

A 2018 Cosmological Redshift Data Set Analysis

douG Snell

Independent Researcher at The Ohio State University Columbus, Ohio 43210 March 2018

Abstract- This paper is an analysis of three data sets modeled to represent the physical processes of single mass points in a well defined geometric space. The First data set represents gravitational contraction modeled as derivative of Einstein Field Equation(s), a well confirmed accepted equation with theory. The Second data set modeled is a reverse of the First data set. This is a model of the physical processes of expansion of single mass points in well defined space using the reverse of First data set's modeled physical processes. The Third data set represents the Hubble Space Telescope Key Project cosmological redshift data set modeled as single mass points, representing the Hubble expansion. All three data sets are modeled with respective physical processes, represented by 3 dimensional cubes with vector arrows plotted, then analyzed by comparison of three individual single dimension slices of each of the three data set models. The three dimensional slices are analyzed, comparing one slice from each of the x, y z axis with each of the three data sets. The Second data set when compared to the Third data set provides significant findings. The Third data set indicates physical processes limited to a single dimension, the z axis, giving a appearance of a self-centered expansion. The First and Second data set Point of View analysis indicate no contradiction, the Third data set Point of View analysis indicates a contradiction.

Introduction

This is an analysis of three data sets representing simple models of the physical processes of mass points in a well defined geometric space.

In each data set model is presented a progression of physical processes. This is modeled as vector arrows (tangent vectors) plotting the function of the physical processes of the mass points within the geometric space. The direction of the mass points is determined and mapped as vector

fields plotted with vector arrows (ref 2 and 4). The magnitude of the mass point is not important for this demonstration. For each single mass point's physical process in each (x,y,z) plane of the geometric space, there is a vector-arrow indicating the direction of the mass point with respect to the other mass points within the model.

The models are simple, indicating the physical processes and the geometric direction of the mass points with respect to each other during the progression of the model.

First data set- The First data set is based on a set of single mass points in a sphere with no initial motion with respect to each other. This data set is then simulated having physical processes in progress to model “contraction” derivative of the Einstein Field Equation(s).

This is an initial data set ‘test’ model representing physical processes of general relativity exhibiting the invariance/covariance of the mass points’ physical processes on each axis. Each point having energy density and its pressure in relations to the other mass points creates the physical process modeled, contraction, indicated by vectors arrows of the model and Exhibit #1.

This First data set is simply to exhibit contraction in accordance with general relativity and the Field Equation(s), a very well verified and accepted theory.

The first data set is labeled “Vectors plotted on a 3d cube representing the physical processes of contraction via the Field Equations”.

This contracting data set modeled is meant to simulate a contracting set of mass points, or, the Universe contracting in accordance with Field Equation without a Field Equation Cosmological Constant:

$$(1) \quad G_{\alpha\beta} = T_{\alpha\beta}$$

Equation (1) indicates Energy Density (left side) equates to Pressure P (right side, ref 3 pg1).

In a sphere of test mass points (particles) at rest relative of each other, we may let $V(t)$ be the volume of the sphere as measured by the mass point at the center of the sphere:

$$(2) \quad \frac{\ddot{V}}{V}|_{t=0} = -\frac{1}{2} \left(\begin{array}{l} \text{flow-of-t-momentum-in-t-direction} + \\ \text{flow-of-x-momentum-in-x-direction} + \\ \text{flow-of-y-momentum-in-y-direction} + \\ \text{flow-of-z-momentum-in-z-direction} \end{array} \right)$$

And the Einstein Equation (ref 3 pg4) may be summarized as:

$$(3) \quad \frac{\ddot{V}}{V}|_{t=0} = -\frac{1}{2} (p + P_x + P_y + P_z)$$

Gravitational Collapse- “Another remarkable feature of Einstein’s equation is the pressure term: it says that not only energy density but also pressure causes gravitational attraction.” Ref 3 pg5. See equation (1) (2) and (3).

Equation(s) (1) (2) and (3) are used to model the First data set. The 3 slices represent each axis and correlate P_x , P_y and P_z respectively. The model’s physical processes in progression are mapped over time.

The model of the First “test” data set, and the First data set cube model, is simply based on the Field Equation(s) and the natural contraction physical processes derivative of the Field Equation(s) without any cosmological constant.

Particularly noticed, represented by equation (3), P Pressure on the x y and z axis has an invariance implied by and in the equation (3) and (1), this invariance/covariance is of particular interest and emphasis in this analysis. Other way to explain this is from Steven Weinberg, the Principle of General Covariance (PGC) and “The equation is generally covariant; that is, it preserves its form under a general coordinate transformation $x \rightarrow x'$.” (ref 21)

Clearly stated, gravity gravitates equally and indiscriminately on all three axis x , y , z of geometric space.

The test modeled sphere (cube) is modeled, measured and analyzed, and has V being the volume of the sphere of mass points in the well defined geometric space. V being the volume of the geometric space with the data set of dust particles or of the Universe.

Definition- This “test” model First data set models the physical processes functionally derivative of the Field Equation, and, is mapped with vector arrows indicating the mass points direction relative the other mass points as time progresses.

Second data set- The second data set is the reverse of the First data set. The second model data set is another “test” data set and simply a reverse of the physical processes of the First “test” data set.

The physical processes of the first data set “contraction” are simply reversed to “expansion” and analyzed the same as the First data set is analyzed.

