Question 444 : Tribonacci Constant and Pi

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abstract

This note presents some formulas related with pi.

1. INTRODUCTION.

- The Tribonacci constant T is defined by

$$T=\frac{1+\sqrt[3]{19+3\sqrt{33}}+\sqrt[3]{19-3\sqrt{33}}}{3}=1+\left(\sqrt[3]{\frac{1}{2}+\sqrt[3]{\frac{1}{2}+\sqrt[3]{\frac{1}{2}+...}}}\right)^{-1} \tag{1}$$

- The number pi is defined by

$$\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...\right)=3.141592... \tag{2}$$

2. Pi in terms of the Tribonacci constant

$$\pi=4\sum_{n=1}^{\infty}\frac{c_n}{n}T^{-n} \tag{3}$$

where

$$c_n=a_n(-1)^{(n-1)/2}+2b_n(-1)^{(n-2)/4} \quad ,n\in\mathbb{N} \tag{4}$$

$$c_n=\{1,2,-1,0,1,-2,-1,0,1,2,-1,0,1,-2,-1,0,...\} \tag{5}$$

Remark: $c_{4n}=0,c_{4n-1}=-1,c_{4n-2}=2(-1)^{n-1},c_{4n-3}=1,n\in\mathbb{N}$.

$$a_n = \begin{cases} 1 & (n-1)/2 \in \mathbb{N} \cup \{0\} \\ 0 & (n-1)/2 \notin \mathbb{N} \cup \{0\} \end{cases} \quad , n \in \mathbb{N} \tag{6}$$

$$a_n = \{1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, ...\} \tag{7}$$

$$b_n = \begin{cases} 1 & (n-2)/4 \in \mathbb{N} \cup \{0\} \\ 0 & (n-2)/4 \notin \mathbb{N} \cup \{0\} \end{cases} \quad , n \in \mathbb{N} \tag{8}$$

$$b_n = \{1, 2, -1, 0, 1, -2, -1, 0, 1, 2, -1, 0, 1, -2, -1, 0, ...\} \tag{9}$$

3. Related formulas

$$\pi = 4\sum_{n=0}^{\infty} T^{-4n-1}\left(\frac{1}{4n+1} + \frac{2(-1)^n T^{-1}}{4n+2} - \frac{T^{-2}}{4n+3}\right) \tag{10}$$

$$\pi = \frac{17}{3T} + \frac{139}{105T^5} + \sum_{n=2}^{\infty} T^{-4n-1}\left(\frac{4}{4n+1} + \frac{4(-1)^n}{4n+2} - \frac{1}{4n+3} - \frac{4(-1)^n}{4n-2} - \frac{2}{4n-1} - \frac{1}{4n-5}\right) \tag{11}$$

4. Representations for Tribonacci constant

$$T = \frac{1}{3} + \frac{1}{3}\sqrt[3]{38 + 12\sqrt[3]{38 + 12\sqrt[3]{38 + ...}}} \tag{12}$$

$$T = \frac{1}{3} + \frac{1}{3}\sqrt{12 + \frac{38}{\sqrt{12 + \frac{38}{\sqrt{12 + ...}}}}} \tag{13}$$

$$\frac{1}{T} = -\frac{1}{3} + \frac{1}{3}\sqrt[3]{34 - 6\sqrt[3]{34 - 6\sqrt[3]{34 - ...}}} \tag{14}$$

$$\frac{1}{T}=\frac{\sqrt[3]{3\sqrt{33}+17}-\sqrt[3]{3\sqrt{33}-17}-1}{3} \tag{15}$$

5. Machin’s formula for pi

$$\pi = 4\tan^{-1}\left(\frac{1}{T}\right)+4\tan^{-1}\left(\frac{1}{T^{2}}\right) \tag{16}$$

References

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