag{14}$$

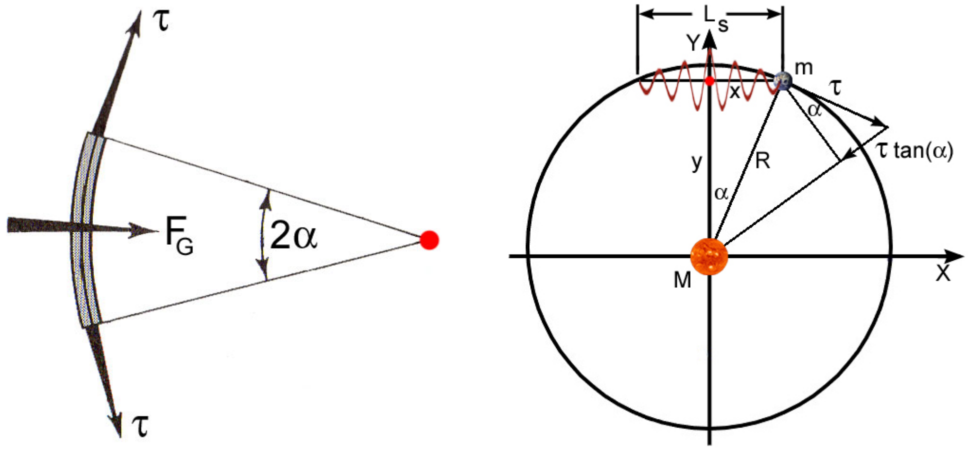


Figure-5, **String Theory Analysis**: Gravitational force from string tension, τ and end-point slope, tan (α)

Finally, string gravitational force of attraction and acceleration to the Sun is found to be.

$$F_{string} = \tau \tan(\alpha) \quad \text{And} \quad g_{string} = \frac{F_{string}}{M_e}. \tag{15}$$

After inserting the required input variables into the equation for $F_{Newtonian}$ and F_{string} a MathCAD analysis predicts an exact match between Newtonian results for gravitational force and acceleration and string theory gravitational force and acceleration.

$$\frac{F_{Newtonian}}{F_{string}} = \frac{F}{Fs} = 1 \quad \text{And} \quad \frac{g_{Newtonian}}{g_{string}} = \frac{G}{Gs} = 1. \tag{16}$$

Validation of the Proposed Relationship Using The Solar System

Figure-6 plots the ratio of gravitational force and acceleration computed using general relativity at the Newtonian limit to the gravitational force and acceleration computed by string theory for each planet in the solar system. Please notice the amazing result that the force of gravity and gravitational acceleration for Newtonian gravity and string theory are equal making the ratio of the gravitational results displayed in Figure-6 identically equal. These amazing results indicate the effects of gravity may be modeled as a heterotic or closed string as a first order model equation solution for a unifying theory for general relativity, quantum mechanics and string theory.

