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ALGORITHMS FOR INTERVAL NEUTROSOPHIC MULTIPLE ATTRIBUTE DECISION-MAKING BASED ON MABAC, SIMILARITY MEASURE, AND EDAS

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In this paper, we define a new axiomatic definition of interval neutrosophic similarity measure, which is presented by interval neutrosophic number (INN). Later, the objective weights of various attributes are determined via Shannon entropy theory; meanwhile, we develop the combined weights, which can show both subjective information and objective information. Then, we present three approaches to solve interval neutrosophic decision-making problems by multi-attributive border approximation area comparison (MABAC), evaluation based on distance from average solution (EDAS), and similarity measure. Finally, the effectiveness and feasibility of algorithms are conceived by two illustrative examples.

KEY WORDS: *similarity measure, combined weights, interval neutrosophic set, MABAC, EDAS*

1. INTRODUCTION

The intuitionistic fuzzy set (IFS), poineered by Atanassov [1], is an extension of the fuzzy set [2] which can deal with the lack of knowledge of nonmembership degrees. IFS is summarized by a membership degree and a nonmembership degree, so it can describe the fuzzy character of data more constitutionally and minutely. However, IFSs can deal only with vague information, but not incongruous information which exists in a real environment. For example, when an expert gives the opinion about a certain statement, he or she may say that the possibility that the statement is true is 0.4, the degree of false statement is 0.5, and the possibility that he or she is not sure is 0.3. For incongruous information, Smarandache [3] initially presented the neutrosophic set from a philosophical point of view. A neutrosophic set (NS) is summarized by a truth-membership degree, an indeterminacy-membership degree, and a falsity-membership degree. It generalizes the concept of the classic set, fuzzy set (FS) [2], and tautological set [3]. Additionally, with regard to the aforementioned example about an expert statement, it can be expressed as (0.4, 0.5, 0.3) by NSs. Later, Rivieccio [4] pointed out that a NS is a set where each element of the universe has a truth-membership, an indeterminacy-membership, and a falsity-membership, and it lies in $]0^-, 1^+[$, the nonstandard unit interval. From a scientific point of view, the NS and set-theoretic operators should be specified. Otherwise, it will be hard to apply in real situations. Hence, Wang et al. [5] proposed a single-valued neutrosophic set (SVNS) which is a variation of a NS, and also introduced the set-theoretic operators. At present, SVNSs have attracted much attention and obtained some achievements [6–19].

In fact, sometimes the degree of truth, falsity, and indeterminacy of a certain statement cannot be defined exactly in the real situations but are denoted by several possible interval values. Hence, Wang et al. [20] proposed the seminal theory of interval neutrosophic sets (INSs) and presented the set-theoretic operators of INSs. Kraipeerapun and Fung [21] introduced an ensemble network and interval neutrosophic sets approach to the problem of binary classification. Kraipeerapun et al. [22] described the integration of neural network ensembles and interval neutrosophic sets using the bagging technique for predicting regional-scale potential for mineral deposits as well as quantifying uncertainty in the predictions. Lupiáñez [23] studied some relations between the interval neutrosophic set and its topology. Zhang et al. [24] proposed some operations and a comparison method for interval neutrosophic numbers (INNs), and applied them to multiple attribute decision-making (MADM). Ye [25] proposed some similarity measures for INSs, and applied them to MADM. Şahin and Karabacak [26] gave a system of axioms for inclusion measure of INSs and also developed a simple inclusion measure for ranking the INSs. Tian et al. [27] established two optimization models to determine the attribute weights in MADM situations where knowledge regarding the weight information is incomplete and the attribute values are INN. Zhang et al. [28] proposed an improved weighted correlation coefficient based on integrated weight for INSs. Zhao et al. [29] presented an interval neutrosophic MADM method based on generalized weighted aggregation operator. Ye [30] introduced an interval neutrosophic MADM method based on the possibility degree ranking method and ordered weighted aggregation operators of INNs. Liu and Tang [31] introduced some power generalized aggregation operators based on INSs. Liu and Wang [32] presented a MADM method based on a interval a neutrosophic prioritized OWA operator. A time-aware approach using INS to select cloud service was proposed by Ma et al. [33]. Zhang et al. [34] proposed a MADM method based on ELECTRE IV for INSs. Meanwhile, Liu et al. [35] also proposed a MADM method based on ELECTRE for INSs. Şahin [36] presented a cross-entropy measure on INSs. By considering credibility on every evaluation value of attributes in interval neutrosophic decision-making, Ye [37] proposed two credibility-induced interval neutrosophic weighted operators, and investigated their properties in detail. Yang et al. [38] generalized a linear assignment method to accommodate the interval neutrosophic sets based on the Choquet integral. Meanwhile, inspired by soft set theory [39], linguistic set theory [40] and hesitant fuzzy set theory [41], some extensional models such as interval neutrosophic soft set [42], interval neutrosophic linguistic set [43,44], and interval neutrosophic hesitant fuzzy set [45,46] are shown. Ye [47] also introduced the concept of simplified neutrosophic sets (SNSs), which can be described by three real numbers in the real unit interval [0, 1], and proposed a multiple attribute decision-making method using the aggregation operators of SNSs. Furthermore, We also introduced the concept of a simplified neutrosophic set (SNS), which is a subclass of a neutrosophic set and includes the concepts of INS and SVNS, and defined some operational laws of SNSs.

In order to compute the similarity measure of two INSs, we propose a new axiomatic definition of the similarity measure, which takes the form of INN. Comparing with the existing studies [25,28,48], our similarity measure can retain more original decision information.

Evaluation based on distance from average solution (EDAS), originally proposed by Ghorabaee et al. [49], is a new MADM method for inventory ABC classification. It is very useful when we have some conflicting parameters. The desirable alternative has a smaller distance from the ideal solution and a greater distance from the nadir solution in these MADM methods. Ghorabaee et al. [50] extended the EDAS method to supplier selection.

The multiattributive border approximation area comparison (MABAC) method is a novel method presented in [51]. It has a systematic process, simple computation procedure, and sound logic. Inspired by Pythagorean fuzzy sets [52,53], Peng and Yang [54] applied the MABAC to R&D project selection for obtaining the best project.

Considering that different attribute weights will determine the ranking results of alternatives, we study a new method to determine the attribute weights by combining the subjective factor with the objective factor. This model is different from the existing methods, which can be divided into two parts: one is the subjective weighting determination methods and the other is the objective weighting determination methods, which can be calculated by the Shannon entropy method [55]. The subjective weighting methods focus on the preference information of the decision-maker [7,24,25,28,29,37], while they ignore the objective information. The objective weighting does not take the preference of the decision-maker into account; in other words, the similar methods fail to take the risk attitude of the decision-maker into account [27]. The function of our model can show both the subjective information and the objective information. Hence, a novel combined model to obtain attribute weights is proposed.

As far as we know, however, the study of the decision-making problem based on proposed similarity measure, EDAS, and MABAC methods has not been reported in the existing academic literature. Therefore, it is a glamorous research topic to apply similarity measure, EDAS, and MABAC methods in decision-making to rank and obtain the best alternative under an interval neutrosophic environment. Meanwhile, through a comparison analysis of the existing algorithms, their objective information is executed, and the approach which maintains consistency of its results is determined.

The remainder of this paper is organized as follows: In Section 2, we review some fundamental concepts of NS, SVNS, and INS. In Section 3, a new axiomatic definition of interval neutrosophic similarity measure and distance measure is investigated. In Section 4, three decision approaches based on MABAC, EDAS, and similarity measure under interval neutrosophic environment are shown. In Section 5, two illustrative examples are proved to state the proposed methods. In Section 6, we compare the novel proposed approaches with the existing interval neutrosophic decision-making approaches. The paper makes a conclusion in Section 7.

2. PRELIMINARIES

In this section, we first recall some basic ideas of NS, SVNS, and INS, and their properties.

2.1 Interval Neutrosophic Set

NS is a portion of neutrosophy, which researches the origin and domain of neutralities, as well as their interactions with diverse ideational scope [3], and is a convincing general formal framework, which extends the presented sets [1,2] from a philosophical point of view. Smarandache [3] introduced the definition of NS as follows:

Definition 1 ([3])**.** Let X be a universe of discourse, with a class of elements in X denoted by x. A NS B in X is summarized by a truth-membership function $T_B(x)$, an indeterminacy-membership function $I_B(x)$, and a falsity-membership function $F_B(x)$. The functions $T_B(x), I_B(x)$, and $F_B(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$. That is, $T_B(x) : X \to]0^-, 1^+[$, $I_B(x) : X \to]0^-, 1^+[$, and $F_B(x) : X \to]0^-, 1^+[$.

There is restriction on the sum of $T_B(x), I_B(x)$, and $F_B(x)$, so $0^- \leq \sup T_B(x) + \sup I_B(x) + \sup F_B(x) \leq 3^+$.

As mentioned above, it is hard to apply the NS to solve some real problems. Hence, Wang et al. [5] presented SVNS, which is a subclass of the NS and mentioned the definition as follows:

Definition 2 ([5])**.** Let X be a universe of discourse, with a class of elements in X denoted by x. A SVNS N in X is summarized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. Then a SVNS N can be denoted as follows:

$$N = \{< x, T_N(x), I_N(x), F_N(x) >| \ x \in X\}, \tag{1}$$

where $T_N(x), I_N(x), F_N(x) \in [0, 1]$ for $\forall x \in X$. Meanwhile, the sum of $T_N(x), I_N(x)$, and $F_N(x)$ fulfills the condition $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$. For a SVNS N in X, the triplet $(T_N(x), I_N(x), F_N(x))$ is called the single-valued neutrosophic number (SVNN). For convenience, we can simply use $x = (T_x, I_x, F_x)$ to represent a SVNN as an element in the SVNS N.

Definition 3 ([20])**.** Let X be a universe of discourse, with a class of elements in X denoted by x. An INS A in X is summarized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then an INS A can be denoted as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >| \ x \in X\}. \tag{2}$$

For each point x in X, $T_A(x) = [T_A^L(x), T_A^U(x)], I_A(x) = [I_A^L(x), I_A^U(x)], F_A(x) = [F_A^L(x), F_A^U(x)] \subseteq [0, 1]$, and $0 \leq T_A^U(x) + I_A^U(x) + F_A^U(x) \leq 3$. For convenience, we can simply use $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ to represent an INN as an element in the INS A.

Definition 4 ([3])**.** An INS N is contained in other INS M, $N \subseteq M$ if and only if $T_N^L(x) \leq T_M^L(x), T_N^U(x) \leq T_M^U(x), I_N^L(x) \geq I_M^L(x), I_N^U(x) \geq I_M^U(x), F_N^L(x) \geq F_M^L(x), F_N^U(x) \geq F_M^U(x)$ for $\forall x$.

Definition 5 ([24])**.** Let $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two INNs, and $\lambda > 0$; then the operations for the INNs are defined as follows:

(1) $\lambda x_1 = ([1 - (1 - T_1^L)^\lambda, 1 - (1 - T_1^U)^\lambda], [(I_1^L)^\lambda, (I_1^U)^\lambda], [(F_1^L)^\lambda, (F_1^U)^\lambda])$;

(2) $x_1^\lambda = ([(T_1^L)^\lambda, (T_1^U)^\lambda], [1-(1-I_1^L)^\lambda, 1-(1-I_1^U)^\lambda], [1-(1-F_1^L)^\lambda, 1-(1-F_1^U)^\lambda]);$

(3) $x_1 \bigoplus x_2 = ([T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L * I_2^L, I_1^U * I_2^U], [F_1^L * F_2^L, F_1^U * F_2^U]);$

(4) $x_1 \bigotimes x_2 = ([T_1^L * T_2^L, T_1^U * T_2^U], [I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U]);$

(5) $x_1^c = ([F_1^L, F_1^U], [1-I_1^U, 1-I_1^L], [T_1^L, T_1^U]).$

Theorem 1 ([24]). *Let* $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ *and* $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ *be two INNs, and* $\lambda, \lambda_1, \lambda_2 > 0$*, then we have*

(1) $x_1 \bigoplus x_2 = x_2 \bigoplus x_1$;

(2) $x_1 \bigotimes x_2 = x_2 \bigotimes x_1$;

(3) $\lambda(x_1 \bigoplus x_2) = \lambda x_1 \bigoplus \lambda x_2$;

(4) $(x_1 \bigotimes x_2)^\lambda = x_1^\lambda \bigotimes x_2^\lambda$;

(5) $\lambda_1 x_1 \bigoplus \lambda_2 x_1 = (\lambda_1 + \lambda_2) x_1$;

(6) $x_1^{\lambda_1} \bigotimes x_1^{\lambda_2} = x_1^{\lambda_1 + \lambda_2}$.

