MAXWELL FIELD EQUATIONS IN EUCLIDEAN RELATIVITY

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Abstract: In this work we formulate Maxwell field equations in Euclidean relativity. Since there is no upper limit for the speed of transmission in Euclidean relativity, the Euclidean relativistic electromagnetic field and the Euclidean relativistic Dirac field may be applied to rectify the EPR paradox in quantum entanglement.

In quantum mechanics, spontaneous emission is a process in which a quantum particle is emitted from an excited quantum mechanical system. The emitted quantum particle may be a photon and the quantum mechanical system may be a hydrogen atom. Even though the current interpretation of radiation in quantum mechanics states that spontaneous emission is a probabilistic quantum process and cannot be explained by the classical theory of electromagnetism is mathematically plausible, it does not prevent us to search for a possible classical connection so that we can understand this quantum phenomenon in a causal manner. If we follow the latter path and regard radiation as a classical process then many questions can be raised within the framework of classical dynamics. In particular, if the emitted photon is regarded as a physical object then we may ask how it will attend its maximum speed from the time it is emitted and what physical laws underneath the physical process. We may then ask a further question about whether the maximum speed is finite or infinite, depending on the underneath physical laws are pseudo-Euclidean or Euclidean relativistic, respectively. In fact, as shown in the following, the maximum speed with a finite value is connected with Maxwell field equations in pseudo-Euclidean relativity in which the law of conservation of energy is presumed and an infinite value of the maximum speed is associated with Maxwell field equations in Euclidean relativity in which energy is created continuously during the transmission of the electromagnetic wave. If the physical laws are pseudo-Euclidean relativistic then as shown in our works on temporal dynamics it is possible to formulate physical laws that govern the speed limit of the photon [1]. We showed that the Planck's quantum of energy $W = h\nu = h/T$ can be written in an integral form as follows

$$W = \frac{h}{T} = \int_{T}^{\infty} \frac{h}{t^2} dt \tag{1}$$

On the other hand, in classical mechanics, the work done W given in Equation (1) by a force **F** that moves an object with velocity **v** from time t_1 to time t_2 is defined as

$$W = \int_{t_1}^{t_2} \mathbf{F}.\mathbf{v}dt \tag{2}$$

From Equations (1) and (2) we obtain the relation

$$\mathbf{F}.\mathbf{v} = \frac{h}{t^2} \tag{3}$$

For the case of the emission of a quantum particle, if we assume $\mathbf{F}.\mathbf{v} = Fv$ and apply Newton's second law $F = m\,dv/dt$, then the following equations are obtained

$$F = \frac{h}{vt^2} \tag{4}$$

$$mv\frac{dv}{dt} = \frac{h}{t^2} \tag{5}$$

If we consider the condition that at the initial time $t_1 = T$ the velocity of the particle is $v = v_0$ then solutions to Equation (5) are found as

$$\frac{mv^2}{2} = \frac{mv_0^2}{2} + h\left[\frac{1}{T} - \frac{1}{t}\right] \tag{6}$$

Taking the positive sign for v from Equation (6) we obtain the causal propulsive force

$$F = \frac{h}{t^2\sqrt{v_0^2 + \frac{2h}{m}\left[\frac{1}{T} - \frac{1}{t}\right]}} \tag{7}$$

The negative sign of v from the solutions (6) may be considered when a quantum particle is being absorbed by a quantum system.

The fact that the maximum speed of the emitted photon has a finite value is also implied in Maxwell field equations in which the law of conservation of energy is complied. In electromagnetism, Faraday induction law can be expressed in the following form

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} \tag{8}$$

where $\mathcal{E}$ is the induced electromotive force and Φ is the magnetic flux. The negative sign specifies Lenz's law which states that the induced current creates an induced magnetic field so that it opposes the change of the original magnetic field. The Faraday induction law is a macroscopic law which is generalised to one of Maxwell field equations written in terms of the electric field **E** and the magnetic field **B** in differential form as