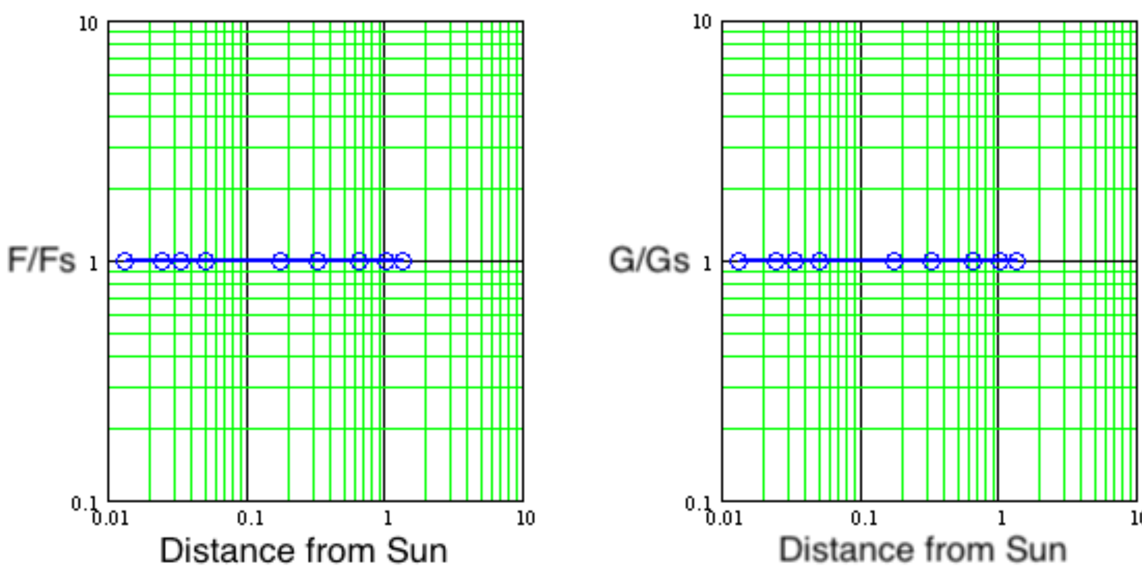


Figure-6, Gravitational force ratio (F/Fs) and acceleration of gravity ratio (G/Gs) verses distance from the Sun

SUMMARY OF RESULTS

In **superstring theory** the force of electromagnetism is caused by the exchange of photon particles between electrons, the weak force is caused by the exchange of W and Z boson particles and the strong force is caused by the exchange of gluon particles. Similarly, **quantum gravity** is the superstring theoretical interpretation that gravity is caused by the exchange of **graviton particles**. Where, the graviton is a massless string best described in Figure-5. This simplified string theory analysis indicates the tangential force acting in tension; τ on a heterotic or closed string having radius equal to R_{sun} from the Sun and planetary mass, m has an inward gravitational force, F_{string} equal to the gravitational force, $F_{Newtonian}$ acting on a planet. The force of gravity is the resulting inward component of the tangential force acting on a closed string in tension. These interesting results were achieved by using **geometrized mass** units for planetary **string length** and then by setting planetary **wave velocity** equal to **escape velocity** at each planetary location, R_{sun}. These results indicate quantum mechanics has an embedded field theory relationship to general relativity and string theory when $|\nu| \ll c$ in this analysis and for $|\nu|$ approaching the speed of light (c) when these results are extended into the relativistic regime. Finally, these results demonstrate that a relationship exits between quantum mechanics and general relativity and that string theory may provide a basis for describing the relationship between gravity, electromagnetism, strong force (nuclear force) and weak force for a theory of everything (TOE).

DERIVATION OF STRING FORCE OF GRAVITY

In Figure-7 tangential force, τ acting on each end of string segment, L_s has a transverse force F_{string} that keeps the string segment in equilibrium and acts as a force of compression toward mass, M. If a hypothetical string having length, L_s is considered an elastic band, the force; F_{string} is acting as a compressive force when the string segment is held in place at radius, R_{sun}. Where, F_{string} is the force of gravitational attraction to mass, M that is equivalent to the force of gravity, $F_{Newtonian}$. The result that Newtonian and string theory forces of attraction are equal strongly suggests there is a relationship between general relativity, Newtonian theory and string theory. The derivation of F_{string}, the force of attraction or gravitational force predicted by string theory follows.

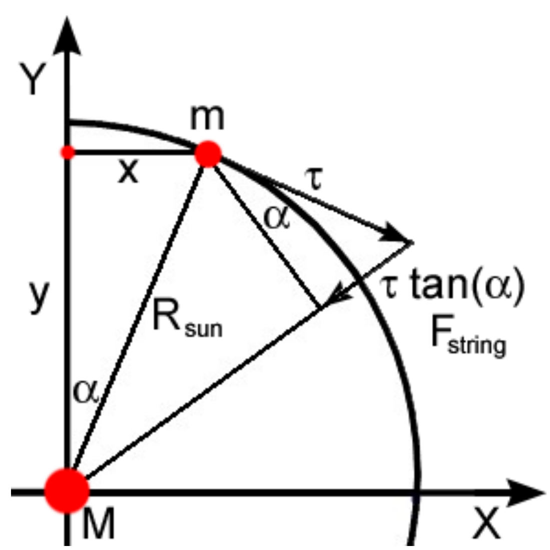


Figure-7, Derivation of F_{string}, the gravitational force of attraction due to string tension, τ