Definition 6 ([25]). Let $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two INNs; then the Hamming distance between x_1 and x_2 can be defined as follows:

$$d_h(x_1, x_2) = \frac{1}{6}(\mid T_1^L - T_2^L \mid + \mid T_1^U - T_2^U \mid + \mid I_1^L - I_2^L \mid + \mid I_1^U - I_2^U \mid + \mid F_1^L - F_2^L \mid + \mid F_1^U - F_2^U \mid). \quad (3)$$

For comparing two INNs, Liu and Tang [31] introduced a cosine similarity measure method for an INN.

Definition 7 ([31]). Let $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ be an INN; then the cosine similarity measure is defined as follows:

$$\cos(x, x^*) = \frac{T^L + T^U}{\sqrt{2((T^L)^2 + (T^U)^2 + (I^L)^2 + (I^U)^2 + (F^L)^2 + (F^U)^2)}}. \quad (4)$$

It measures the cosine similarity measure between $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ and the ideal solution $x^* = ([1,1],[0,0],[0,0])$ for the comparison of INNs. Suppose that two INNs $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$; if $\cos(x_1, x^*) \le \cos(x_2, x^*)$, then $x_1 \le x_2$.

However, we can find some drawbacks of $\cos(x, x^*)$ when we compare two INNs.

(1) For two INNs x_1 and x_2, if $x_1 = ([0,0],[0,0],[0,0])$ and $x_2 = ([0,0],[0,0],[0,0])$, then $\cos(x_1, x^*)$ and $\cos(x_2, x^*)$ are undefined or insignificant. In this case, one cannot apply it to compare x_1 and x_2. In fact, if $x_1 = ([0,0],[0,0],[0,0])$ and $x_2 = ([0,0],[0,0],[0,0])$, then $x_1 = x_2$.

(2) For two INNs $x_1 = ([0,0],[0,0],[1,1])$ and $x_2 = ([0,0],[1,1],[0,0])$, then $\cos(x_1, x^*) = \cos(x_2, x^*) = 0$. In fact, $x_1 \ne x_2$.

We also can find more unreasonable results when $T_1^L = T_2^L$ and $T_1^U = T_2^U$, meanwhile, by obtaining the special values of $(I_1^L)^2 + (I_1^U)^2 + (F_1^L)^2 + (F_1^U)^2 = (I_2^L)^2 + (I_2^U)^2 + (F_2^L)^2 + (F_2^U)^2$. For example, $x_1 = ([0.4, 0.5],[0.6, 0.8],[0.6, 0.7])$ and $x_2 = ([0.4, 0.5],[0,1],[0.6, 0.7])$; based on Liu and Tang [31], $\cos(x_1, x^*) = \cos(x_2, x^*)$, but in fact, $x_1 \ne x_2$.

(3) For two INNs $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $x_2 = ([k * T_1^L, k * T_1^U], [k * I_1^L, k * I_1^U], [k * F_1^L, k * F_1^U])(0 < k < 1)$, we can know that $\cos(x_1, x^*) = \cos(x_2, x^*)$. But in fact, $x_1 \ne x_2$.

From the above discussion, it is unreasonable to apply to MADM. In order to solve these disadvantages, we propose a score function (improved similarity measure) in the following.

Definition 8. Let $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ be an INN; then the proposed score function $s_{1,1}(x)$ is defined as follows:

$$s_{1,1}(x) = \frac{2}{3} + \frac{T^L + T^U}{6} - \frac{I^L + I^U}{6} - \frac{F^L + F^U}{6}. \quad (5)$$

It measures the Hamming similarity $(1 - d_h(x, x^*))$ between $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ and the ideal solution $x^* = ([1, 1], [0, 0], [0, 0])$ for the comparison of INNs.

It also brings us to the problem of cases 2 and 3, so we can add two parameters α and β to adjust the results. In our intuition, we hope that the smaller of the $[I^L, I^U]$ (more indeterminate or inconsistent information) and the bigger of the $[F^L, F^U]$ when $[T^L, T^U]$ are equal, so the INN is bigger. Hence, we define a better score function as follows:

$$s_{\alpha,\beta}(x) = \frac{2}{3} + \frac{T^L + T^U}{6} - \alpha\frac{I^L + I^U}{6} - \beta\frac{F^L + F^U}{6}, \quad (6)$$

where $0 < \beta < \alpha \leq 1, 0 \leq s_{\alpha,\beta}(x) \leq 1$. Meanwhile, when the equal condition $(s_{\alpha,\beta}(x_1) = s_{\alpha,\beta}(x_2))$, in fact, it cannot hold $(x_1 \neq x_2)$. We can adjust the parameters α and β to obtain ideal results. In the following section, we set $\alpha = 0.5, \beta = 0.3$.

Definition 9 ([24])**.** Let $x_j (j = 1, 2, \cdots, n)$ be a series of the INNs, and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $x_j (i = 1, 2, \cdots, n)$; then an interval neutrosophic weighted averaging (INWA) operator is a mapping INWA, $X^n \to X$, where

$$\text{INWA}(x_1, x_2, \cdots, x_n) = \bigoplus_{j=1}^{n}(w_j x_j)$$
$$= \left(\left[1 - \prod_{j=1}^{n}(1 - T_j^L)^{w_j}, 1 - \prod_{j=1}^{n}(1 - T_j^U)^{w_j}\right], \left[\prod_{j=1}^{n}(I_j^L)^{w_j}, \prod_{j=1}^{n}(I_j^U)^{w_j}\right], \left[\prod_{j=1}^{n}(F_j^L)^{w_j}, \prod_{j=1}^{n}(F_j^U)^{w_j}\right]\right). \quad (7)$$

However, we can see that the INWA operator has drawbacks in some cases, described as follows.

Let $x_j (j = 1, 2, \cdots, n)$ be a series of INNs. If there is i such that $x_i = ([1, 1], [0, 0], [0, 0])$, then based on Eq. (7), we can have $\text{INWA}(x_1, x_2, \cdots, x_n) = ([1, 1], [0, 0], [0, 0])$. This result may cause counterintuitive phenomena in MADM. In other words, it only determines by x_i to make a decision and the decision information of others can be neglected.

Moreover, based on Eq. (7), if there is an INN such that $x_i = ([T_i^L, T_i^U], [0, 0], [0, 0])$, the aggregated value is $\text{INWA}(x_1, x_2, \cdots, x_n) = ([T^L, T^U], [0, 0], [0, 0])$. In other words, the indeterminacy-membership degree and the falsity-membership degree of aggregated value must be zero. This result may cause counterintuitive phenomena in some cases.

Hence, it is unreasonable and unsuitable to apply Eq. (7) to aggregate the information in MADM when meeting the special cases mentioned above. Meanwhile, for interval neutrosophic Hamacher average operators [24], interval neutrosophic Einstein average operators [24], interval neutrosophic prioritized average operators [32], and interval neutrosophic power average operators [31], they have the same drawbacks with interval neutrosophic average operators.

Definition 10 ([24])**.** Let $x_j (j = 1, 2, \cdots, n)$ be a series of the INNs, and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $x_j (i = 1, 2, \cdots, n)$; then an interval neutrosophic weighted geometric (INWG) operator is a mapping INWG: $X^n \to X$, where

$$\text{INWG}(x_1, x_2, \cdots, x_n) = \bigotimes_{j=1}^{n} x_j^{w_j} \quad (8)$$
$$= \left(\left[\prod_{j=1}^{n}\left(T_j^L\right)^{w_j}, \prod_{j=1}^{n}\left(T_j^U\right)^{w_j}\right], \left[1 - \prod_{j=1}^{n}\left(1 - I_j^L\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - I_j^U\right)^{w_j}\right], \left[1 - \prod_{j=1}^{n}\left(1 - F_j^L\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - F_j^U\right)^{w_j}\right]\right).$$

However, we can see that the INWG operator also has drawbacks in some cases, described as follows.

Let $x_j (j = 1, 2, \cdots, n)$ be a series of INNs. If there is i such that $x_i = ([0, 0], [1, 1], [1, 1])$, then based on Eq. (8), we can have $\text{INWG}(x_1, x_2, \cdots, x_n) = ([0, 0], [1, 1], [1, 1])$. This result may cause counterintuitive phenomena in MADM. In other words, it only determines by x_i to make a decision and the decision information of others can be neglected.

Moreover, based on Eq. (8), if there is an INN such that $x_i = ([0, 0], [I_i^L, I_i^U], [F_i^L, F_i^U])$, the aggregated value is $\text{INWA}(x_1, x_2, \cdots, x_n) = ([0, 0], [I^L, I^U], [F^L, F^U])$. In other words, the truth-membership degree of aggregated value must be zero. This result may cause counterintuitive phenomena in some cases.

Hence, it is unreasonable and unsuitable to apply Eq. (8) to aggregate the information in MADM when meeting the special cases mentioned above. Meanwhile, for interval neutrosophic Hamacher geometric operators [24], interval neutrosophic Einstein geometric operators [24], interval neutrosophic prioritized geometric operators [32], and interval neutrosophic power geometric operators [31], they have the same drawbacks with interval neutrosophic geometric operators.

For solving the above drawbacks, we propose a revised aggregation operator in the following.

Definition 11. Let $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ be an INN; then

$$x_T^\Delta = \begin{cases} \left([\Delta, \Delta], \left[\mid I^L - \frac{\Delta}{2} \mid, \mid I^U - \frac{\Delta}{2} \mid\right], \left[\mid F^L - \frac{\Delta}{2} \mid, \mid F^U - \frac{\Delta}{2} \mid\right]\right), & \text{if } T^L = T^U = 0, \\ \left([\Delta, T^U], \left[\mid I^L - \frac{\Delta}{2} \mid, I^U\right], \left[\mid F^L - \frac{\Delta}{2} \mid, F^U\right]\right), & \text{if } T^L = 0, T^U \neq 0, \\ ([T^L, T^U], [I^L, I^U], [F^L, F^U]), & \text{if } T^L \neq 0, \end{cases} \tag{9}$$

$$x_I^\Delta = \begin{cases} \left(\left[\mid T^L - \frac{\Delta}{2} \mid, \mid T^U - \frac{\Delta}{2} \mid\right], [\Delta, \Delta], \left[\mid F^L - \frac{\Delta}{2} \mid, \mid F^U - \frac{\Delta}{2} \mid\right]\right), & \text{if } I^L = I^U = 0, \\ \left(\left[\mid T^L - \frac{\Delta}{2} \mid, T^U\right], [\Delta, I^U], \left[\mid F^L - \frac{\Delta}{2} \mid, F^U\right]\right), & \text{if } I^L = 0, I^U \neq 0, \\ ([T^L, T^U], [I^L, I^U], [F^L, F^U]), & \text{if } I^L \neq 0, F^L \neq 0, \\ \left(\left[\mid T^L - \Delta \mid, \mid T^U - \Delta \mid\right], \left[\frac{\Delta}{2}, \frac{\Delta}{2}\right], \left[\frac{\Delta}{2}, \frac{\Delta}{2} \mid\right]\right), & \text{if } I^L = I^U = F^L = F^U = 0, \end{cases} \tag{10}$$

$$x_F^\Delta = \begin{cases} \left(\left[\mid T^L - \frac{\Delta}{2} \mid, \mid T^U - \frac{\Delta}{2} \mid\right], \left[\mid F^L - \frac{\Delta}{2} \mid, \mid F^U - \frac{\Delta}{2} \mid\right], [\Delta, \Delta]\right), & \text{if } F^L = F^U = 0, \\ \left(\left[\mid T^L - \frac{\Delta}{2} \mid, T^U\right], \left[\mid I^L - \frac{\Delta}{2} \mid, I^U\right], [\Delta, F^U]\right), & \text{if } F^L = 0, F^U \neq 0, \\ ([T^L, T^U], [I^L, I^U], [F^L, F^U]), & \text{if } F^L \neq 0, I^L \neq 0, \\ \left(\left[\mid T^L - \Delta \mid, \mid T^U - \Delta \mid\right], \left[\frac{\Delta}{2}, \frac{\Delta}{2}\right], \left[\frac{\Delta}{2}, \frac{\Delta}{2} \mid\right]\right), & \text{if } I^L = I^U = F^L = F^U = 0, \end{cases} \tag{11}$$

where Δ is a positive fuzzy number and far less than any nonzero $T^L, T^U, I^L, I^U, F^L, F^U$. Then x_T^Δ is called the Δ-revised interval neutrosophic geometric number of x; x_I^Δ and x_F^Δ are called the Δ-revised interval neutrosophic averaging number of x.