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{9}$$

The negative sign in the Maxwell equation given in Equation (9) is significant because it indicates the law of conservation of energy. Despite the fact that we do not know the intrinsic mechanism underneath the classical dynamics of the photon, which may be determined by the relationship between the geometric structures of the electric field **E** and the magnetic field **B**, it is reasonable to infer that since energy is not being created during the transmission of the electromagnetic wave the speed of transmission should remain constant. Even though the law of conservation of energy has been demonstrated to be one of the most fundamental laws in physics, there still remain questions whether the law is universal, in particular, due to the uncertainty principle it cannot be applied rigorously in the quantum domain. Even in macroscopic phenomena the conservation law of energy cannot be applied thoroughly unless unknown and unobservable physical processes are assumed. For example, it has been observed that the observable universe is expanding with an accelerating rate. According to classical mechanics, this requires a supply of energy and it has been suggested that the source of the required energy could be dark matter. However, instead of assuming the existence of dark matter, we may explain the accelerating rate of the expansion of the observable universe by assuming that the law of conservation of energy is not applied in this case but rather the accelerating rate is due to unknown dynamical relationships between established physical objects that are responsible for the expansion. It could be that the macroscopic expansion is due to the microscopic interactions between quantum objects. Furthermore, if we adhere to Einstein's ideas expressed in the general theory of relativity that physical laws are manifestations of the dynamical relationships between intrinsic geometric structures of the spacetime continuum then matter and energy should be regarded as geometrical objects that determine the structures of spacetime manifold. It could even be that matter and dark matter are simply different forms of geometrical structures. With this general relativistic view, the electric field **E** and the magnetic field **B** in the Maxwell equation given in Equation (9) are more likely to be related to the rates of change of intrinsic geometric objects, such as the Ricci scalar curvature, of the spacetime continuum and the identification of physical entities with mathematical objects allows us to formulate physics purely in terms of differential geometry and topology [2]. It may also be suggested that a complete picture of quantum dynamics would require a mathematical formulation of quantum particles as three-dimensional differentiable manifolds in which physical laws should be established from the Ricci flow and the quantum particles classified according to Thurston geometries [3,4].

In the case of Euclidean relativity in which the speeds of transmission have no upper values and if we assume that the speeds of transmission is related to the overall energy of the wave then we may ask what change should be made in order to have energy being continuously created during the transmission so that the speed of transmission can increase continuously to attend its infinite value? From Faraday induction law, it can be seen that this can be done by converting Equation (9) into the following equation by ignoring the negative sign

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \tag{10}$$

The relationship between the electric and magnetic fields given in Equation (10) states that the magnetic field that creates the induced current and the induced magnetic field have the same direction and they would add up to form a stronger magnetic field and create more energy. Since energy is being continuously created during the process of transmission it could be suggested that the created energy is the source for the speed of transmission to increase and attend infinite value, which is allowed by Euclidean relativity. In fact, we will show in the following that the change from Equation (9) to Equation (10) is the only change that is needed in order to convert Maxwell field equations in pseudo-Euclidean relativity to Maxwell field equations in Euclidean relativity. And this change is implied in our formulation of Euclidean relativistic Maxwell field equations from a general system of linear first order partial differential equations. A general system of linear first order partial differential equation can be written as follows [5,6]

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{r}\frac{\partial\psi_i}{\partial x_j} = k_1\sum_{l=1}^{n} b_l^r\,\psi_l + k_2 c^r, \qquad r = 1, 2, \dots, n \tag{11}$$

The system of equations given in Equation (1) can be rewritten in a matrix form as

$$\left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)\psi = k_1\sigma\psi + k_2 J \tag{12}$$

where $\psi = (\psi_1, \psi_2, \dots, \psi_n)^T$, $\partial\psi/\partial x_i = (\partial\psi_1/\partial x_i, \partial\psi_2/\partial x_i, \dots, \partial\psi_n/\partial x_i)^T$, A_i, σ and J are matrices representing the quantities a_{ij}^k, b_l^r and c^r, and k_1 and k_2 are undetermined constants. Now, if we apply the operator $\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}$ on the left on both sides of Equation (12) then we have

$$\left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)\left(\sum_{j=1}^{n} A_j\frac{\partial}{\partial x_j}\right)\psi = \left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)(k_1\sigma\psi + k_2 J) \tag{13}$$

If we assume further that the coefficients a_{ij}^k and b_l^r are constants and $A_i\sigma = \sigma A_i$, then Equation (13) can be rewritten in the following form

$$\left(\sum_{i=1}^{n} {A_i}^2\frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{n}\sum_{j>i}^{n}\left(A_iA_j + A_jA_i\right)\frac{\partial^2}{\partial x_i\partial x_j}\right)\psi = k_1^2\sigma^2\psi + k_1k_2\sigma J + k_2\sum_{i=1}^{n} A_i\frac{\partial J}{\partial x_i} \tag{14}$$

