A Precise Value of the Hubble Constant in the Planck Model

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The matter density parameter Ω_m and Hubble constant H_0 appear together twice in the equations of the Planck Model. First, the entropy of the Hubble sphere is given by the product of Ω_m and the Bekenstein-Hawking entropy of a black hole of Hubble radius c/H_0. When Ω_m and H_0 take realistic values the entropy approximates closely to a special value within a binary scheme formulated for black holes. With the entropy set exactly to that special value, one may write down an equation in which H_0 is a function of Ω_m. Second, the dark energy density has a definite value that is related to the Bohr radius. Consequently, one may write down a second equation in which H_0 is a function of Ω_m. Solving the two equations simultaneously one finds that $\Omega_m = 0.358$ and $H_0 = 75.4$ km/s/Mpc.

Introduction

Planck CMB observations have resulted in a low-uncertainty evaluation of the Hubble constant H_0 at 67.8 ± 0.9 km/s/Mpc [1]. This value is at variance with that of Riess et al from HST observations of Cepheid variables and the measurement of redshifts and distances [2]: four distance calibrations resulted in values of H_0 between 72.04 and 76.18, with a central value of 73.24 ± 1.74 km/s/Mpc. The Planck Model—see [3] for an introduction—offers a way of determining the precise value of H_0.

The Entropy of the Hubble Sphere

As a measure of the information content within the Hubble horizon we have calculated the entropy of the Hubble sphere as the Bekenstein-Hawking entropy S_H of a black hole of Hubble radius c/H_0 multiplied by the matter density parameter Ω_m [4]. In natural units ($c = G = \hbar = 1$),

$$S_H = \frac{A}{4}.\Omega_m \tag{1}$$

where $A = 4\pi/{H_0}^2$. We can write

$$S_H = 8\pi.\frac{1}{8{H_0}^2}.\Omega_m \tag{2}$$

where the factor 8π is Bekenstein's area quantum [5]. We conjectured that, in (2),

$$\frac{1}{8{H_0}^2}.\Omega_m = 2^n \tag{3}$$

where 2^n is the number of states of n quantum bits. With $H_0 = 70$ km/s/Mpc and $\Omega_m = 0.3$—middling experimental values—we find that n = 399.96. The value n = 400 fits well in the binary scheme formulated for black holes, in which values of n that are multiples of 5 and especially 25 are favoured [4]. Powers that are multiples of 25 are a recurrent feature of the Planck Model [3]. With n = 400,

$$H_0 = \left(\frac{\Omega_m}{2^{403}}\right)^{1/2} \tag{4}$$

The Dark Energy Density

The dark energy density ρ_Λ has been conjectured to be equal to the zero point energy at Planck scale—notionally ½ in natural units—diluted in a 5-sphere of Bohr radius a_0 [6]:

$$\rho_\Lambda = ½ \, . \, a_0^{-5} \tag{5}$$

which has the value 1.32886×10^{-123} in natural units. Note that $a_0 = (\pi/2)^{125}$ in natural units [7].

Since $\rho_\Lambda = \Omega_\Lambda \, . \, 3H_0^2/8\pi$ and $\Omega_\Lambda = (1 - \Omega_m)$ we can write

$$\rho_\Lambda = (1 - \Omega_m) \, . \, 3H_0^2/8\pi \tag{6}$$

With H_0 = 70 km/s/Mpc and $\Omega_m = 0.3$ we find that $\rho_\Lambda = 1.25 \times 10^{-123}$. Setting ρ_Λ to 1.32886×10^{-123}, as in (5),

$$H_0 = \left[\frac{8\pi}{3} \, . \, \frac{1.32886 \times 10^{-123}}{(1 - \Omega_m)}\right]^{½} \tag{7}$$

The Value of the Hubble Constant in the Planck Model

Equations (4) and (7) are solved simultaneously in Figure 1. The Planck Model values of Ω_m and H_0 are found at the intersection of the two curves. We find that $\Omega_m = 0.358$ and H_0 = 75.4 km/s/Mpc.

References

1. Planck Collaboration, Planck 2015 Results. XIII. Cosmological Parameters, arXiv:1502.01589; Astronomy & Astrophysics 594, A13 (2016)
2. A. G. Riess et al, A 2.4% Determination of the Local Value of the Hubble Constant, arXiv:1604.01424
3. B. F. Riley, The Planck Model, viXra:1311.0053
4. B. F. Riley, Bekenstein-Hawking Entropy from Holography, viXra:1602.0305
5. J. D. Bekenstein, The Quantum Mass Spectrum of the Kerr Black Hole, Lett. Nuovo Cim. **11**, 467 (1974)
6. B. F. Riley, The Cosmological Constant from $AdS_5 \times S^5$, viXra:1307.0107
7. B. F. Riley, The Scales of Atomic and Particle Physics in Relation to Planck Scale, viXra:1305.0061

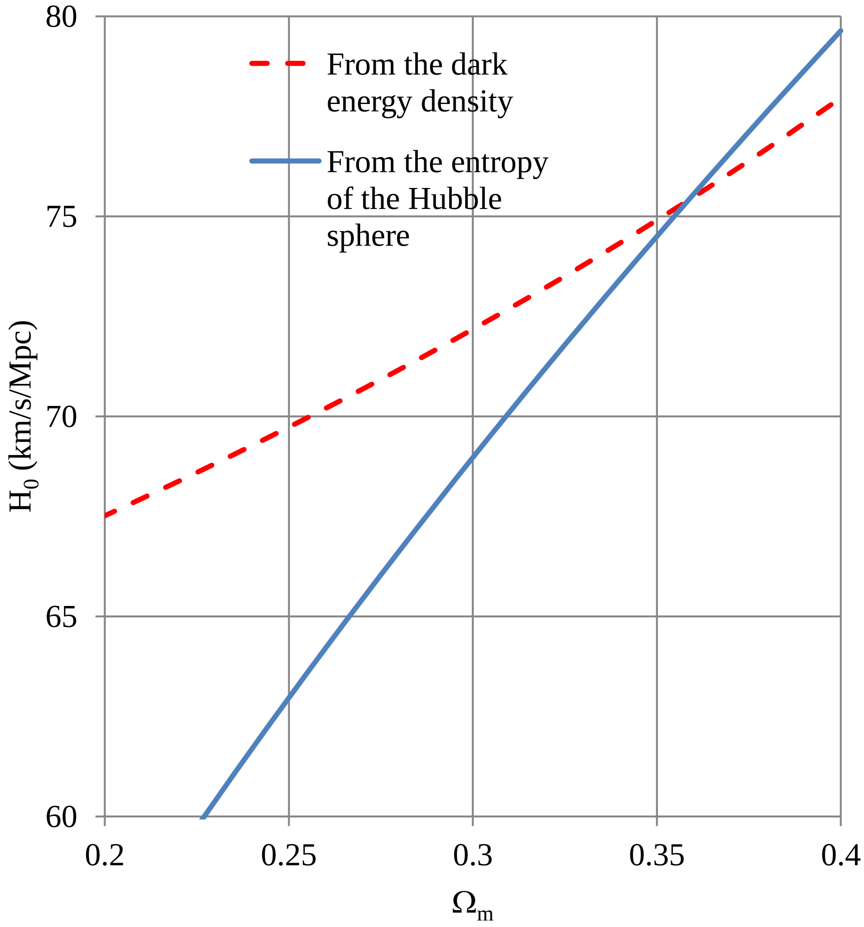

Figure 1: The Hubble constant H_0 as a function of the matter density parameter Ω_m from (i) the entropy of the Hubble sphere and (ii) the dark energy density