The equation for a circle[6] that represents a **heterotic** or closed string of mass, m circling around the Sun of mass, M follows.

$$x^2 + y^2 = R^2. \tag{17}$$

Solving for variable, y the equation for the Sun-orbiting circle becomes.

$$y = (R^2 - x^2)^{1/2}. \tag{18}$$

Slope of string end-points is determined by first substituting, u for the variable, y.

$$u = y. \tag{19}$$

String end-point slope is determined using equation-8 on page 328 of reference 6.

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}. \tag{20}$$

After finding the derivative the slope of each string end-point is.

$$\frac{dy}{dx} = -\frac{x}{\sqrt{R^2 - x^2}}. \tag{21}$$

Forces acting on the string toward the Sun are determined using string end-point slope.

$$\tau \tan\alpha = \tau \sin\alpha. \tag{22}$$

String end-point slope for a small angle approximation ($\alpha \to 0$) is.

$$\tan\alpha = \frac{dy}{dx}. \tag{23}$$

Numerically, string end-point slope is determined using Equation-21 and Equation-23.

$$tan(\alpha) = \frac{x}{\sqrt{{R_{sun}}^2 - x^2}}\ . \text{ Where, } x = \frac{L_s}{2}. \tag{24}$$

Then, after a little algebra, the string theory force of gravitational attraction, Equation-25 between the Sun and each planet in the solar system is easily determined. Equation-25 is used to determine string gravitational forces of attraction for each planet orbiting the Sun and compared to forces of gravitational attraction determined by general relativity and Newtonian theory when $|\nu| \ll c$. Finally, gravitational forces of attraction determined by general relativity are compared to string theory results as the ratio, $F_{Newtonian}/F_{string}$. Please see Figure-6, which illustrates an exact match between string theory and classical methods where the plotted ratio of gravitational forces of attraction and gravitational acceleration toward the Sun is exactly 1. These results indicate that classical methods of gravity are related to quantum mechanics and string theory.

$$F_{string} = \tau\ tan(\alpha). \tag{25}$$

When the following six substitutions is made into Equation-25 the final form for string theory force of gravitational attraction is finally displayed in Equation-27.

$$tan(\alpha), \tau = \mu\, \nu^2, \mu = \frac{M_e}{L_s}, \nu = V_{escape}, L_s = \frac{G\, M_e}{c^2}, \text{ and } V_{escape} = \sqrt{\frac{2\, G\, M_{sun}}{R_{sun}}}. \tag{26}$$

$$F_{string} = \frac{M_e \frac{G\, M_{sun}}{R_{sun}}}{\sqrt{{R_{sun}}^2 - \left(\frac{G\, M_e}{2\, c^2}\right)^2}}. \tag{27}$$

The new string theory force of gravitational attraction derived in Equation-27 is identical to the Newtonian force of gravitational attraction in Equation-6 except for the vanishingly small term in the denominator. These results indicate Newtonian, general relativity and string theory forces of gravitational attraction are exactly equal. The classical Newtonian equation of gravity is repeated for comparison to the string theory result in Equation-27.

$$F_{Newtonian} = M_e \frac{G\, M_{sun}}{{R_{sun}}^2}. \tag{6}$$

STRING THEORY PREDICTION

ROTATING BLACK HOLE (KERR) INNER/OUTER EVENT HORIZONS

The event horizon of a nonrotating black hole is called the Schwarzschild radius, r. Where the Schwarzschild radius for a nonrotating black hole is.

$$r = 2m. \tag{28}$$

Where, m is now the geometrized form of mass, M and not simply mass.