Theorem 2. *Let $x_j = ([T_j^L, T_j^U], [I_j^L, I_j^U], [F_j^L, F_j^U])$ be a series of INNs; x_T^Δ, x_I^Δ, and x_F^Δ are still INNs.*

Proof.

(1) When $T_j^L \neq 0, I_j^L \neq 0$, and $F_j^L \neq 0$, it meets the condition in Definition 3.

(2) When $T_j^L = 0, T_j^U \neq 0$, or $I_j^L = 0, I_j^U \neq 0$, or $F_j^L = 0, F_j^U \neq 0$, it meets the condition in Definition 3.

(3) When $T_j^L = 0, T_j^U = 0$, the revised INN $x_T^\Delta = ([\Delta, \Delta], [| I_j^L - \Delta/2 |, | I_j^U - \Delta/2 |], [| F_j^L - \Delta/2 |, | F_j^U - \Delta/2 |])$. Because Δ is far less than any nonzero I_j^U, F_j^U, then $\Delta + | I_j^U - \Delta/2 | + | F_j^U - \Delta/2 | = \Delta + I_j^U - \Delta/2 + F_j^U - \Delta/2 = I_j^U + F_j^U \leq 3$. Similarly, when $I_j^L = 0, I_j^U = 0$ or $F_j^L = 0, F_j^U = 0$, it also meets the condition in Definition 3.

Based on above analysis, we can conclude that x_T^Δ, x_I^Δ, and x_F^Δ are still INNs. □

Definition 12. Let $x_j = ([T_j^L, T_j^U], [I_j^L, I_j^U], [F_j^L, F_j^U])(j = 1, 2, \cdots, n)$ be a series of the INNs, then the Δ-revised interval neutrosophic weighted geometric operator (R-INWG) is defined as follows:

$$\text{R-INWG}(x_1, x_2, \cdots, x_n) = \bigoplus_{j=1}^{n} (x_j^\Delta)^{w_j}, \tag{12}$$

where w_j is the weight of $x_j(i = 1, 2, \cdots, n), w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$, x_j^Δ is the Δ-revised INN of x_j.

Theorem 3. *Let $x_j(j = 1, 2, \cdots, n)$ be a series of the INNs, and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $x_j(i = 1, 2, \cdots, n)$; then a revised interval neutrosophic weighted geometric (R-INWG) operator is a mapping R-INWG: $X^n \to X$, where*

$$\text{R-INWG}(x_1, x_2, \cdots, x_n) = \left(\left[\prod_{j=1}^{n} \left((T_j^L)^\Delta \right)^{w_j}, \prod_{j=1}^{n} \left((T_j^U)^\Delta \right)^{w_j} \right], \right. \tag{13}$$
$$\left. \left[1 - \prod_{j=1}^{n} \left(1 - (I_j^L)^\Delta \right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - (I_j^U)^\Delta \right)^{w_j} \right], \left[1 - \prod_{j=1}^{n} \left(1 - (F_j^L)^\Delta \right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - (F_j^U)^\Delta \right)^{w_j} \right] \right).$$

Theorem 4. *Let $x_j = ([T_j^L, T_j^U], [I_j^L, I_j^U], [F_j^L, F_j^U])(j = 1, 2, \cdots, n)$ be a series of the INNs, $x_j \neq ([0, 0], [0, 0], [0, 0])(j = 1, 2, \cdots, n)$ and the aggregation result by Eq.(13) is $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$, then T^L and T^U are monotonically increasing when Δ is monotonically increasing and $I_j^L, I_j^U, F_j^L, F_j^U$ are monotonically decreasing when Δ is monotonically increasing.*

Proof. Divide the possible value into two cases (case 1 and case 2) when $T_j^L = T_j^U = 0$. Case 1 has k_1 IVNNs which has revised by Eq.(9) with $j \in m_{k_1}$. Case 2 has k_2 IVNNs which keep original values with $j \in m_{k_2}$. It is obvious that $k_1 + k_2 = n$.

Based on Eq. (13), we can have

$$\text{R-INWG}(x_1, x_2, \cdots, x_n) = \left(\left[\prod_{j=1}^{n} \left((T_j^L)^\Delta \right)^{w_j}, \prod_{j=1}^{n} \left((T_j^U)^\Delta \right)^{w_j} \right], \left[1 - \prod_{j=1}^{n} \left(1 - (I_j^L)^\Delta \right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - (I_j^U)^\Delta \right)^{w_j} \right], \right.$$
$$\left. \left[1 - \prod_{j=1}^{n} \left(1 - (F_j^L)^\Delta \right)^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - (F_j^U)^\Delta \right)^{w_j} \right] \right) = \left(\left[\prod_{j \in m_{k_1}} \Delta^{w_j} \prod_{j \in m_{k_2}} (T_j^L)^{w_j}, \prod_{j \in m_{k_1}} \Delta^{w_j} \prod_{j \in m_{k_2}} (T_j^U)^{w_j} \right], \right.$$
$$\left[1 - \prod_{j \in m_{k_1}} \left(1 - |I_j^L - \frac{\Delta}{2}| \right)^{w_j} \prod_{j \in m_{k_2}} (1 - I_j^L)^{w_j}, 1 - \prod_{j \in m_{k_1}} \left(1 - |I_j^U - \frac{\Delta}{2}| \right)^{w_j} \prod_{j \in m_{k_2}} (1 - I_j^U)^{w_j} \right],$$
$$\left. \left[1 - \prod_{j \in m_{k_1}} \left(1 - |F_j^L - \frac{\Delta}{2}| \right)^{w_j} \prod_{j \in m_{k_2}} (1 - F_j^L)^{w_j}, 1 - \prod_{j \in m_{k_1}} \left(1 - |F_j^U - \frac{\Delta}{2}| \right)^{w_j} \prod_{j \in m_{k_2}} (1 - F_j^U)^{w_j} \right] \right).$$

For truth-membership function T, and letting $\Delta_1 < \Delta_2$, then

$$\prod_{j\in m_{k_1}}\Delta_1^{w_j}\prod_{j\in m_{k_2}}\left(T_j^L\right)^{w_j}-\prod_{j\in m_{k_1}}\Delta_2^{w_j}\prod_{j\in m_{k_2}}\left(T_j^L\right)^{w_j}=\left(\prod_{j\in m_{k_1}}\Delta_1^{w_j}-\prod_{j\in m_{k_1}}\Delta_2^{w_j}\right)\prod_{j\in m_{k_2}}\left(T_j^L\right)^{w_j}\leq 0,$$

$$\prod_{j\in m_{k_1}}\Delta_1^{w_j}\prod_{j\in m_{k_2}}\left(T_j^U\right)^{w_j}-\prod_{j\in m_{k_1}}\Delta_2^{w_j}\prod_{j\in m_{k_2}}\left(T_j^U\right)^{w_j}=\left(\prod_{j\in m_{k_1}}\Delta_1^{w_j}-\prod_{j\in m_{k_1}}\Delta_2^{w_j}\right)\prod_{j\in m_{k_2}}\left(T_j^U\right)^{w_j}\leq 0.$$

Hence, we can conclude that T^L and T^U are monotonically increasing when Δ is monotonically increasing. □

For indeterminacy-membership function I, and letting $\Delta_1 < \Delta_2$, then

$$1-\prod_{j\in m_{k_1}}\left(1-|I_j^L-\frac{\Delta_1}{2}|\right)^{w_j}\prod_{j\in m_{k_2}}\left(1-I_j^L\right)^{w_j}-\left[1-\prod_{j\in m_{k_1}}\left(1-|I_j^L-\frac{\Delta_2}{2}|\right)^{w_j}\prod_{j\in m_{k_2}}\left(1-I_j^L\right)^{w_j}\right]$$
$$\leq\prod_{j\in m_{k_2}}\left(1-I_j^L\right)^{w_j}\left[\prod_{j\in m_{k_1}}\left(1-|I_j^L-\frac{\Delta_2}{2}|\right)^{w_j}-\prod_{j\in m_{k_1}}\left(1-|I_j^L-\frac{\Delta_1}{2}|\right)^{w_j}\right]$$

$(\Delta_1, \Delta_2$ are real numbers and far less than any nonzero I_j, i.e., $I_j > \Delta_1, I_j > \Delta_2)$

$$\leq\prod_{j\in m_{k_2}}\left(1-I_j^L\right)^{w_j}\left[\prod_{j\in m_{k_1}}\left(1-I_j^L+\frac{\Delta_2}{2}\right)^{w_j}-\prod_{j\in m_{k_1}}\left(1-I_j^L+\frac{\Delta_1}{2}\right)^{w_j}\right]\geq 0,$$

$$1-\prod_{j\in m_{k_1}}\left(1-|I_j^U-\frac{\Delta_1}{2}|\right)^{w_j}\prod_{j\in m_{k_2}}\left(1-I_j^U\right)^{w_j}-\left[1-\prod_{j\in m_{k_1}}\left(1-|I_j^U-\frac{\Delta_2}{2}|\right)^{w_j}\prod_{j\in m_{k_2}}\left(1-I_j^U\right)^{w_j}\right]$$
$$\leq\prod_{j\in m_{k_2}}\left(1-I_j^U\right)^{w_j}\left[\prod_{j\in m_{k_1}}\left(1-|I_j^U-\frac{\Delta_2}{2}|\right)^{w_j}-\prod_{j\in m_{k_1}}\left(1-|I_j^U-\frac{\Delta_1}{2}|\right)^{w_j}\right]$$
$$\leq\prod_{j\in m_{k_2}}\left(1-I_j^U\right)^{w_j}\left[\prod_{j\in m_{k_1}}\left(1-I_j^U+\frac{\Delta_2}{2}\right)^{w_j}-\prod_{j\in m_{k_1}}\left(1-I_j^U+\frac{\Delta_1}{2}\right)^{w_j}\right]\geq 0.$$

Hence, we can conclude that I_j^L and I_j^U are monotonically decreasing when Δ is monotonically increasing.

Similarly, for falsity-membership function F and letting $\Delta_1 < \Delta_2$, then we can conclude that F_j^L and F_j^U are monotonically decreasing when Δ is monotonically increasing. In this paper, we set $\Delta = 0.0001$.

Definition 13. Let $x_j = ([T_j^L, T_j^U], [I_j^L, I_j^U], [F_j^L, F_j^U])(j = 1, 2, \cdots, n)$ be a series of the INNs, then the Δ-revised interval neutrosophic weighted averaging (R-INWA) operator is defined as follows:

$$\text{R-INWA}(x_1, x_2, \cdots, x_n) = \bigoplus_{j=1}^{n} w_j x_j^{\Delta}, \tag{14}$$

where w_j is the weight of $x_j(i = 1, 2, \cdots, n), w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$, x_j^{Δ} is the Δ-revised INN of x_j.