This Second data set is modeling expansion in accordance with reverse general relativity or a “reverse” of the Field Equation(s) “contraction”. The implications of expansion comes with its own constraints requiring unknown ‘energy density’ opposing gravity, Dark Energy the hypothesis cause of Hubble expansion (ref 8).

The physical processes of the First data set are reversed and then mapped showing the vectors arrows the opposite or reverse of the First data set. The second data set is labeled “ Vectors plotted representing expansion via reversing the General Relativity physical processes of Data Set #1”

Please note the expansion is in reverse of what is naturally indicated by the Field Equation(s) physical processes, and, the expansion physical processes would require some ‘unknown’ force opposing gravity.

This Second “test” data set is intended to indicate expansion via the reverse of the Field Equation(s) natural contraction physical processes.

Third data set- The Third data set is a model of the Hubble Space Telescope Key Project data set. This data set is the subject of our analysis, and, concerns cosmological redshift.

The cosmological redshift of the HST Key Project data set is modeled as mass points expanding similar to the Second data set expanding. The Third data set’s modeled physical processes vector arrows are plotted for analysis.

The physical processes of the mass points under Hubble expansion, The Hubble Constant (ref 1), is the data modeled then analyzed in the Third analysis. “This is convenient because the redshift is observable and usually has a great effect on the rates of physical processes.” (ref 6 pg 2)

The SoO- The SoO Sphere of Observation, the Hubble sphere, is similar to the Observable Universe. The term SoO is used for reasons of being a more exacting description and limits the confusion (ref 5).

Definition- The SoO is the physical geometric sphere from which light reaches the observer, including any observer of any geometric position. The SoO term brings focus to our Sphere of Observation, being a geometric sphere, with the pedagogue’s understanding of light’s physical processes within the SoO.

Aim- Although this analysis analyzes modeled mass points and physical processes of data sets, the implications of analyzing 'physical process within a geometric space' correlates to our SoO and what is observed and interpreted. This analysis aim is to improve interpretations of cosmological redshift to improve photometric estimates.

Clearly highlighted by the HST Key Project's data set, the data is regarding cosmological redshift and recession velocities, an effect of physical processes affecting light within our SoO (ref 6).

The interpretation of cosmological redshift has our attention.

Hubble's Constant- The Third data set is from the Hubble Space Telescope Key Project to analyze the Hubble Constant. The HST Key Project concluded the best estimate for Hubble's Constant expansion from a weighted average estimated at: " $H_0 = 72 \pm 8.0$ units" (Ref 1 Section 7 Table 12).

The Hubble expansion based on the Hubble Constant is directly from cosmological redshift, described as: " Thus, this distance increases (or decreases) with the scale factor $a(t)$. The information about if $a(t)$ is increasing, decreasing or constant comes from astronomical observations, and these ones tell us that there is a (red)shift in the spectral lines from distant galaxies. Then, under the hypothesis of expansion of the universe, the proper distances $D(t)$ between distant galaxies (located in fixed co-moving positions χ) must be increasing with time, because the scale factor $a(t)$ is growing. If $a(t)$ is increasing, then there will be a redshift in frequency given by $a_0/a(t)$, conventionally denoted by $1 + z$ and shown" (ref 6 pg3).

The z of $(1 + z)$ is directly associated to the z axis (ref 1, 6, 11, 15, 17, 18, 19, 20). Hubble's expansion is on the z axis, and, the observer regardless its location will perceive themselves as the center of the Hubble expansion. "So (cosmological) redshift is related to the expansion factor of the Universe" (ref 7 pg4) "Therefore, just angular

distance and its derivative (in time) are the most natural characteristics of the Hubble flow from the observer's point of view." (ref 20)

The Robertson-Walker Metric equation is of interest to this topic. In RW metric equation the evolution of the Universe is governed by the scale-factor $R(t)$. This scale-factor $R(t)$ within the RW metric (ref 9) is also relative to the z component discussed above (ref 7, 10 & 15).

Required Unknowns- Both the Third data set and the Second data set would require an 'unknown' force, the cause of expansion and mass points or galaxies' recession velocities, opposing gravity (ref 8), and, reversing the First data set's natural physical progression in conformity with the Field Equations and Equations (1), (2) and (3).

Clearly expansion comes with its own unknowns. Dark Energy (DE) opposing gravity is one hypothesized cause of Hubble expansion (ref 8). Significant unknowns as components of the Hubble expansion explanations infer tension or contradictions (ref 5, 15, 18). "In the context of the new standard Λ CDM cosmology we point out confusions regarding the particle horizon, the event horizon, the "observable universe" and the Hubble sphere (distance at which recession velocity = c)." (Ref 5 pg1)

That recession velocity = c is of the z axis of the Hubble's expansion (model). Hubble expansion gives perspective to every observer, and, that each observer is the center of the Hubble expansion (ref 16, 17, 18, 19, 20). This becomes our focus and concern, being a possible source of tension and contradiction. As the observer moves so does their Hubble sphere, the center of expansion, and, the observer regardless of their location is perceived as the center of the HST Key Project data set expansion (ref 17, 18, 19 20), call it self-centered expansion.