In order for the above systems of partial differential equations to be used to describe physical phenomena, the matrices A_i must be determined. As for the case of Dirac equation, Equation (14) reduces to the following equation

$$\left(\sum_{i=1}^{n} {A_i}^2\frac{\partial^2}{\partial x_i^2}\right)\psi = k_1^2\sigma^2\psi + k_1k_2\sigma J + k_2\sum_{i=1}^{n} A_i\frac{\partial J}{\partial x_i} \tag{15}$$

if the matrices A_i satisfy the following commutation relations [7]

$${A_i}^2 = \pm 1 \tag{16}$$

$$A_iA_j + A_jA_i = 0 \tag{17}$$

On the other hand, for the case of Maxwell field equations in pseudo-Euclidean relativity, the matrices A_i must take a form so that Equation (14) also reduces to a wave equation for the components of the wavefunction $\psi = (\psi_1, \psi_2, \dots, \psi_n)^T$ similar to Equation (15) even though the matrices A_i do not satisfy the commutation relations given in Equations (16-17). As in the case of formulation of Maxwell field equations from a general system of linear first order partial differential equations [8], we will seek to formulate Maxwell field equations in Euclidean relativity from the equation given in Equation (12) written the following matrix form

$$\left(A_0\frac{\partial}{\partial t} + A_1\frac{\partial}{\partial x} + A_2\frac{\partial}{\partial y} + A_3\frac{\partial}{\partial z}\right)\psi = A_4 J \tag{18}$$

where $\psi = (\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6)^T$, $J = (j_1, j_2, j_3, 0,0,0)^T$ with the matrices A_i are now given in the following forms

$$A_0 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_4 = \begin{pmatrix} \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{19}$$

From the matrix forms given in Equation (19) we obtain

$$A_0^2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad A_1^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad A_2^2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_3^2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_4^2 = \begin{pmatrix} \mu^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_1A_2 + A_2A_1 = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_1A_3 + A_3A_1 = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

$$A_2A_3 + A_3A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$A_0A_i + A_iA_0 = 0 \quad \text{for} \quad i = 1, 2, 3 \tag{20}$$

Now, if we apply the differential operator $(A_0\,\partial/\partial t + A_1\,\partial/\partial x + A_2\,\partial/\partial y + A_3\,\partial/\partial z)$ to Equation (18) then we arrive at

$$\left(\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\frac{\partial^2}{\partial t^2} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\frac{\partial^2}{\partial x^2} + \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\frac{\partial^2}{\partial y^2} + \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\frac{\partial^2}{\partial z^2} \right.$$
$$+ \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\frac{\partial^2}{\partial x \partial y} + \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}\frac{\partial^2}{\partial x \partial z}$$
$$\left. + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}\frac{\partial^2}{\partial y \partial z}\right)\psi = -\left(\begin{pmatrix} \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\frac{\partial}{\partial t}\right)J \tag{21}$$

From the equation given in Equation (21), we obtain the following equations for the Euclidean relativistic electric field $\mathbf{E} = (E_x, E_y, E_z) = (\psi_1, \psi_2, \psi_3)$

$$\frac{\partial^2\psi_1}{\partial t^2} + \frac{\partial^2\psi_1}{\partial y^2} + \frac{\partial^2\psi_1}{\partial z^2} - \frac{\partial}{\partial x}\left(\frac{\partial\psi_2}{\partial y} + \frac{\partial\psi_3}{\partial z}\right) = -\mu\frac{\partial j_1}{\partial t} \tag{22}$$

$$\frac{\partial^2\psi_2}{\partial t^2} + \frac{\partial^2\psi_2}{\partial x^2} + \frac{\partial^2\psi_2}{\partial z^2} - \frac{\partial}{\partial y}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_3}{\partial z}\right) = -\mu\frac{\partial j_2}{\partial t} \tag{23}$$

$$\frac{\partial^2\psi_3}{\partial t^2} + \frac{\partial^2\psi_3}{\partial x^2} + \frac{\partial^2\psi_3}{\partial y^2} - \frac{\partial}{\partial z}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}\right) = -\mu\frac{\partial j_3}{\partial t} \tag{24}$$