Theorem 5. *Let $x_j(j = 1, 2, \cdots, n)$ be a series of the INNs, and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $x_j(i = 1, 2, \cdots, n)$; then a revised interval neutrosophic weighted averaging (R-INWA) operator is a mapping R-INWA, $X^n \to X$, where*

$$\text{R-INWA}(x_1, x_2, \cdots, x_n) = \left(\left[1 - \prod_{j=1}^{n}\left(1 - \left(T_j^L\right)^{\Delta}\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \left(T_j^U\right)^{\Delta}\right)^{w_j}\right],\right.$$
$$\left.\left[\prod_{j=1}^{n}\left(\left(I_j^L\right)^{\Delta}\right)^{w_j}, \prod_{j=1}^{n}\left(\left(I_j^U\right)^{\Delta}\right)^{w_j}\right], \left[\prod_{j=1}^{n}\left(\left(F_j^L\right)^{\Delta}\right)^{w_j}, \prod_{j=1}^{n}\left(\left(F_j^U\right)^{\Delta}\right)^{w_j}\right]\right). \tag{15}$$

Theorem 6. *Let $x_j = ([T_j^L, T_j^U], [I_j^L, I_j^U], [F_j^L, F_j^U])(j = 1, 2, \cdots, n)$ be a series of the INNs, $x_j \neq ([0,0], [0,0], [0,0])$ $(j = 1, 2, \cdots, n)$ and the aggregation result by Eq. (15) is $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$; then T^L and T^U are monotonically decreasing when Δ is monotonically increasing and $I_j^L, I_j^U, F_j^L, F_j^U$ are monotonically increasing when Δ is monotonically increasing.*

3. A NEW INTERVAL NEUTROSOPHIC DISTANCE MEASURE AND SIMILARITY MEASURE

Definition 14. Let N_1, N_2, and N_3 be three INSs on X. A distance measure $D^{\Delta}(N_1, N_2)$ is a mapping D^{Δ}, INS$(X) \times$ INS$(X) \rightarrow$ INN, possessing the following properties:

(1) $D^{\Delta}(N_1, N_2)$ is an INN;

(2) $D^{\Delta}(N_1, N_2) = ([0,0], [1,1], [1,1])$, if $N_1 = N_2$;

(3) $D^{\Delta}(N_1, N_2) = D^{\Delta}(N_2, N_1)$;

(4) If $N_1 \subseteq N_2 \subseteq N_3$, then $D^{\Delta}(N_1, N_2) \subseteq D^{\Delta}(N_1, N_3)$ and $D^{\Delta}(N_2, N_3) \subseteq D^{\Delta}(N_1, N_3)$.

Theorem 7. *Let N_i and N_k be two INSs; then $D^{\Delta}(N_i, N_k)$ is a distance measure.*

$$D^{\Delta}(N_i, N_k) = \left(\left[\sum_{j=1}^{n} w_j \min\left\{\mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid\right\}, \sum_{j=1}^{n} w_j \max\left\{\mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid\right\}\right],\right.$$
$$\left[1 - \sum_{j=1}^{n} w_j \max\left\{\mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid\right\}, 1 - \sum_{j=1}^{n} w_j \min\left\{\mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid\right\}\right],$$
$$\left.\left[1 - \sum_{j=1}^{n} w_j \max\{\mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid\}, 1 - \sum_{j=1}^{n} w_j \min\left\{\mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid\right\}\right]\right), \tag{16}$$

where w_j is the weight of the jth INN.

Proof. In order for $D^{\Delta}(N_i, N_k)$ to be qualified as a sensible distance measure for INSs, it must satisfy (1)–(4) of axiomatic requirements.

(1) Since

$$0 \leq \sum_{j=1}^{n} w_j \min\left\{\mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid\right\} \leq \sum_{j=1}^{n} w_j \max\left\{\mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid\right\} \leq 1,$$
$$0 \leq 1 - \sum_{j=1}^{n} w_j \max\left\{\mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid\right\} \leq 1 - \sum_{j=1}^{n} w_j \min\left\{\mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid\right\} \leq 1,$$
$$0 \leq 1 - \sum_{j=1}^{n} w_j \min\left\{\mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid\right\} \leq 1 - \sum_{j=1}^{n} w_j \max\left\{\mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid\right\} \leq 1,$$

so $D^{\Delta}(I_i, I_k)$ is an INN.

(2) **Necessity:**
Since $D^{\Delta}(N_i, N_k) = ([0,0], [1,1], [1,1])$, we have

$$\sum_{j=1}^{n} w_j \text{min}\left\{ \mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid \right\} = 0, \quad \sum_{j=1}^{n} w_j \text{max}\left\{ \mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid \right\} = 0,$$
$$1 - \sum_{j=1}^{n} w_j \text{max}\left\{ \mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid \right\} = 1, \quad 1 - \sum_{j=1}^{n} w_j \text{min}\left\{ \mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid \right\} = 1,$$
$$1 - \sum_{j=1}^{n} w_j \text{max}\left\{ \mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid \right\} = 1, \quad 1 - \sum_{j=1}^{n} w_j \text{min}\left\{ \mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid \right\} = 1.$$

Based on the randomicity of w_j, we can have $T_{ij}^L = T_{kj}^L, T_{ij}^U = T_{kj}^U, I_{ij}^L = I_{kj}^L, I_{ij}^U = I_{kj}^U, F_{ij}^L = F_{kj}^L, F_{ij}^U = F_{kj}^U$ for $\forall j$.
Hence, $N_i = N_k$.

Sufficiency:
Since $N_i = N_k$, we have $T_{ij}^L = T_{kj}^L, T_{ij}^U = T_{kj}^U, I_{ij}^L = I_{kj}^L, I_{ij}^U = I_{kj}^U, F_{ij}^L = F_{kj}^L, F_{ij}^U = F_{kj}^U$.
Furthermore,

$$\sum_{j=1}^{n} w_j \text{min}\left\{ \mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid \right\} = 0, \quad \sum_{j=1}^{n} w_j \text{max}\left\{ \mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid \right\} = 0,$$
$$1 - \sum_{j=1}^{n} w_j \text{max}\left\{ \mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid \right\} = 1, \quad 1 - \sum_{j=1}^{n} w_j \text{min}\left\{ \mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid \right\} = 1,$$
$$1 - \sum_{j=1}^{n} w_j \text{max}\left\{ \mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid \right\} = 1, \quad 1 - \sum_{j=1}^{n} w_j \text{min}\left\{ \mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid \right\} = 1.$$

Hence, $D^{\Delta}(N_i, N_k) = ([0, 0], [1, 1], [1, 1])$.

(3) It is straightforward.

(4) If $N_1 \subseteq N_2 \subseteq N_3$, then $\forall j, T_{1j}^L \le T_{2j}^L \le T_{3j}^L, T_{1j}^U \le T_{2j}^U \le T_{3j}^U, I_{1j}^L \ge I_{2j}^L \ge I_{3j}^L, I_{1j}^U \ge I_{2j}^U \ge I_{3j}^U, F_{1j}^L \ge F_{2j}^L \ge F_{3j}^L$ and $F_{1j}^U \ge F_{2j}^U \ge F_{3j}^U$.
Hence, $\mid T_{1j}^L - T_{2j}^L \mid \le \mid T_{1j}^L - T_{3j}^L \mid, \mid T_{1j}^U - T_{2j}^U \mid \le \mid T_{1j}^U - T_{3j}^U \mid, \mid I_{1j}^L - I_{2j}^L \mid \ge \mid I_{1j}^L - I_{3j}^L \mid, \mid I_{1j}^U - I_{2j}^U \mid \ge \mid I_{1j}^U - I_{3j}^U \mid, \mid F_{1j}^L - F_{2j}^L \mid \ge \mid F_{1j}^L - F_{3j}^L \mid, \mid F_{1j}^U - F_{2j}^U \mid \ge \mid F_{1j}^U - F_{3j}^U \mid$.
Furthermore,

$$\sum_{j=1}^{n} w_j \text{min}\left\{ \mid T_{1j}^L - T_{2j}^L \mid, \mid T_{1j}^U - T_{2j}^U \mid \right\} \le \sum_{j=1}^{n} w_j \text{min}\left\{ \mid T_{1j}^L - T_{3j}^L \mid, \mid T_{1j}^U - T_{3j}^U \mid \right\},$$
$$\sum_{j=1}^{n} w_j \text{max}\left\{ \mid T_{1j}^L - T_{2j}^L \mid, \mid T_{1j}^U - T_{2j}^U \mid \right\} \le \sum_{j=1}^{n} w_j \text{max}\left\{ \mid T_{1j}^L - T_{3j}^L \mid, \mid T_{1j}^U - T_{3j}^U \mid \right\},$$
$$1 - \sum_{j=1}^{n} w_j \text{max}\left\{ \mid I_{1j}^L - I_{2j}^L \mid, \mid I_{1j}^U - I_{2j}^U \mid \right\} \ge 1 - \sum_{j=1}^{n} w_j \text{max}\left\{ \mid I_{1j}^L - I_{3j}^L \mid, \mid I_{1j}^U - I_{3j}^U \mid \right\},$$
$$1 - \sum_{j=1}^{n} w_j \text{min}\left\{ \mid I_{1j}^L - I_{2j}^L \mid, \mid I_{1j}^U - I_{2j}^U \mid \right\} \ge 1 - \sum_{j=1}^{n} w_j \text{min}\left\{ \mid I_{1j}^L - I_{2j}^L \mid, \mid I_{1j}^U - I_{2j}^U \mid \right\},$$
$$1 - \sum_{j=1}^{n} w_j \text{max}\left\{ \mid F_{1j}^L - F_{2j}^L \mid, \mid F_{1j}^U - F_{2j}^U \mid \right\} \ge 1 - \sum_{j=1}^{n} w_j \text{max}\left\{ \mid F_{1j}^L - F_{3j}^L \mid, \mid F_{1j}^U - F_{3j}^U \mid \right\},$$
$$1 - \sum_{j=1}^{n} w_j \text{min}\left\{ \mid F_{1j}^L - F_{2j}^L \mid, \mid F_{1j}^U - F_{2j}^U \mid \right\} \ge 1 - \sum_{j=1}^{n} w_j \text{min}\left\{ \mid F_{1j}^L - F_{2j}^L \mid, \mid F_{1j}^U - F_{2j}^U \mid \right\}.$$

Consequently, $D^{\Delta}(N_1, N_2) \subseteq D^{\Delta}(N_1, N_3)$.
Similarly, $D^{\Delta}(N_2, N_3) \subseteq D^{\Delta}(N_1, N_3)$. □

Definition 15. Let N_1, N_2, and N_3 be three INSs on X. A similarity measure $S^{\Delta}(N_1, N_2)$ is a mapping S^{Δ}, $\text{INS}(X) \times \text{INS}(X) \to \text{INN}$, possessing the following properties:

(1) $S^{\Delta}(N_1, N_2)$ is an INN;

(2) $S^{\Delta}(N_1, N_2) = ([1,1],[0,0],[0,0])$, if $N_1 = N_2$;

(3) $S^{\Delta}(N_1, N_2) = S^{\Delta}(N_2, N_1)$;

(4) If $N_1 \subseteq N_2 \subseteq N_3$, then $S^{\Delta}(N_1, N_3) \subseteq S^{\Delta}(N_1, N_2)$ and $S^{\Delta}(N_1, N_3) \subseteq S^{\Delta}(N_2, N_3)$.

Theorem 8. *Let N_i and N_k be two INSs; then $S^{\Delta}(N_i, N_k)$ is a similarity measure.*

$$\begin{aligned} S^{\Delta}(N_i, N_k) = \Bigg(&\left[1 - \sum_{j=1}^{n} w_j \max\left\{\mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid\right\}, 1 - \sum_{j=1}^{n} w_j \min\left\{\mid T_{ij}^L - T_{kj}^L \mid, \mid T_{ij}^U - T_{kj}^U \mid\right\}\right], \\ &\left[\sum_{j=1}^{n} w_j \min\left\{\mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid\right\}, \sum_{j=1}^{n} w_j \max\left\{\mid I_{ij}^L - I_{kj}^L \mid, \mid I_{ij}^U - I_{kj}^U \mid\right\}\right], \\ &\left[\sum_{j=1}^{n} w_j \min\left\{\mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid\right\}, \sum_{j=1}^{n} w_j \max\left\{\mid F_{ij}^L - F_{kj}^L \mid, \mid F_{ij}^U - F_{kj}^U \mid\right\}\right]\Bigg), \end{aligned} \tag{17}$$

where w_j is the weight of the jth INN.