This center of the z axis Hubble expansion has a correlation of the observer perceiving self-centered expansion. "What transforms this bland diagram into a profound discovery is an understanding that the pattern Hubble found is exactly what you would expect for any observer

in a universe expanding in all directions. Hubble's diagram does not imply that we are at the center of the universe” (Ref 19, 20). Being significant, the observer “appears” and perceive itself as the center of Hubble expansion according to the Third data set, yet that does not imply they are the center of the entire Universe (ref 15, 19, 20).

The mass points in the HST study’s Hubble sphere data set are modeled similar the single mass points represented in the First & Second data sets.

The Third data set model is labeled “Vectors plotted representing the cosmological redshift physical processes of Hubble’s expansion” in the exhibit#3

The labeling and plotting of the Third data set are self evident from Hubble expansion theory, by the references and the method and analysis process of the First and Second ‘test’ data sets.

Method- Each data set is analyzed by examining the physical processes plotted as vector-arrows of the single mass points on all three axis of each data set during the progression of the model over time, and, Point of View analysis:

Data sets-

First- “Test” data set of general relativity contraction, mapping and plotting vector arrows.

Second- “Test” data set being the reverse of the First data set contraction, or, expansion under relativity constrains, mapping and plotting vector arrows and,

Third- Hubble’s expansion exhibited by modeling the HST Key Project data set and mapping and plotting vector arrows.

Analysis- 1- analyzing the data set model on all three dimensions being the x, y, z axis, with particular attention given to the invariance/covariance of energy density and P Pressure of the Field Equation(s) regarding P_x , P_y and P_z equation (3).

2- Point of View analysis on the Second & Third sets.

The three physical processes of the data sets are modeled, exhibited by cube(s) of the space with vector arrows, exhibit #1, #2 and #3, and then analyzed by taking sample slices of each cube. Each of the three slices of the three data sets represents one slice each of the x, y, and z axis, nine slices analyzed in total or 3 from each data set model. The point of view analysis simply plots two observers and analysis what each observer perceives of the mass points physical processes in relation to the other observer and mass points.

Each of the three single dimension slices exhibit the physical processes evident as each data set progresses over time simply by the correlating vectors (arrows) on each slice for each axis. Comparisons of each of the data sets' slices are made by analysis of the physical processes of the mass points on each axis of each data set.

Findings and conclusions are self evident from the comparison analysis of each data set slice for each axis. The interesting results exhibited by diagrams of each slice representing each axis for each of the three data sets, clearly exhibit the physical processes modeled in the well defined space. Each x, y, z, axis exhibited by slice in each of the three data sets exhibits a modeled physical process, or lack thereof; please be reminded of gravity's invariance, gravity acts indiscriminately.

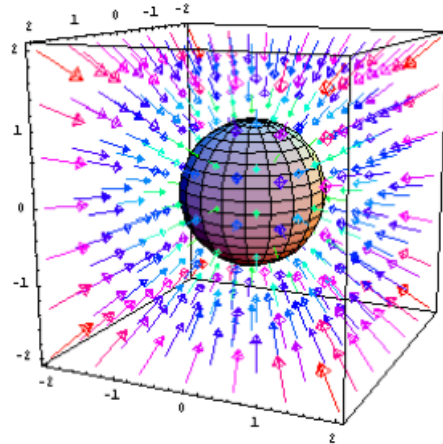
The Data Sets Modeled

First- The First data set is simply a data set "test" model to represent a sphere of single mass points proceeding with physical processes in accordance with general relativity. More broadly this data set may represent the Universe occupied by baryonic matter contracting via the Field Equation(s). The vectors of the mass points, as the model progresses contracting, are plotted (arrows) to exhibit the physical processes of the mass points as time passes.

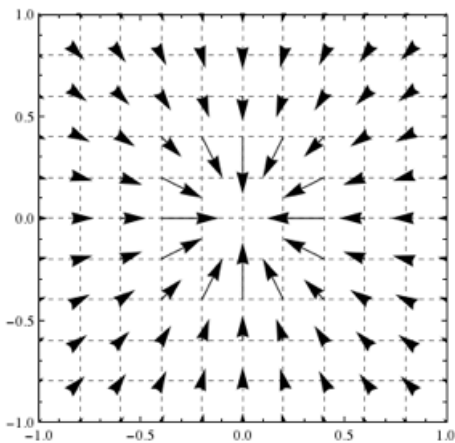
Exhibit #1 exhibits the First data set model progress by plotting the indicated vectors:

Vectors plotted on a 3d cube representing the physical processes of contraction via the Field Equation(s)

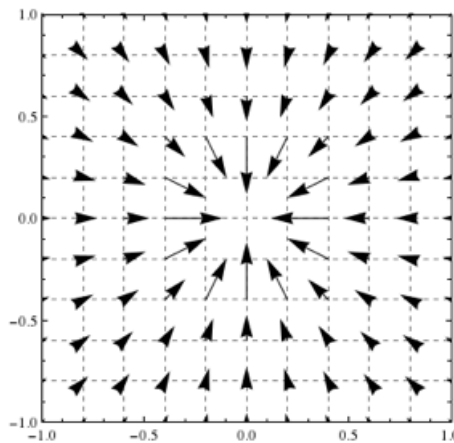
**Contraction via
General Relativity
Physical processes**