$$m = \frac{GM}{c^2}. \tag{29}$$

For a rotating (Kerr) black hole[4, 7] there are two event horizons according to general relativity. The outer event horizon of a Kerr black hole is located at.

$$r_+ = m + \sqrt{m^2 - a^2}. \tag{30}$$

And the inner event horizon called the Cauchy event horizon is located at.

$$r_- = m - \sqrt{m^2 - a^2}. \tag{31}$$

Using Equation-31, the angular momentum per unit mass of a rotating black hole is $a = \sqrt{3}m/2$ when the inner event horizon is located at $r_- = m/2$. The corresponding outer event horizon is located at $r_+ = 1.5\, m$ for black hole angular momentum, a. Interestingly, the string theory force of gravitational attraction expressed by Equation-27 for a **particle-wave** predicts the same inner event horizon and outer event horizon locations predicted by **general relativity** for a rotating black hole, when.

$$R_{sun} = \frac{GM_e}{2\, c^2} = \frac{m}{2}. \tag{32}$$

In these equations the angular momentum per unit mass, a is the angular momentum, J of a rotating black hole divided by black hole mass, M. Please see Figure-8 to see where the inner event horizon and outer event horizon for a rotating black hole are located. In addition, please notice the singularity at r = 0 is a disk and not a point for a true three-dimensional rotating black hole.

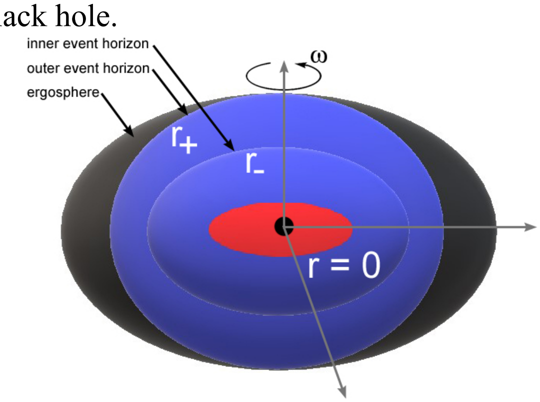


Figure-8, Rotating black hole structure, Inner/Outer event horizons predicted by string theory

POSSIBILITIES FOR QUANTUM GRAVITY TO EXPLAIN THE PASSAGE OF TIME AND SPACE

If general relativity and quantum mechanics or string theory were unified into one unified theory of **quantum gravity** an explanation for the concept and meaning of **time**[8] could allow a new unified theory to see within the singularity of black holes, the Big Bang and beyond the Big Bang into alternate universes. A unified theory of quantum gravity will allow general relativity to accommodate quantum mechanics in those regions or singularities where general relativity is not valid like the singularity of a black hole, the Big Bang or even peer into alternate universes. In fact, Stephen Hawking says information isn't destroyed when entering a black hole because information never disappears into the singularity. Instead, information is trapped at the **event horizon** or boundary of the black hole. At the event horizon, Stephen Hawking says information is stored as a 2-D hologram, where a hologram is a 2-D description of a 3-D object. Therefore, we could actually be living in a holographic universe that is reflected on the surface of a 3-D bubble of spacetime where the event horizon is where we live in what we call "present time".

To explain the passage of time, the author believes the mechanism "that causes the front edge of the universe to push forward" in time is due to certain quantum effects where the uncertainty of the future changes to the certainty of the past as the "uncertain future crystalizes into the past through a sequence of microscopic quantum events". Therefore, the present is defined as the point on the leading edge of spacetime where an infinite number of particles are being "observed" causing the leading edge of spacetime to collapse into a single randomly determined identity, the present. The precise state of a particle, its position and energy, is determined on the leading edge of the universe where a particle transforms from the uncertain quantum state to the measured quantum state having a rigid identity, the present. Therefore, a unified theory could explain the concept of time and so much more…

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NOTE: This is a work in progress and will be revised as new information is discovered