Especially, for any two INNs x_1 and x_2, the similarity measure between $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ is defined as follows:

$$\begin{aligned} S^{\Delta}(N_i, N_k) = \big(&\left[1 - \max\left\{\mid T_1^L - T_2^L \mid, \mid T_1^U - T_2^U \mid\right\}, 1 - \min\left\{\mid T_1^L - T_2^L \mid, \mid T_1^U - T_2^U \mid\right\}\right], \\ &\left[\min\left\{\mid I_1^L - I_2^L \mid, \mid I_1^U - I_2^U \mid\right\}, \max\left\{\mid I_1^L - I_2^L \mid, \mid I_1^U - I_2^U \mid\right\}\right], \\ &\left[\min\left\{\mid F_1^L - F_2^L \mid, \mid F_1^U - F_2^U \mid\right\}, \max\left\{\mid F_1^L - F_2^L \mid, \mid F_1^U - F_2^U \mid\right\}\right]\big). \end{aligned} \tag{18}$$

If $x_1 = x_2$, then $S^{\Delta}(x_1, x_2) = ([1,1],[0,0],[0,0])$, i.e., the similarity is the biggest; if $x_1 = ([1,1],[0,0],[0,0])$, $x_2 = ([0,0],[1,1],[1,1])$ or $x_1 = ([0,0],[1,1],[1,1])$, $x_2 = ([1,1],[0,0],[0,0])$, then $S^{\Delta}(x_1, x_2) = ([0,0],[1,1],[1,1])$, i.e., the similarity is the smallest.

Based on the above Definitions 14 and 15, and Theorems 7 and 8, a direct argument proves the following proposition.

Proposition 1. Let N_i and N_k be two INSs; then

(1) $S^{\Delta}(N_i, N_k) = S^{\Delta}(N_i \cap N_k, N_i \cup N_k)$;

(2) $S^{\Delta}(N_i, N_i \cap N_k) = S^{\Delta}(N_k, N_i \cup N_k)$;

(3) $S^{\Delta}(N_i, N_i \cup N_k) = S^{\Delta}(N_k, N_i \cap N_k)$;

(4) $D^{\Delta}(N_i, N_k) = D^{\Delta}(N_i \cap N_k, N_i \cup N_k)$;

(5) $D^{\Delta}(N_i, N_i \cap N_k) = D^{\Delta}(N_k, N_i \cup N_k)$;

(6) $D^{\Delta}(N_i, N_i \cup N_k) = D^{\Delta}(N_k, N_i \cap N_k)$.

4. THREE ALGORITHMS FOR INTERVAL NEUTROSOPHIC DECISION-MAKING

4.1 Problem Description

Let $A = \{A_1, A_2, ..., A_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, ..., C_n\}$ be a series of n attributes, and $W = \{w_1, w_2, ..., w_n\}$ be weight vector assigned for the attributes by the decision-makers with $w_j \in [0,1], \Sigma_{j=1}^n w_j = 1$. Assume that the evaluation of the alternative A_i with respect to attribute C_j is represented by interval neutrosophic matrix $R = (r_{ij})_{m\times n} = ([T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U])_{m\times n} (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$. The values united with the alternatives for MADM problems can be shown in Table 1.

TABLE 1: The interval neutrosophic MADM matrix

	C_1	C_2	$\cdots$	C_n
A_1	r_{11}	r_{12}	$\cdots$	r_{1n}
A_2	r_{21}	r_{22}	$\cdots$	r_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	r_{m1}	r_{m2}	$\cdots$	r_{mn}

4.2 The Method of Computing the Combined Weights

4.2.1 Determining the Objective Weights: Shannon Entropy Method

Shannon entropy [55] evaluates the expected information content of a certain message. The degree of uncertainty in information can be measured using the entropy concept. The information entropy idea can regulate the decision making process because it is able to measure existent contrasts between sets of data and thus clarify the intrinsic information for the decision-maker.

The following procedure should be employed to determine integrated weights through Shannon entropy under an interval neutrosophic environment.

Step 1. Normalize decision matrix $R = (r_{ij})_{m\times n}$ into $\widetilde{R} = (\widetilde{r}_{ij})_{m\times n} = ([\widetilde{T}_{ij}^L, \widetilde{T}_{ij}^U], [\widetilde{I}_{ij}^L, \widetilde{I}_{ij}^U], [\widetilde{F}_{ij}^L, \widetilde{F}_{ij}^U])_{m\times n}$ by Eq. (19):

$$\widetilde{r}_{ij} = \begin{cases} ([T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U]), & C_j \text{ is benefit attribute,} \\ ([F_{ij}^L, F_{ij}^U], [1 - I_{ij}^U, 1 - I_{ij}^L], [T_{ij}^L, T_{ij}^U]), & C_j \text{ is cost attribute.} \end{cases} \quad (19)$$

Step 2. Compute the score function $s_{\alpha,\beta}(\widetilde{r}_{ij})$ of $\widetilde{r}_{ij}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ by Eq. (6).

Step 3. Normalize the score function $s_{\alpha,\beta}(\widetilde{r}_{ij})$ by $P_{ij} = s_{\alpha,\beta}(\widetilde{r}_{ij})/\sum_{i=1}^{m} s_{\alpha,\beta}(\widetilde{r}_{ij})$.

Step 4. Calculate the entropy measure of the score function of the normalized decision matrix as follows:

$$E_j = -\frac{1}{\ln m}\sum_{i=1}^{m} P_{ij} \ln P_{ij}. \quad (20)$$

Step 5. Obtain objective weights ω_j as follows:

$$\omega_j = \frac{1 - E_j}{\sum_{j=1}^{n}(1 - E_j)}. \quad (21)$$

4.2.2 Determining the Combined Weights: The Linear Weighted Comprehensive Method

Suppose that the vector of the subjective weight, given by the decision-makers directly, is $w = \{w_1, w_2, \cdots, w_n\}$, where $\sum\limits_{j=1}^{n} w_j = 1, 0 \le w_j \le 1$. The vector of the objective weight, computed by Eq. (19) directly, is $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$, where $\sum_{j=1}^{n} \omega_j = 1, 0 \le \omega_j \le 1$.

Therefore, the vector of the combined weight $\varpi = \{\varpi_1, \varpi_2, \cdots, \varpi_n\}$ can be defined as follows:

$$\varpi_j = \lambda w_j + (1 - \lambda)\omega_j, \quad (22)$$

where λ is the key degree (based on the real decision cases; in this paper, we set $\lambda = 0.5$), $\sum_{j=1}^{n} \varpi_j = 1, 0 \le \varpi_j \le 1$.

The objective weight and subjective weight are aggregated by the linear weighted comprehensive method. According to the addition effect, the larger the value of the subjective weight and the objective weight, the larger the combined weight is, or vice versa. At the same time, we can obtain that Eq. (22) overcomes the limitation of only considering either subjective or objective factor influence. The advantage of Eq. (22) is that the attribute weights and rankings of alternatives can show both subjective information and objective information.

4.3 Three Interval Neutrosophic Approaches in MADM

4.3.1 The Interval Neutrosophic MADM Approach Based MABAC

The MABAC is a new MADM method presented in [49]. Due to its straightforward computation procedure and the steadiness (consistency) of solution, the MABAC method is a particularly practical and credible tool for decision-making. In this subsection, a modified MABAC method within the interval neutrosophic environment is introduced to help decision-makers.

Algorithm 1: MABAC

Step 1. Identify the alternatives and attributes, and obtain the interval neutrosophic matrix $R = (r_{ij})_{m\times n}\,(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ which is shown in Table 1.

Step 2. Normalize decision matrix $R = (r_{ij})_{m\times n}$ into $\widetilde{R} = (\widetilde{r}_{ij})_{m\times n} = ([\widetilde{T}^L_{ij}, \widetilde{T}^U_{ij}], [\widetilde{I}^L_{ij}, \widetilde{I}^U_{ij}], [\widetilde{F}^L_{ij}, \widetilde{F}^U_{ij}])_{m\times n}$ by Eq. (19).

Step 3. Compute relative weight ϖ_j of attribute C_j by Eq. (22).

Step 4. Compute the weighted matrix $T = (\ t_{ij})_{m\times n}$ by Eq. (23).

$$
\begin{aligned}
t_{ij} &= \left(\left[{T'_{ij}}^L, {T'_{ij}}^U\right], \left[{I'_{ij}}^L, {I'_{ij}}^U\right], \left[{F'_{ij}}^L, {F'_{ij}}^U\right]\right) = \varpi_j \widetilde{r}_{ij} \\
&= \left(\left[1-\left(1-\widetilde{T}^L_{ij}\right)^{\varpi_j}, 1-\left(1-\widetilde{T}^U_{ij}\right)^{\varpi_j}\right], \left[\left(\widetilde{I}^L_{ij}\right)^{\varpi_j}, \left(\widetilde{I}^U_{ij}\right)^{\varpi_j}\right], \left[\left(\widetilde{F}^L_{ij}\right)^{\varpi_j}, \left(\widetilde{F}^U_{ij}\right)^{\varpi_j}\right]\right). \quad (23)
\end{aligned}
$$

Step 5. Compute the border approximation area (BAA) matrix $G = (g_j)_{1\times n}$. The BAA for each attribute is obtained by Eq. (24).

$$
\begin{aligned}
g_j = \prod_{i=1}^{m} (t_{ij})^{1/m} = &\left(\left[\prod_{i=1}^{m}\left({T'_{ij}}^L\right)^{1/m}, \prod_{i=1}^{m}\left({T'_{ij}}^U\right)^{1/m}\right], \left[1-\prod_{i=1}^{m}\left(1-{I'_{ij}}^L\right)^{1/m}, 1-\prod_{i=1}^{m}\left(1-{I'_{ij}}^U\right)^{1/m}\right],\right. \\
&\left.\left[1-\prod_{i=1}^{m}\left(1-{F'_{ij}}^L\right)^{1/m}, 1-\prod_{i=1}^{m}\left(1-{F'_{ij}}^U\right)^{1/m}\right]\right) \quad (24)
\end{aligned}
$$

Step 6. Reckon the distance matrix $D = (d_{ij})_{m\times n}$ by Eq. (25).

$$
d_{ij} = \begin{cases} d_h(t_{ij}, g_j), & \text{if } t_{ij} > g_j, \\ 0, & \text{if } t_{ij} = g_j, \\ -d_h(t_{ij}, g_j), & \text{if } t_{ij} < g_j, \end{cases} \quad (25)
$$

where distance measure d_h is defined in Eq. (3).

Especially, alternative A_i will pertain to the BAA (G) if $d_{ij} = 0$, upper approximation area (G^+) if $d_{ij} > 0$, and lower approximation area (G^-) if $d_{ij} < 0$. The upper approximation area (G^+) is the area which includes the ideal alternative (A^+) while the lower approximation area (G^-) is the area which includes the anti-ideal alternative (A^-) (see Fig. 1, [51]). For choosing alternative A_i as the best from the set, there is need of as many attributes as possible pertaining to the upper approximate area (G^+).