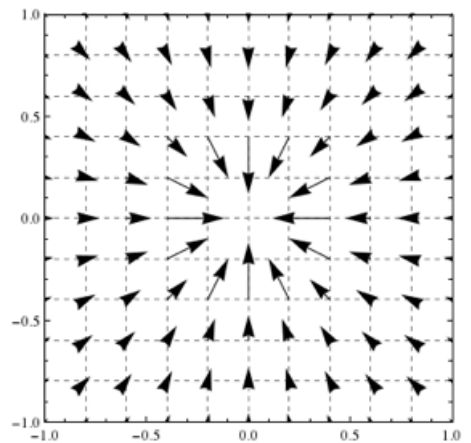
**Slice of 3d cube
for each axis**



x axis



y axis



z axis

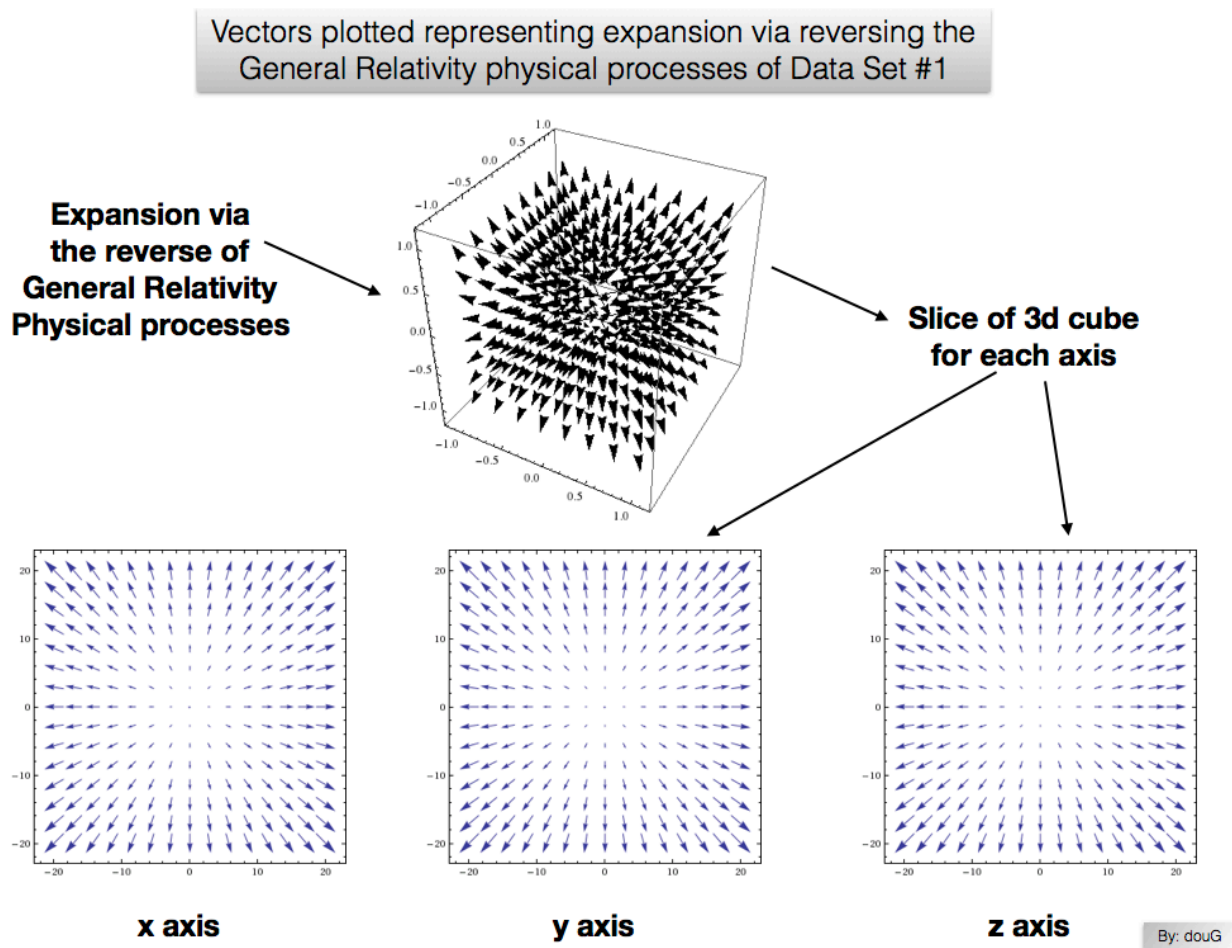
By: douG

The sphere-mass in the center of the above 3d cube is for visual effect only, to represent the accumulation of baryonic matter mass points in the center of the data set during progression of the physical processes.

Second-The Second data set is a test model cube to represent a sphere of single mass points. Generally this data set is intended to represent the Universe occupied by baryonic matter expanding via reversing the Field Equation(s) natural physical process progressions. This data set mass points are modeled to start with a state of no motion with respect to each

other. Then this model progresses in reverse of the First data sets' physical processes. This model progresses in reverse of General Relativity contraction, i.e. expanding via relativity reversed. The vectors of the mass points as the model progresses expanding are plotted by arrows to exhibit the physical processes of the mass points as time progresses. Magnitude is of no concern, direction is the concern.