It is seen from Equations (22-24) that if the components (ψ_1, ψ_2, ψ_3) obey Gauss's law

$$\boldsymbol{\nabla}.\mathbf{E} = \frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y} + \frac{\partial\psi_3}{\partial z} = \frac{\rho_e}{\epsilon} \tag{25}$$

then the wave equation for the electric field $\mathbf{E} = (E_x, E_y, E_z) = (\psi_1, \psi_2, \psi_3)$ can be written as follows

$$\frac{\partial^2\mathbf{E}}{\partial t^2} + \nabla^2\mathbf{E} = \nabla\left(\frac{\rho_e}{\epsilon}\right) - \mu\frac{\partial\mathbf{J}_e}{\partial t} \tag{26}$$

where $\mathbf{J}_e = (j_1, j_2, j_3)$. Similarly for the magnetic field $\mathbf{B} = (B_x, B_y, B_z) = (\psi_4, \psi_5, \psi_6)$ we obtain the following equations

$$\frac{\partial^2\psi_4}{\partial t^2}+\frac{\partial^2\psi_4}{\partial y^2}+\frac{\partial^2\psi_4}{\partial z^2}-\frac{\partial}{\partial x}\left(\frac{\partial\psi_5}{\partial y}+\frac{\partial\psi_6}{\partial z}\right)=0 \tag{27}$$

$$\frac{\partial^2\psi_5}{\partial t^2}+\frac{\partial^2\psi_5}{\partial x^2}+\frac{\partial^2\psi_5}{\partial z^2}-\frac{\partial}{\partial y}\left(\frac{\partial\psi_4}{\partial x}+\frac{\partial\psi_6}{\partial z}\right)=0 \tag{88}$$

$$\frac{\partial^2\psi_6}{\partial t^2}+\frac{\partial^2\psi_6}{\partial x^2}+\frac{\partial^2\psi_6}{\partial y^2}-\frac{\partial}{\partial z}\left(\frac{\partial\psi_4}{\partial x}+\frac{\partial\psi_5}{\partial y}\right)=0 \tag{29}$$

$$\boldsymbol{\nabla}.\mathbf{B}=\frac{\partial\psi_4}{\partial x}+\frac{\partial\psi_5}{\partial y}+\frac{\partial\psi_6}{\partial z}=0 \tag{30}$$

$$\frac{\partial^2\mathbf{B}}{\partial t^2}+\nabla^2\mathbf{B}=0 \tag{31}$$

In terms of the components $(\psi_1,\psi_2,\psi_3,\psi_4,\psi_5,\psi_6)$, with the forms of the matrices A_i given in Equation (19), Maxwell field equations in Euclidean relativity are written out as follows

$$-\frac{\partial\psi_1}{\partial t}+\frac{\partial\psi_6}{\partial y}-\frac{\partial\psi_5}{\partial z}=\mu j_1 \tag{32}$$

$$-\frac{\partial\psi_2}{\partial t}+\frac{\partial\psi_4}{\partial z}-\frac{\partial\psi_6}{\partial x}=\mu j_2 \tag{33}$$

$$-\frac{\partial\psi_3}{\partial t}+\frac{\partial\psi_5}{\partial x}-\frac{\partial\psi_4}{\partial y}=\mu j_3 \tag{34}$$

$$\frac{\partial\psi_4}{\partial t}-\frac{\partial\psi_3}{\partial y}+\frac{\partial\psi_2}{\partial z}=0 \tag{35}$$

$$\frac{\partial\psi_5}{\partial t}-\frac{\partial\psi_1}{\partial z}+\frac{\partial\psi_3}{\partial x}=0 \tag{36}$$

$$\frac{\partial\psi_6}{\partial t}-\frac{\partial\psi_2}{\partial x}+\frac{\partial\psi_1}{\partial y}=0 \tag{37}$$

In vector form, with $\epsilon\mu=1$, Maxwell field equations of the electromagnetic field in Euclidean relativity can be written as

$$\boldsymbol{\nabla}.\mathbf{E}=\frac{\rho_e}{\epsilon} \tag{38}$$

$$\boldsymbol{\nabla}.\mathbf{B}=0 \tag{39}$$

$$\boldsymbol{\nabla}\times\mathbf{E}-\frac{\partial\mathbf{B}}{\partial t}=0 \tag{40}$$

$$\boldsymbol{\nabla}\times\mathbf{B}-\frac{\partial\mathbf{E}}{\partial t}=\mu\mathbf{j}_e \tag{41}$$

where the charge density ρ_e and the current density $\mathbf{j}_e$ satisfy the conservation law

$$\nabla . \mathbf{j}_e + \frac{\partial \rho_e}{\partial t} = 0 \tag{42}$$

It is noted that the only difference between Maxwell field equations in pseudo-Euclidean relativity and Maxwell field equations in Euclidean relativity is negative sign in front of the term $\partial \mathbf{B}/\partial t$ in Equation (40). This change of sign allows energy to be created continuously during the process of transmission of the Euclidean relativistic electromagnetic wave.

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