Step 7. Rank the alternatives by $Q_i(i = 1, 2, \cdots, m)$. The most desired alternative is the one with the biggest value of Q_i.

$$
Q_i = \sum_{j=1}^{n} d_{ij}, \quad i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n. \quad (26)
$$

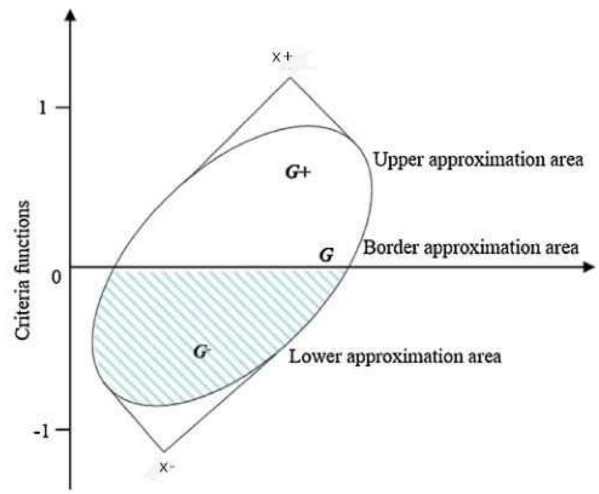

FIG. 1: Exhibition of the upper (G^{+}), lower (G^{-}), and border (G) approximation areas

4.3.2 The Interval Neutrosophic MADM Approach Based Similarity Measure

In this section, we present a novel method for solving the MADM problem by the proposed similarity measure between INSs. The concept of ideal point has been used to help obtain the best alternative in the decision process. Although the ideal alternative does not exist in real problems, it does provide a useful theoretical construct against which to appraise alternatives. Therefore, we define the ideal alternative A^* as the INN $a_j^* = ([(T^*)^L, (T^*)^U], [(I^*)^L, (I^*)^U], [(F^*)^L, (F^*)^U]) = ([1,1],[0,0],[0,0])$ for $\forall j$.

Hence, by applying Eq. (17), the proposed similarity measure S^{Δ} between an alternative A_i and the ideal alternative A^* represented by the INSs is defined by

$$
\begin{aligned}
S^{\Delta}(A_i, A^*) = &\left(\left[1-\sum_{j=1}^{n} w_j \max\{|T_{ij}^L - (T^*)^L|, |T_{ij}^U - (T^*)^U|\}, 1-\sum_{j=1}^{n} w_j \min\{|T_{ij}^L - (T^*)^L|, |T_{ij}^U - (T^*)^U|\}\right],\right.\\
&\left[\sum_{j=1}^{n} w_j \min\{| I_{ij}^L - (I^*)^L |, | I_{ij}^U - (I^*)^U |\}, \sum_{j=1}^{n} w_j \max\{| I_{ij}^L - (I^*)^L |, | I_{ij}^U - (I^*)^U |\}\right],\\
&\left.\left[\sum_{j=1}^{n} w_j \min\{| F_{ij}^L - (F^*)^L |, | F_{ij}^U - (F^*)^U |\}, \sum_{j=1}^{n} w_j \max\{| F_{ij}^L - (F^*)^L |, | F_{ij}^U - (F^*)^U |\}\right]\right)\\
= &\left(\left[\sum_{j=1}^{n} w_j T_{ij}^L, \sum_{j=1}^{n} w_j T_{ij}^U\right], \left[\sum_{j=1}^{n} w_j I_{ij}^L, \sum_{j=1}^{n} w_j I_{ij}^U\right], \left[\sum_{j=1}^{n} w_j F_{ij}^L, \sum_{j=1}^{n} w_j F_{ij}^U\right]\right). \qquad (27)
\end{aligned}
$$

Algorithm 2: Similarity measure

Steps 1–3. There are the same as Steps 1–3 in Algorithm 1.

Step 4. Calculate the similarity measure $S^{\Delta}(A_i, A^*)(i = 1, 2, \cdots, m)$ by Eq. (27).

Step 5. Compute each alternative of the score function $s_{\alpha,\beta}(S^{\Delta}(A_i, A^*))$ by Eq. (6).

Step 6. Rank the alternatives by $s_{\alpha,\beta}(S^{\Delta}(A_i, A^*))(i = 1, 2, \cdots, m)$. The most desired alternative is the one with the biggest value of $s_{\alpha,\beta}(S^{\Delta}(A_i, A^*))$.

4.3.3 The Interval Neutrosophic MADM Approach Based EDAS

Algorithm 3: EDAS

Steps 1–3. These are the same as Steps 1–3 in Algorithm 1.

Step 4. Determine the average solution according to all alternatives, shown as follows:

$$AV = (AV_j)_{1\times n}, \tag{28}$$

where

$$\begin{aligned} AV_j &= \frac{1}{m}\bigoplus_{i=1}^{m}\widetilde{r}_{ij} = \frac{1}{m}\left(\left[1-\prod_{i=1}^{m}\left(1-\widetilde{T}_{ij}^{L}\right), 1-\prod_{i=1}^{m}\left(1-\widetilde{T}_{ij}^{U}\right)\right], \left[\prod_{i=1}^{m}\widetilde{I}_{ij}^{L}, \prod_{i=1}^{m}\widetilde{I}_{ij}^{U}\right], \left[\prod_{i=1}^{m}\widetilde{F}_{ij}^{L}, \prod_{i=1}^{m}\widetilde{F}_{ij}^{U}\right]\right) \\ &= \left(\left[1-\left(\prod_{i=1}^{m}\left(1-\widetilde{T}_{ij}^{L}\right)\right)^{1/m}, 1-\left(\prod_{i=1}^{m}\left(1-\widetilde{T}_{ij}^{U}\right)\right)^{1/m}\right], \left[\left(\prod_{i=1}^{m}\widetilde{I}_{ij}^{L}\right)^{1/m}, \left(\prod_{i=1}^{m}\widetilde{I}_{ij}^{U}\right)^{1/m}\right],\right. \\ &\quad \left.\left[\left(\prod_{i=1}^{m}\widetilde{F}_{ij}^{L}\right)^{1/m}, \left(\prod_{i=1}^{m}\widetilde{F}_{ij}^{U}\right)^{1/m}\right]\right). \end{aligned} \tag{29}$$

Step 5. Compute the positive distance from average (PDA) with PDA $= (P_{ij})_{m\times n}$ and the negative distance from average (NDA) with NDA $= (N_{ij})_{m\times n}$ matrixes according to the type of attributes, shown as follows:

$$P_{ij} = \begin{cases} \dfrac{\max\{0, s_{\alpha,\beta}(r_{ij}) - s_{\alpha,\beta}(AV_j)\}}{s_{\alpha,\beta}(AV_j)}, & C_j \text{ is benefit attribute}, \\ \dfrac{\max\{0, s_{\alpha,\beta}(AV_j) - s_{\alpha,\beta}(r_{ij})\}}{s_{\alpha,\beta}(AV_j)}, & C_j \text{ is cost attribute}, \end{cases} \tag{30}$$

$$N_{ij} = \begin{cases} \dfrac{\max\{0, s_{\alpha,\beta}(AV_j) - s_{\alpha,\beta}(\widetilde{r}_{ij})\}}{s_{\alpha,\beta}(AV_j)}, & C_j \text{ is benefit attribute}, \\ \dfrac{\max\{0, s_{\alpha,\beta}(\widetilde{r}_{ij}) - s_{\alpha,\beta}(AV_j)\}}{s_{\alpha,\beta}(AV_j)}, & C_j \text{ is cost attribute}, \end{cases} \tag{31}$$

where $s_{\alpha,\beta}(AV_j)$ and $s_{\alpha,\beta}(\widetilde{r}_{ij})$ are the score functions of AV_j and $\widetilde{r}_{ij}$, respectively.

Step 6. Determine the weighted sum of PDA and NDA for all alternatives, shown as follows:

$$SP_i = \sum_{j=1}^{n} w_j P_{ij}, \tag{32}$$

$$SN_i = \sum_{j=1}^{n} w_j N_{ij}. \tag{33}$$

Step 7. Normalize the values of SP_i and SN_i for all alternatives, shown as follows:

$$NSP_i = \frac{SP_i}{\max_i\{SP_i\}}, \tag{34}$$

$$NSN_i = 1 - \frac{SN_i}{\max_i\{SN_i\}}. \tag{35}$$

Step 8. Calculate the appraisal score $AS_i (i = 1, 2, \cdots, m)$ for all alternatives, shown as follows:

$$AS_i = \frac{1}{2}(NSP_i + NSN_i), \tag{36}$$

where $0 \leq AS_i \leq 1$.

Step 9. Rank the alternatives according to the decreasing values of AS_i. The alternative with the biggest AS_i is the best alternative.

5. TWO ILLUSTRATIVE EXAMPLES

In this section, we give two illustrative examples to illustrate the implementation process and availability of the proposed three approaches.

The examples are the investment selection problem and the assessment of C Programming Language teaching effect that include subjective weight data and incomplete weight determined information.

5.1 Two Illustrative Examples

Example 1 ([29]). Consider that an investment company wants to select an excellent project. There are four possible alternatives in which to invest, expressed as $\{A_1, A_2, A_3, A_4\}$, where A_1 is a bookshop, A_2 is a chemical plant, A_3 is a supermarket, and A_4 is a food company. The experts evaluate the alternatives in the following three attributes: C_1 is the earning estimate analysis, C_2 is the growth analysis, and C_3 is the environmental impact analysis for the alternatives; C_1 and C_2 are benefit attributes, while C_3 is a cost attribute. The weight vector of the attribute is given by $w = (0.55, 0.25, 0.2)$. The four possible alternatives are evaluated according to the above three attributes by INSs, as shown in the following interval neutrosophic decision matrix $R = (r_{ij})_{4\times3} = ([T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U])_{4\times3}$. The assessments for projects arising from questionnaire investigation to the experts are shown in Table 2.

TABLE 2: The evaluation values of four possible alternatives with respect to the three attributes

	C_1	C_2	C_3
A_1	$([0.4, 0.5], [0.2, 0.3], [0.3, 0.4])$	$([0.4, 0.6], [0.1, 0.3], [0.2, 0.4])$	$([0.7, 0.9], [0.2, 0.3], [0.4, 0.5])$
A_2	$([0.6, 0.7], [0.1, 0.2], [0.2, 0.3])$	$([0.6, 0.7], [0.1, 0.2], [0.2, 0.3])$	$([0.3, 0.6], [0.3, 0.5], [0.8, 0.9])$
A_3	$([0.3, 0.6], [0.2, 0.3], [0.3, 0.4])$	$([0.5, 0.6], [0.2, 0.3], [0.3, 0.4])$	$([0.4, 0.5], [0.2, 0.4], [0.7, 0.9])$
A_4	$([0.7, 0.8], [0.0, 0.1], [0.1, 0.2])$	$([0.6, 0.7], [0.1, 0.2], [0.1, 0.3])$	$([0.6, 0.7], [0.3, 0.4], [0.8, 0.9])$

Algorithm 4: MABAC

Step 1. The interval neutrosophic decision matrix $R = (r_{ij})_{4\times3} = ([T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U])_{4\times3}$ which is shown in Table 2.

Step 2. Normalize the interval neutrosophic decision matrix $R = (r_{ij})_{4\times3}$ into $\widetilde{R} = (\widetilde{r}_{ij})_{4\times3}$ by Eq. (19), which is shown in Table 3.

Step 3. Compute relative weight ϖ_j of attribute C_j by Eq. (22) as follows:

$$\varpi_1 = 0.4288, \varpi_2 = 0.1647, \varpi_3 = 0.4066.$$

Step 4. Calculate the weighted matrix $T = (t_{ij})_{4\times3}$ by Eq. (23), which is shown in Table 4.

Step 5. The BAA $G = (g_j)_{1\times3}$ is determined according to Eq. (24); we can get

$$g_1 = ([0.2459, 0.3600], [0.3717, 0.5251], [0.5251, 0.6184]),$$

$$g_2 = ([0.1143, 0.1587], [0.7076, 0.7954], [0.7645, 0.8413]),$$

$$g_3 = ([0.3597, 0.4846], [0.8152, 0.8918], [0.7650, 0.8729]).$$

Step 6. Calculate the distance matrix $D = (d_{ij})_{4\times 3}$ by Eq. (25), which is shown in Table 5.
Step 7. Rank the alternatives by $Q_i (i = 1, 2, 3, 4)$ as follows:

$$Q_1 = -0.211839, \quad Q_2 = 0.142991, \quad Q_3 = -0.047243, \quad Q_4 = 0.269572.$$

Hence, $A_4 > A_2 > A_3 > A_1$; i.e., the best alternative is A_4.