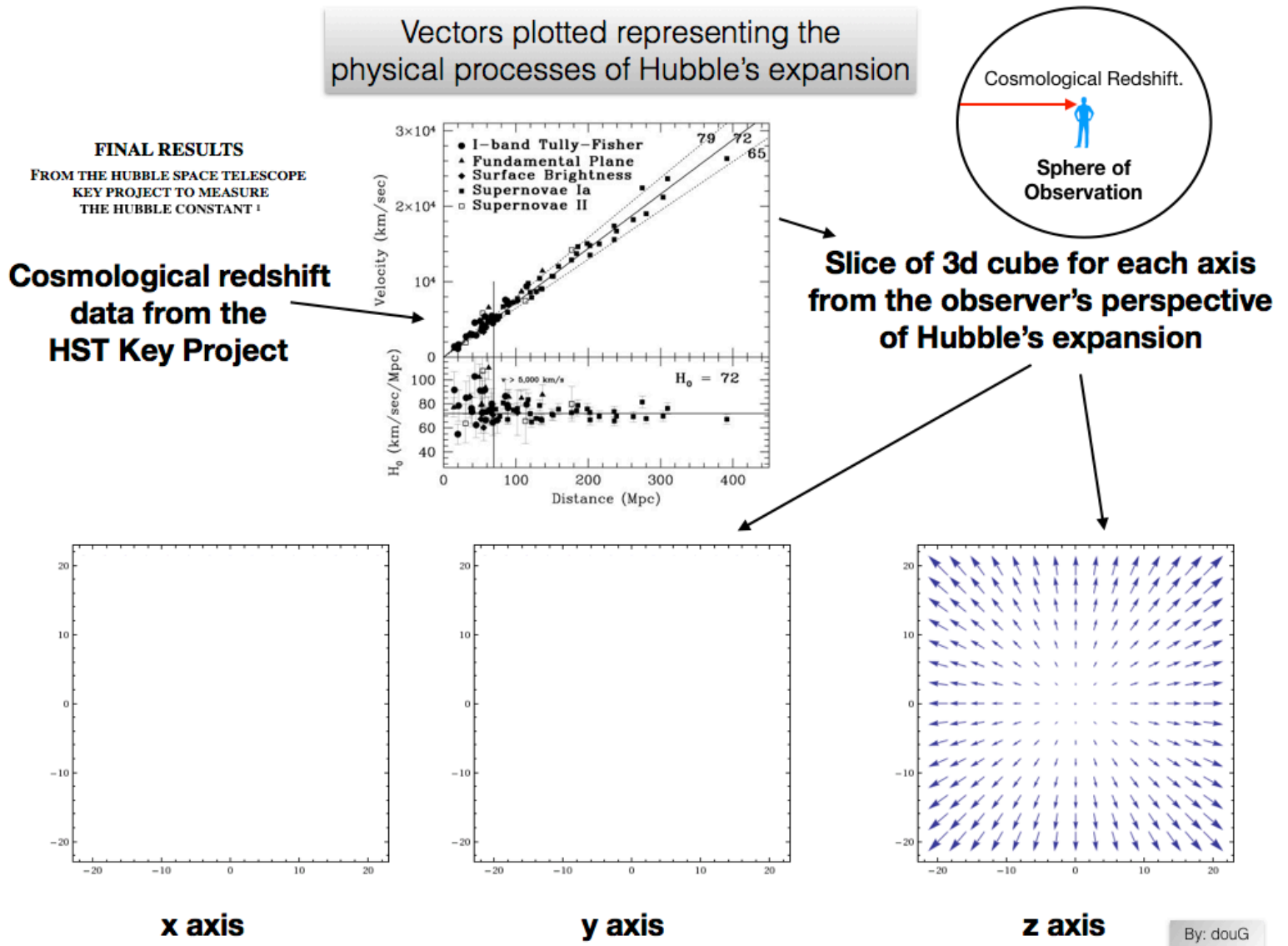
Exhibit #2 exhibits the Second data set model progression by plotting the indicated vector arrows of the mass points:



Third- The third data set is a model to represent the mass points physical process interpreted via Hubble's expansion and the HST Key Project data set of cosmological redshift. In this data set the cosmological redshift data set of the HST Key Project is plotted as a uniform set of mass points in the well defined area representing our

SoO. In this instance it can be properly inferred our SoO is a fair representative sample area of the Universe indicating present physical processes.

Exhibit #3 exhibits the Third data set modeled physical processes by plotting the indicated vector arrows:



Analysis of x-y-z Axis

The analysis of the modeled data sets is comparatively simple. A comparison of the x, y, z, axis slices from each data set can be compared. In particular, the slices from the Second and Third data sets are compared for significant reason, both are of modeled expansion.

Please notice the absence of the physical processes of Hubble expansion on the x and y axis in the Third modeled HST Key Project data set.

Analysis of Point of View

The analysis of the Second and Third data set models is extended to the Point of View of all observers within the mass points geometric space.

The famous balloon analogy to explain the Hubble expansion and the HST Key Project data sets' physical process has considerable literature available explaining the modeled expansion. Hubble expansion is a well known and generally accepted theory, (see ref 15 (pg12-18), 14, 13, 12, 11).

