New coordinate vacuum solution in general relativity theory

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ABSTRACT

In the general relativity theory, we discover new vacuum solution by Einstein's gravity field equation. We investigate the new coordinate in general relativity theory.

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1. Introduction

We solve new vacuum solution by gravity field equation in general relativity theory.

New spherical coordinate is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dr^2 + V(t,r)\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

$$V(t,r) = C_1(act+br)^2, \quad C_1 = \frac{1}{b^2 - a^2}$$

a, b, C_1 is constant, c is light's velocity. (1)

In this time, Einstein's gravity equation is

$$R_{tt} = \frac{\ddot{V}}{V} - \frac{\dot{V}^2}{2V^2}$$

$$= \frac{2a^2}{(act+br)^2} - \frac{1}{2}\frac{4a^2}{(act+br)^2} = 0 \quad (2)$$

$$R_{rr} = \frac{V''}{V} - \frac{1}{2}\frac{V'^2}{V^2}$$

$$= \frac{2b^2}{(act+br)^2} - \frac{1}{2}\frac{4b^2}{(act+br)^2} = 0$$

(3)

$$R_{\theta\theta} = -\frac{\ddot{V}}{2} + \frac{V''}{2} - 1$$

$$= -C_1a^2 + C_1b^2 - 1 = 0 \quad (4)$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} = 0 \quad (5)$$

$$R_{tr} = \frac{\dot{V}'}{V} - \frac{\dot{V}V'}{2V^2}$$

$$= \frac{2C_1ab}{(act+br)^2} - \frac{1}{2}\frac{4C_1ab}{(act+br)^2} = 0 \quad (6)$$

In this time,

$$V' = 2C_1b(act+br), \dot{V} = 2C_1a(act+br), V'' = 2C_1b^2, \ddot{V} = 2C_1a^2$$

$$A' = \frac{\partial A}{\partial r}, \dot{A} = \frac{1}{c}\frac{\partial A}{\partial t}$$

2. New vacuum solution in general relativity theory

Hence, new vacuum solution is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dr^2 + \frac{1}{b^2 - a^2}(act + br)^2\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

a, b, C_1 are constant, c is light's velocity. (7)

In this time, if r' is

$$r' = \frac{1}{\sqrt{b^2 - a^2}}(act + br)$$

As

$$dr' = \frac{1}{\sqrt{b^2 - a^2}}(acdt + bdr)$$

Or

$$dr = \frac{\sqrt{b^2 - a^2}}{b}dr' - \frac{a}{b}cdt \tag{8}$$

If new solution Eq(7) is inserted by transformation Eq(8),

$$dr^2 = \frac{b^2 - a^2}{b^2}dr'^2 - 2\frac{a}{b^2}\sqrt{b^2 - a^2}dr' cdt + \frac{a^2}{b^2}c^2dt^2 \tag{9}$$

In this time, if α_0 is

$$\alpha_0 = \frac{a}{b} \tag{10}$$

Hence, proper time $d\tau$ of new solution is

$$d\tau^2 = (1 - {\alpha_0}^2)dt^2 + 2\alpha_0\sqrt{1 - {\alpha_0}^2}dr'\frac{dt}{c} - \frac{1}{c^2}[(1 - {\alpha_0}^2)dr'^2 + r'^2\{d\theta^2 + \sin^2\theta d\phi^2\}] \tag{11}$$

In this time, if dt' is

$$dt' = \sqrt{1 - {\alpha_0}^2}dt \tag{12}$$

Therefore, new solution is

$$d\tau^2 = dt'^2 + 2\alpha_0 dr'\frac{dt'}{c} - \frac{1}{c^2}[(1 - {\alpha_0}^2)dr'^2 + r'^2\{d\theta^2 + \sin^2\theta d\phi^2\}] \tag{13}$$

If we rewrite dt, dr, r instead of dt', dr', r', the proper time $d\tau$ of new solution is

$$d\tau^2 = dt^2 + 2\alpha_0 dr\frac{dt}{c} - \frac{1}{c^2}[(1 - {\alpha_0}^2)dr^2 + r^2\{d\theta^2 + \sin^2\theta d\phi^2\}] \tag{14}$$

3. Conclusion

Therefore, new spherical solution in general relativity theory is

$$d\tau^2 = dt^2 + 2\alpha_0 dr \frac{dt}{c} - \frac{1}{c^2}[(1-\alpha_0{}^2)dr^2 + r^2\{d\theta^2 + \sin^2\theta d\phi^2\}]$$

$\alpha_0 \neq 1$, α_0 is constant

(15)

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