TABLE 3: The normalized interval neutrosophic evaluation values

	C_1	C_2	C_3
A_1	$([0.4, 0.5], [0.2, 0.3], [0.3, 0.4])$	$([0.4, 0.6], [0.1, 0.3], [0.2, 0.4])$	$([0.4, 0.5], [0.7, 0.8], [0.7, 0.9])$
A_2	$([0.6, 0.7], [0.1, 0.2], [0.2, 0.3])$	$([0.6, 0.7], [0.1, 0.2], [0.2, 0.3])$	$([0.8, 0.9], [0.5, 0.7], [0.3, 0.6])$
A_3	$([0.3, 0.6], [0.2, 0.3], [0.3, 0.4])$	$([0.5, 0.6], [0.2, 0.3], [0.3, 0.4])$	$([0.7, 0.9], [0.6, 0.8], [0.4, 0.5])$
A_4	$([0.7, 0.8], [0.0, 0.1], [0.1, 0.2])$	$([0.6, 0.7], [0.1, 0.2], [0.1, 0.3])$	$([0.8, 0.9], [0.6, 0.7], [0.6, 0.7])$

TABLE 4: The weighed interval neutrosophic matrix $T = (t_{ij})_{4\times 3}$

	C_1
A_1	$([0.1967, 0.2571], [0.5015, 0.5968], [0.5968, 0.6751])$
A_2	$([0.3249, 0.4032], [0.3726, 0.5015], [0.5015, 0.5968])$
A_3	$([0.1418, 0.3249], [0.5015, 0.5968], [0.5968, 0.6751])$
A_4	$([0.4032, 0.4985], [0.0000, 0.3726], [0.3726, 0.5015])$
	C_2
A_1	$([0.0807, 0.1400], [0.6845, 0.8202], [0.7672, 0.8600])$
A_2	$([0.1400, 0.1798], [0.6845, 0.7672], [0.7672, 0.8202])$
A_3	$([0.1079, 0.1400], [0.7672, 0.8202], [0.8202, 0.8600])$
A_4	$([0.1400, 0.1798], [0.6845, 0.7672], [0.6845, 0.8202])$
	C_3
A_1	$([0.1875, 0.2456], [0.8650, 0.9133], [0.8650, 0.9581])$
A_2	$([0.4802, 0.6079], [0.7544, 0.8650], [0.6129, 0.8125])$
A_3	$([0.3871, 0.6079], [0.8125, 0.9133], [0.6890, 0.7544])$
A_4	$([0.4802, 0.6079], [0.8125, 0.8650], [0.8125, 0.8650])$

TABLE 5: The interval neutrosophic matrix $D = (d_{ij})_{4\times 3}$

	C_1	C_2	C_3
A_1	-0.080325	-0.020250	-0.111265
A_2	0.032001	0.020334	0.090657
A_3	-0.078175	-0.030637	0.061570
A_4	0.181568	0.033229	0.054775

Algorithm 5: Similarity measure

Steps 1–3. These are the same as Algorithm 4 in Steps 1–3.
Step 4. Calculate the similarity measure $S(A_i, A^*) (i = 1, 2, 3, 4)$ by Eq. (27).

$$S(A_1, A^*) = ([0.400000, 0.516466], [0.386826, 0.503292], [0.446168, 0.603292]),$$
$$S(A_2, A^*) = ([0.681317, 0.781317], [0.262633, 0.403292], [0.240658, 0.421975]),$$
$$S(A_3, A^*) = ([0.495565, 0.721975], [0.362633, 0.503292], [0.340658, 0.440658]),$$
$$S(A_4, A^*) = ([0.724193, 0.824193], [0.260416, 0.360416], [0.303292, 0.419757]).$$

Step 5. Each alternative of score function $s_{0.5,0.3}(S(A_i, A^*))$ is shown as follows:

$$s_{0.5,0.3}(S(A_1, A^*)) = 0.692755,$$

$$s_{0.5,0.3}(S(A_2, A^*)) = 0.821807,$$

$$s_{0.5,0.3}(S(A_3, A^*)) = 0.758357,$$

$$s_{0.5,0.3}(S(A_4, A^*)) = 0.836836.$$

Step 6. Rank the alternatives by $s_{\alpha,\beta}(S(A_i, A^*))(i = 1, 2, 3, 4)$ as follows: $A_4 > A_2 > A_3 > A_1$; i.e., the best alternative is A_4.

Algorithm 6: EDAS

Steps 1–3. These are the same as Algorithm 4 in Steps 1–3.

Step 4. Determine the average solution according to all attributes by Eq. (29), shown as follows:

$$AV_1 = ([0.5262, 0.6690], [0.0000, 0.2060], [0.2060, 0.3130]),$$

$$AV_2 = ([0.5319, 0.6536], [0.1189, 0.2449], [0.1861, 0.3464]),$$

$$AV_3 = ([0.7087, 0.8505], [0.5958, 0.7483], [0.4738, 0.6593]).$$

Step 5. Calculate the positive distance from average PDA $= (P_{ij})_{4\times3}$ and the negative distance from average NDA $= (N_{ij})_{4\times3}$ matrices by Eqs. (30) and (31), shown as follows:

$$\text{PDA} = (P_{ij})_{4\times3}\begin{pmatrix} 0.0000 & 0.0000 & 0.0000 \\ 0.0129 & 0.0322 & 0.0622 \\ 0.0000 & 0.0000 & 0.0182 \\ 0.0858 & 0.0384 & 0.0248 \end{pmatrix},$$

$$\text{NDA} = (N_{ij})_{4\times3}\begin{pmatrix} 0.1006 & 0.0462 & 0.1929 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.1006 & 0.0421 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \end{pmatrix}.$$

Step 6. Determine the weighted sum of PDA and NDA for all alternatives by Eqs. (32) and (33), respectively, shown as follows:

$$SP_1 = 0.0000,\ \ SP_2 = 0.0361,\ \ SP_3 = 0.0074,\ \ SP_4 = 0.0532,$$

$$NP_1 = 0.1292,\ \ NP_2 = 0.0000,\ \ NP_3 = 0.0501,\ \ NP_4 = 0.0000.$$

Step 7. Normalize the values of SP_i and SN_i for all alternatives by Eqs. (34) and (35), respectively, shown as follows:

$$NSP_1 = 0.0000,\ \ NSP_2 = 0.6788,\ \ NSP_3 = 0.1392,\ \ NSP_4 = 1.0000,$$

$$NSN_1 = 0.0000,\ \ NSN_2 = 1.0000,\ \ NSN_3 = 0.6125,\ \ NSN_4 = 1.0000.$$

Step 8. Calculate the appraisal score $AS_i(i = 1, 2, 3, 4)$ for all alternatives by Eq. (36), shown as follows:

$$AS_1 = 0.0000,\ \ AS_2 = 0.8394,\ \ AS_3 = 0.3759,\ \ AS_4 = 1.0000.$$

Step 9. Rank the software development projects x_i according to the decreasing values of AS_i as follows:

$$A_4 \succ A_2 \succ A_3 \succ A_1.$$

Obviously, among them, A_4 is the best investment project.

According to Algorithms 4–6, we can conclude that the optimal results are the same; i.e., A_4 is the most desirable investment project. Hence, the three algorithms proposed above are effective and feasible.

Example 2. Consider that a school wants to select an excellent C Programming Language teacher. The teacher experts give four feasible excellent teachers $A_i(i = 1, 2, 3, 4)$. Suppose that three attributes C_1 (the environment of teaching and studying), C_2 (the management of teaching information), and C_3 (the empathy and the teaching practice), then the weight vector of the corresponding attribute $C_j(j = 1, 2, 3)$ is $w = (0.33, 0.34, 0.33)^T$. Meanwhile, the attributes are all benefit attributes. Assume that the teacher $A_i(i = 1, 2, 3, 4)$ with respect to the attribute $C_j(j = 1, 2, 3)$ is given by the interval neutrosophic matrix $R = (r_{ij})_{4\times3} = ([T^L_{ij}, T^U_{ij}], [I^L_{ij}, I^U_{ij}], [F^L_{ij}, F^U_{ij}])_{4\times3}$. The assessments for teachers arising from questionnaire investigation to the teacher experts are shown in Table 6.

TABLE 6: The interval neutrosophic matrix given by experts

	C_1	C_2	C_3
A_1	$([0.9, 0.9], [0.1, 0.1], [0.1, 0.2])$	$([0.1, 0.2], [0.1, 0.2], [0.1, 0.1])$	$([0.1, 0.2], [0.1, 0.2], [0.1, 0.2])$
A_2	$([1.0, 1.0], [0.1, 0.1], [0.1, 0.2])$	$([0.1, 0.2], [0.1, 0.2], [0.1, 0.1])$	$([0.1, 0.2], [0.1, 0.2], [0.1, 0.2])$
A_3	$([1.0, 1.0], [0.1, 0.1], [0.1, 0.2])$	$([1.0, 1.0], [0.1, 0.2], [0.1, 0.1])$	$([0.0, 0.0], [0.1, 0.2], [0.1, 0.2])$
A_4	$([1.0, 1.0], [0.1, 0.1], [0.1, 0.2])$	$([0.1, 0.2], [0.1, 0.2], [0.1, 0.1])$	$([0.0, 0.0], [0.1, 0.2], [0.1, 0.2])$

Algorithm 7: MABAC

Step 1. The interval neutrosophic decision matrix $R = (r_{ij})_{4\times3} = ([T^L_{ij}, T^U_{ij}], [I^L_{ij}, I^U_{ij}], [F^L_{ij}, F^U_{ij}])_{4\times3}$ which is shown in Table 6.

Step 2. There is no need to normalize the decision matrix based on the above condition.

Step 3. Compute relative weight ϖ_j of attribute C_j by Eq. (22) as follows:

$$\varpi_1 = 0.1693, \varpi_2 = 0.6378, \varpi_3 = 0.1929.$$

Step 4. Calculate the weighted matrix $T = (t_{ij})_{4\times3}$ by Eq. (23), which is shown in Table 7.

Step 5. The BAA $G = (g_j)_{1\times3}$ is determined according to the Eq. (24), we can get

$$g_1 = ([0.7537, 0.7537], [0.6772, 0.6772], [0.6772, 0.7615]),$$

TABLE 7: The weighed interval neutrosophic matrix $T = (t_{ij})_{4\times3}$

	C_1
A_1	$([0.3228, 0.3228], [0.6772, 0.6772], [0.6772, 0.7615])$
A_2	$([1.0000, 1.0000], [0.6772, 0.6772], [0.6772, 0.7615])$
A_3	$([1.0000, 1.0000], [0.6772, 0.6772], [0.6772, 0.7615])$
A_4	$([1.0000, 1.0000], [0.6772, 0.6772], [0.6772, 0.7615])$
	C_2
A_1	$([0.0650, 0.1327], [0.2303, 0.3583], [0.2303, 0.2303])$
A_2	$([0.0650, 0.1327], [0.2303, 0.3583], [0.2303, 0.2303])$
A_3	$([1.0000, 1.0000], [0.2303, 0.3583], [0.2303, 0.2303])$
A_4	$([0.0650, 0.1327], [0.2303, 0.3583], [0.2303, 0.2303])$
	C_3
A_1	$([0.0201, 0.0421], [0.6413, 0.7331], [0.6413, 0.7331])$
A_2	$([0.0201, 0.0421], [0.6413, 0.7331], [0.6413, 0.7331])$
A_3	$([0.0000, 0.0000], [0.6413, 0.7331], [0.6413, 0.7331])$
A_4	$([0.0000, 0.0000], [0.6413, 0.7331], [0.6413, 0.7331])$

$$g_2 = ([0.1287, 0.2198], [0.2303, 0.3583], [0.2303, 0.2303]),$$

$$g_3 = ([0.0000, 0.0000], [0.6413, 0.7331], [0.6413, 0.7331]).$$

Step 6. Calculate the distance matrix $D = (d_{ij})_{4\times 3}$ by Eq. (25), which is shown in Table 8.
Step 7. Rank the alternatives by $Q_i (i = 1, 2, 3, 4)$ as follows:

$$Q_1 = -0.158429, \quad Q_2 = 0.067320, \quad Q_3 = 0.357334, \quad Q_4 = 0.056942.$$

Hence, $A_3 > A_2 > A_4 > A_1$, i.e., the best C Programming Language teacher is A_3.