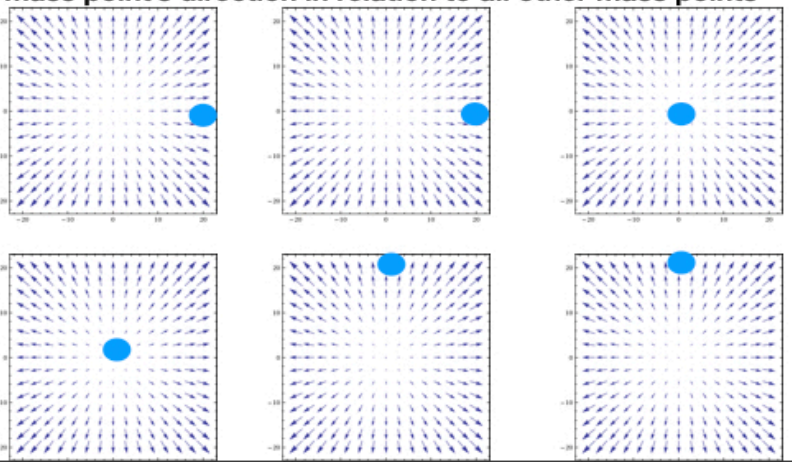
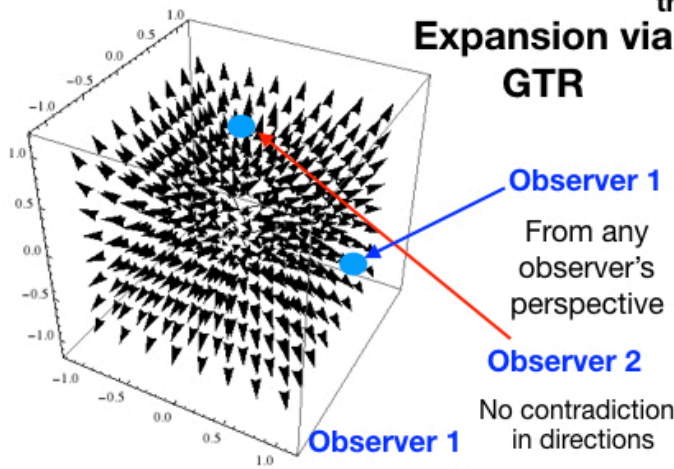
Clearly the Third data set expansion is as if all observers observe themselves as the center on the expansion. It is as if all/any observers simultaneously are the center of the Universal expansion. Essentially all observers are the perceived center of the famous 'balloon expanding' as explained and modeled (ref 16, 17, 18, 19, 20).

This phenomenon of observer self-centered Universal expansion now has our attention. This analysis turns to the physical processes of the self-centered expansion model from the Third data set of the HST Key Project.

The following exhibit is to help describe 1- the Second and Third data set observer's perspective and perception, and, 2- the actual verses "perceived" physical processes of the single mass points in relation to all other mass points.

Point of View plotted representing the Second and Third Data Sets

Each Observers observes the same of each mass point and the mass point's direction in relation to all other mass points

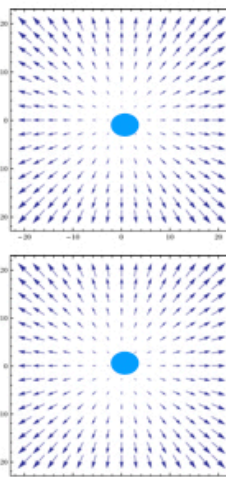
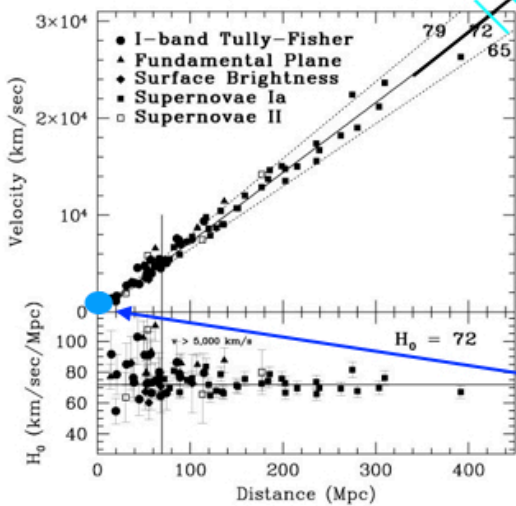


Each Observers perceives themselves as the center of expansion with distinct mass point directions in relation to all other mass points (observers)

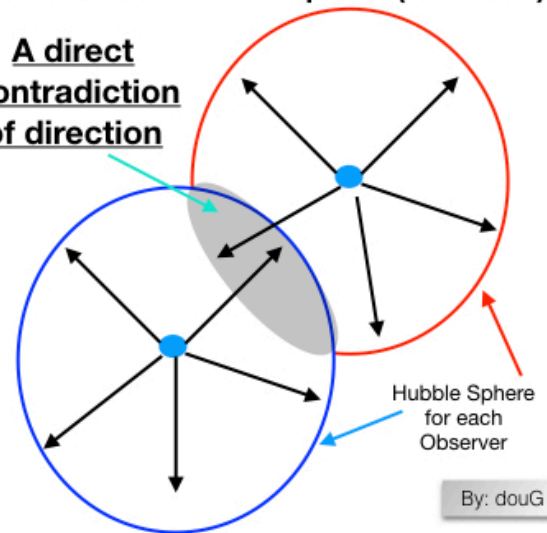
Hubble data set

Observer 2
a Self-centered Expansion Perception from each observer's perspective

Observer 1



A direct Contradiction of direction



By: douG

When plotting individual observers on the Second data set model, regardless of location of the observer, all mass points' physical

processes, their motion of directions and magnitude (vectors), are preserved in relation to all other mass points. All observers observe the same as any observer in relation to each other.

When plotting the individual observers on the Third data set model, the preservation the physical processes is lost, and the motion of direction and magnitude (vectors) of all mass points in relation to each other depends AND changes on the location of the observer(s). (ref 10 & 11)

Exhibited in the diagram above, the mass points within the grey shaded area would have contradicting directions and magnitudes (vectors), nearly the exact opposite, based strictly on the observer. This ‘contradiction’ within the Third data set modeled Point of View is significant and harbors considerable concern.

The vectors of the mass points in the Third data set model is based on the locations of the different observers modeled. The vectors of the Second data set is based on the vectors of the mass points in relation to all other mass points, regardless of the observer’s position within the mass points. The vector arrows of the Third data set mass points seem based on the observer’s self-centered expansion perceptions, rather than based on the magnitude and direction (vectors) of all mass points in relation to each other.

Conclusion- questions presented by the Author

Please ponder these questions:

- 1- How best might this analysis be used to facilitate photometric estimates and the interpretations of cosmological redshift?
- 2-Is the cosmological redshift of the HST Key Project data set the result of the red-shifting of light because of:
 - A- a unique Hubble expansion of the Universe and the physical processes of receding baryonic mass points from each other, or the expansion of space, restricted to the z axis in the defined

geometric space, observed as cosmological redshift and Hubble expansion, or,

B- might we reinterpret cosmological redshift as being merely an affect on light, observed on the z axis, having no geometric space expansion significance or recession velocity significance?

3- Is the Third data set modeled deducing that all observers should always be the 'perceived center' of Hubble expansion regardless the observer's location, and, is that significant?

4- Does the (direction) vector arrow of the observed mass points in relation to all other mass points, depend strictly on the observer's location in the Third data sets' expansion? Does that indicate that the Third data set observer's perception of physical processes (vectors) of the single mass point depend NOT on the direction (vector arrows) of the mass points in relation to all other mass points?

How are answers to questions #1-4 supported?

Could revolutionary improvements to photometric estimates and cosmological dynamics come by way of analysis & self correction?

"In a time of universal deceit - telling the truth is a revolutionary act."

(George Orwell)

References:

1- The Hubble Space Telescope Key Project -FINAL RESULTS FROM THE HUBBLE SPACE TELESCOPE KEY PROJECT TO MEASURE THE HUBBLE CONSTANT ¹

https://ned.ipac.caltech.edu/level5/Sept01/Freedman/Freedman_contents.html Wendy L. Freedman ², Barry F. Madore ^{2,3}, Brad K. Gibson ⁴, Laura Ferrarese ⁵, Daniel D. Kelson ⁶, Shoko Sakai ⁷, Jeremy R. Mould ⁸, Robert C. Kennicutt, Jr. ⁹, Holland C. Ford ¹⁰, John A. Graham ⁶, John P. Huchra ¹¹, Shaun M.G. Hughes ¹², Garth D. Illingworth ¹³, Lucas M. Macri ¹¹ and Peter B. Stetson ^{14,15} ¹ Based on observations with the NASA/ESA *Hubble Space Telescope*, obtained at the Space Telescope Science Institute, which is operated by AURA, Inc., under NASA Contract No. NAS 5-26555. etc

2- - Vectors and diagrams of vector arrows. A study guide from The Ohio State University <https://ximera.osu.edu/mooculus/calculus3/vectorFields/digInVectorFields>

3- The Meaning of Einstein's Equation John C. Baez* and Emory F. Bunn† January 4, 2006
*Department of Mathematics, University of California, Riverside, California 92521, USA. †Physics Department, University of Richmond, Richmond, VA 23173, USA

4- Source on Vectors and Vector Fields, and diagrams. A study guide from the Cambridge University Press <http://planning.cs.uiuc.edu/node379.html>

5- Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe <https://arxiv.org/abs/astro-ph/0310808> Tamara M. Davis
1 Charles H. Lineweaver 1 1 University of New South Wales, Sydney, Australia, 2052

6- What do we talk about when we speak of cosmological redshift? <https://arxiv.org/abs/1802.02444> Gabriel R. Bengochea¹, * ¹Instituto de Astronomía y Física del Espacio (IAFE), CONICET - Universidad de Buenos Aires, (1428) Buenos Aires, Argentina

7- The Redshift - “cosmological redshift”, “R-W metric”, “Lemaitre Equation” referenced descriptions. <http://burro.astr.cwru.edu/Academics/Astr328/Notes/Redshift/redshift.html>

8- Dark energy two decades after: Observables, probes, consistency tests Feb 5, 2018
Dragan Huterer¹ and Daniel L Shafer² ¹ Department of Physics, University of Michigan, 450 Church Street, Ann Arbor, MI 48109, USA ² Department of Physics and Astronomy, Johns Hopkins University, 3400 North Charles Street, Baltimore, MD 21218, USA <https://arxiv.org/pdf/1709.01091.pdf>

9-The Robertson-Walker Metric In this equation the evolution of the Universe is governed by the scale-factor $R(t)$ <http://www.jb.man.ac.uk/~jpl/cosmo/RW.html>