TABLE 8: The interval neutrosophic matrix $D = (d_{ij})_{4\times 3}$

	C_1	C_2	C_3
A_1	−0.143660	−0.025147	0.010378
A_2	0.082089	−0.025147	0.010378
A_3	0.082089	0.275245	0.000000
A_4	0.082089	−0.025147	0.000000

Algorithm 8: Similarity measure

Steps 1–3. These are the same as Algorithm 7 in Steps 1–3.
Step 4. Calculate the similarity measure $S(A_i, A^*)(i = 1, 2, 3, 4)$ by Eq. (27).

$$S(A_1, A^*) = ([0.235388, 0.318464], [0.100000, 0.183077], [0.100000, 0.136165]),$$

$$S(A_2, A^*) = ([0.252311, 0.335387], [0.100000, 0.183077], [0.100000, 0.136165]),$$

$$S(A_3, A^*) = ([0.807587, 0.807587], [0.100000, 0.183077], [0.100000, 0.136165]),$$

$$S(A_4, A^*) = ([0.233070, 0.296905], [0.100000, 0.183077], [0.100000, 0.136165]).$$

Step 5. Each alternative of score function $s_{0.5,0.3}(S(A_i, A^*))$ is shown as follows:

$$s_{0.5,0.3}(S(A_1, A^*)) = 0.723571,$$

$$s_{0.5,0.3}(S(A_2, A^*)) = 0.729212,$$

$$s_{0.5,0.3}(S(A_3, A^*)) = 0.900458,$$

$$s_{0.5,0.3}(S(A_4, A^*)) = 0.719591.$$

Step 6. Rank the alternatives by $s_{\alpha,\beta}(S(A_i, A^*))(i = 1, 2, 3, 4)$ as follows: $A_3 > A_2 > A_1 > A_4$; i.e., the best C Programming Language teacher is A_3.

Algorithm 9: EDAS

Steps 1–3. These arethe same as Algorithm 7 in Steps 1–3.
Step 4. Determine the average solution according to all attributes by Eq. (29), shown as follows:

$$AV_1 = ([1.0000, 1.0000], [0.1000, 0.1000], [0.1000, 0.2000]),$$

$$AV_2 = ([1.0000, 1.0000], [0.1000, 0.2000], [0.1000, 0.1000]),$$

$$AV_3 = ([0.0513, 0.1056], [0.1000, 0.2000], [0.1000, 0.2000]).$$

Step 5. Calculate the positive distance from average PDA $= (P_{ij})_{4\times 3}$ and the negative distance from average NDA $= (N_{ij})_{4\times 3}$ matrices by Eqs. (30) and (31), shown as follows:

$$\text{PDA} = (P_{ij})_{4\times 3} \begin{pmatrix} 0.0000 & 0.0000 & 0.0365 \\ 0.0000 & 0.0000 & 0.0365 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \end{pmatrix},$$

$$\text{NDA} = (N_{ij})_{4\times 3} \begin{pmatrix} 0.0344 & 0.2936 & 0.0000 \\ 0.0000 & 0.2936 & 0.0000 \\ 0.0000 & 0.0000 & 0.0401 \\ 0.0000 & 0.2936 & 0.0401 \end{pmatrix}.$$

Step 6. Determine the weighted sum of PDA and NDA for all alternatives by Eqs. (32) and (33), respectively, shown as follows:

$$SP_1 = 0.0070, \quad SP_2 = 0.0070, \quad SP_3 = 0.0000, \quad SP_4 = 0.0000,$$

$$NP_1 = 0.1931, \quad NP_2 = 0.1873, \quad NP_3 = 0.0077, \quad NP_4 = 0.1950.$$

Step 7. Normalize the values of SP_i and SN_i for all alternatives by Eqs. (34) and (35), respectively, shown as follows:

$$NSP_1 = 1.0000, \quad NSP_2 = 1.0000, \quad NSP_3 = 0.0000, \quad NSP_4 = 0.0000,$$

$$NSN_1 = 0.0098, \quad NSN_2 = 0.0396, \quad NSN_3 = 0.9604, \quad NSN_4 = 0.0000.$$

Step 8. Calculate the appraisal score $AS_i (i = 1, 2, 3, 4)$ for all alternatives by Eq. (36), shown as follows:

$$AS_1 = 0.5049, \quad AS_2 = 0.5198, \quad AS_3 = 0.5202, \quad AS_4 = 0.0000.$$

Step 9. Rank the software development projects x_i according to the decreasing values of AS_i as follows:

$$A_3 \succ A_2 \succ A_1 \succ A_4.$$

Obviously, among them A_3 is the best C Programming Language teacher.

By means of the Algorithms 7–9, we can find that the final results are the same; i.e., A_3 is the best C Programming Language teacher. Hence, the three algorithms discussed above are effective and feasible.

6. COMPARISON OF THE NEWLY PROPOSED APPROACHES WITH THE OTHER APPROACHES TO INTERVAL NEUTROSOPHIC SET BASED DECISION-MAKING

6.1 Comparison of the Newly Proposed Three Approaches with Their Own Advantages

(1) Comparison of computational complexity

We know that Algorithms 1 and 3 will consume more computational complexity than Algorithm 2, especially in Step 4 (Algorithm 3) and in Step 5 (Algorithm 1). So if we take the computational complexity into consideration, Algorithm 2 is given priority to make decisions.

(2) Comparison of discrimination

Comparing the results in Algorithms 1 and 2 with Algorithm 3, we can find that the results of Algorithm 2 are quite close and vary from 0.6927 to 0.8218 and 0.7196 to 0.9004. These results of decision values cannot clearly distinguish; in other words, the results obtained from Algorithm 2 are not very convincing (or at least not applicable). That is to say, the Algorithm 1 has a clear ability to distinguish. So if we take the discrimination into consideration, the Algorithms 1 and 3 are given priority to make decisions.

6.2 Comparison of the Newly Proposed Three Approaches with Other Approaches

In order to further verify the practicability of the proposed algorithms based on the MABAC, EDAS, and similarity measure of INSs, a comparison study with some existing algorithms is now bulit. The decision data are adopted from Tian et al. [27] and Example 2.

6.2.1 A Comparison Analysis from Tian [27]

We take the example adopted from Tian [27], and are the interval neutrosophic decision matrix is shown in Table 9. The corresponding weight information is $W = (0.15, 0.15, 0.375, 0.325)$.

If the existing methods in Ye [12,25,30,37], Zhang et al. [24,28,34], Chi and Liu [56], Liu and Tang [31], Liu and Wang [32], Tian et al. [27], and the proposed three methods are applied to solve the MADM problem in Tian et al. [27], then the results can be achieved and shown in Table 10.

From the above results shown in Table 10, we can know that the ranking order of the four alternatives and optimal alternative are in agreement with the results of [12,24,25,27,28,30,32,34,37,56]. For Liu and Tang [31], the optimal alternative may be different when λ is assigned different values. That is to say, it will not obtain a convincing result when the experts of the corresponding field make decisions.

6.2.2 A Comparison Analysis from Example 2

If the existing methods in Ye [12,25,30,37], Zhang et al. [24,28,34], Chi and Liu [56], Liu and Wang [32], Tian et al. [27], and the proposed three methods are applied to solve the MADM problem in Example 2, then the results can be obtained and are shown in Table 11.

From the above results shown in Table 11, we can know that the final ranking of the four alternatives and optimal alternative are in agreement with the results of [12,25,27,28,30,32,34,37,56]. For Zhang et al. [24], the final ranking and the optimal alternative cannot be obtained due to their equal values discussed in Definition 9. The optimal alternative is A_2 in [24], which is different from some existing methods and our three Algorithms. It is unreasonable, which we have discussed in Definition 10.

TABLE 9: The interval neutrosophic matrix adopted from [27]

	C_1	C_2
A_1	$([0.7, 0.8], [0.5, 0.7], [0.1, 0.2])$	$([0.6, 0.8], [0.4, 0.5], [0.3, 0.3])$
A_2	$([0.6, 0.8], [0.4, 0.6], [0.1, 0.3])$	$([0.5, 0.7], [0.3, 0.5], [0.1, 0.3])$
A_3	$([0.4, 0.6], [0.2, 0.2], [0.2, 0.4])$	$([0.6, 0.7], [0.4, 0.6], [0.3, 0.4])$
A_4	$([0.4, 0.5], [0.5, 0.6], [0.4, 0.4])$	$([0.5, 0.6], [0.3, 0.4], [0.4, 0.5])$
A_5	$([0.6, 0.7], [0.4, 0.5], [0.4, 0.5])$	$([0.8, 0.9], [0.3, 0.4], [0.1, 0.2])$
	C_3	C_4
A_1	$([0.8, 0.8], [0.4, 0.6], [0.1, 0.2])$	$([0.7, 0.9], [0.3, 0.4], [0.2, 0.2])$
A_2	$([0.6, 0.6], [0.2, 0.3], [0.4, 0.5])$	$([0.6, 0.8], [0.4, 0.4], [0.2, 0.4])$
A_3	$([0.7, 0.8], [0.6, 0.7], [0.1, 0.2])$	$([0.5, 0.6], [0.5, 0.6], [0.2, 0.3])$
A_4	$([0.6, 0.7], [0.7, 0.8], [0.2, 0.3])$	$([0.8, 0.9], [0.3, 0.4], [0.1, 0.2])$
A_5	$([0.7, 0.8], [0.5, 0.6], [0.1, 0.2])$	$([0.5, 0.7], [0.5, 0.5], [0.2, 0.3])$

TABLE 10: A comparison study with some existing methods

Method	Final ranking	Optimal alternative
Ye [30]	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Liu and Tang [31]		
$\lambda = 0.1$	$A_5 \succ A_1 \succ A_2 \succ A_4 \succ A_3$	A_5
$\lambda = 3.2$	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$	A_1
Liu and Wang [32]	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Ye [12]	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Ye [37]	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$	A_1
Zhang et al. [28]	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Zhang et al. ($p = 0.2$ and $q = 0.1$) [34]	$A_1 \succ A_2 \succ \{A_3, A_4, A_5\}$	A_1
Zhang et al. [24]		
Method based on the INWA operator	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$	A_1
Method based on the INWG operator	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Ye [25]		
Similarity measure based on the Hamming distance	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$	A_1
Similarity measure based on the Euclidean distance	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Chi and Liu [56]	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$	A_1
Tian et al. [27]	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Algorithm 1	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Algorithm 2	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1
Algorithm 3	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	A_1

TABLE 11: A comparison study with some existing methods in Example 2

Method	Final ranking	Optimal alternative
Ye [30]	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Liu and Wang [32]	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Ye [12]	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Ye [37]	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Zhang et al. [28]	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Zhang et al. ($p = 0.2$ and $q = 0.1$) [34]	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Zhang et al. [24]		
Method based on the INWA operator	$\{A_2, A_3, A_4\} \succ A_1$	$*$
Method based on the INWG operator	$A_2 \succ A_1 \succ \{A_3, A_4\}$	A_2
Ye [25]		
Similarity measure based on the Hamming distance	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Similarity measure based on the Euclidean distance	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Chi and Liu [56]	$A_3 \succ A_2 \succ A_4 \succ A_1$	A_3
Tian et al. [27]	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Algorithm 1	$A_3 \succ A_2 \succ A_4 \succ A_1$	A_3
Algorithm 2	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3
Algorithm 3	$A_3 \succ A_2 \succ A_1 \succ A_4$	A_3

"$*$" presents no sure result.

7. CONCLUSIONS

This paper introduces three new approaches for MADM under an interval neutrosophic environment. First, we define a new axiomatic definition of interval neutrosophic distance measure and similarity measure. Comparing with the existing literature [10,12,18,22], our distance measure or similarity measure can keep more original decision information. Later, a novel score function is proposed. Comparing with the existing score functions [31], we can overcome their drawbacks. Meanwhile, the combined weight model is proposed to solve the weight information which is too objective [27] or subjective [7,24,25,28,29,37]. Then, three approaches (EDAS, MABAC, similarity measure) are proposed to deal with the real MADM problems. Finally, the effectiveness and feasibility of approaches are demonstrated by two examples. Meanwhile, a comparison analysis is presented in Tables 10 and 11.

In the future, we shall apply the similarity measure of INSs to other sets [57–73].

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