10- Friedman-like Robertson-Walker model in generalized metric space-time with weak anisotropy. Oct 27 2007 P.C.Stavrinos *, Department of Mathematics University of Athens 15784 Greece A.P.Kouretsis†, 5 Festou St. 16674 Glifada Athens Greece M.Stathakopoulos‡, 1 Anastasiou Genadiou St. 11474 Athens Greece <https://arxiv.org/pdf/gr-qc/0612157.pdf>

11-A CRUCIAL DIPOLE TEST OF THE EXPANSION CENTER UNIVERSE BASED ON HIGH-Z SCP UNION & UNION2 SUPERNOVAE, ECM paper XV by Luciano Lorenzi by merging the SAI 2011 ECM paper X with the EWASS 2012 ECM paper XII <https://arxiv.org/pdf/1105.3697.pdf>

12-Celestial Ephemerides in an Expanding Universe, Pg 16-18. Sergei M. Kopeikin Department of Physics & Astronomy, University of Missouri, 322 Physics Bldg., Columbia, MO 65211, USA* <https://arxiv.org/pdf/1207.3873.pdf>

13- Where is the centre of the universe? Philip Gibbs 1997, <http://math.ucr.edu/home/baez/physics/Relativity/GR/centre.html>

14- The Expanding Universe, 1922 Sir Arthur Eddington, M.A., D.Sc., F.R.S. Plumian Professor of Astronomy and Director of the Observatory, University of Cambridge http://www-history.mcs.st-and.ac.uk/Extras/Eddington_Universe.html

15- Introduction to Cosmology, 2nd Edition 2017, Barbara Ryden, Professor at The Ohio State University, Cambridge University Press ISBN 978-1-107-15483-4

16- I - THE EXPANDING UNIVERSE from the HUGE VOID CENTER THEORY & MODELING - 1st. version: August, 1997 - 2nd. ver.: January, 1998 - 3rd. ver.: January, 1999 - by Luciano Lorenzi former astronomer at the Astronomical Observatory of Turin - Italy -<https://arxiv.org/pdf/astro-ph/9906290.pdf>

17- Hubble, Hubble's law and the Expanding Universe J. S. Bagla Harish-Chandra Research Institute Chhatnag Road, Jhusi, Allahabad 211019, India February 18, 2013 <https://arxiv.org/pdf/0904.2633.pdf>

18- Sample variance in the local measurements of the Hubble constant Hao-Yi Wu¹ and Dragan Huterer² ¹California Institute of Technology, MC 367-17, Pasadena, CA 91125, USA ²Department of Physics, University of Michigan, 450 Church Street, Ann Arbor, MI 48109, USA <https://arxiv.org/pdf/1706.09723.pdf>

19- Hubble's diagram and cosmic expansion Robert P. Kirshner* [Proc Natl Acad Sci U S A](https://doi.org/10.1073/pnas.2536799100). 2004 Jan 6; 101(1): 8–13. Published online 2003 Dec 26. doi: [10.1073/pnas.2536799100](https://doi.org/10.1073/pnas.2536799100) <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC314128/>

20- **The Hubble flow: an observer's perspective** Alexey V. Toporensky, Sergei B. Popov (Lomonosov Moscow State University, Sternberg Astronomical Institute) Nov 12th 2013 <https://arxiv.org/pdf/1311.2472.pdf>

21- **Approximation to the Second Order Approximation of Einstein Field Equations with a Cosmological Constant in a Flat Background** C. J. de Matos* February 4, 2008 <https://arxiv.org/pdf/gr-qc/0609116.pdf>

Note: **Abstract of HST Key Project.** We present here the final results of the Hubble Space Telescope (HST) Key Project to measure the Hubble constant. We summarize our method, the results and the uncertainties, tabulate our revised distances, and give the implications of these results for cosmology. Our results are based on a Cepheid calibration of several secondary distance methods applied over the range of about 60 to 400 Mpc. The analysis presented here benefits from a number of recent improvements and refinements, including (1) a larger LMC Cepheid sample to define the fiducial period-luminosity (PL) relations, (2) a more recent HST Wide Field and Planetary Camera 2 (WFPC2) photometric calibration, (3) a correction for Cepheid metallicity, and (4) a correction for incompleteness bias in the observed Cepheid PL samples. We adopt a distance modulus to the LMC (relative to which the more distant galaxies are measured) of $\mu_0(\text{LMC}) = 18.50 \pm 0.10$ mag, or 50 kpc. New, revised distances are given for the 18 spiral galaxies for which Cepheids have been discovered as part of the Key Project, as well as for 13 additional galaxies with published Cepheid data. The new calibration results in a Cepheid distance to NGC 4258 in better agreement with the maser distance to this galaxy. Based on these revised Cepheid distances, we find values (in km/sec/Mpc) of $H_0 = 71 \pm 2_r$ (random) $\pm 6_s$ (systematic) (type Ia supernovae), $H_0 = 71 \pm 3_r \pm 7_s$ (Tully-Fisher relation), $H_0 = 70 \pm 5_r \pm 6_s$ (surface brightness fluctuations), $H_0 = 72 \pm 9_r \pm 7_s$ (type II supernovae), and $82 \pm 6_r \pm 9_s$ (fundamental plane). We combine these results for the different methods with 3 different weighting schemes, and find good agreement and consistency with $H_0 = 72 \pm 8$ units. Finally, we compare these results with other, global methods for measuring H_0 .