# Continuity,
# Non-Constant Rate of Ascent,
# & The Beal Conjecture

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**Abstract**

The Beal Conjecture considers positive integers $A$, $B$, and $C$ having respective positive integer exponents $X$, $Y$, and $Z$ all greater than 2, where bases $A$, $B$, and $C$ must have a common prime factor. Taking the general form $A^X + B^Y = C^Z$, we explore a small opening in the conjecture through reformulation and substitution to create two new variables. One we call '$C$ dot' ($\dot{C}$) representing and replacing $C$ and the other we call '$Z$ dot' ($\dot{Z}$) representing and replacing $Z$. With this, we show that $\dot{C}$ and $\dot{Z}$ are separate continuous functions, with argument $\left(A^X + B^Y\right)$, that achieve all positive integers during their continuous non-constant rates of infinite ascent. Possibilities for each base and exponent in the reformulated general equation $A^X + B^Y = \dot{C}^{\dot{Z}}$ are examined using a binary table along with analyzing user input restrictions and $\dot{C}$'s value relative to $A$ and $B$. Lastly, an indirect proof is made, where conclusively we find the continuity theorem to hold over the conjecture.

*Keywords:* Beal, Diophantine, Continuity
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Beal Conjecture:

> If $A^X + B^Y = C^Z$, where $A$, $B$, $C$, $X$, $Y$, and $Z$ are positive integers and $X$, $Y$, and $Z$, are all greater than 2, then $A$, $B$, and $C$ must have a common prime factor.

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## 1. Main Results

The Beal Conjecture considers positive integers $A$, $B$, and $C$ having respective positive integer exponents $X$, $Y$, and $Z$ all greater than 2, where bases $A$, $B$, and $C$ must have a common prime factor.

In this paper we reformulate two variables held in the conjectures equation $A^X + B^Y = C^Z$ through construction of a '$C$ dot' ($\dot{C}$) representing and replacing positive base $C$, and a '$Z$ dot' ($\dot{Z}$) representing and replacing positive exponent $Z$. Where our reformulation shows solutions always exist for $\dot{C}$ and $\dot{Z}$. We further show that $\dot{C}$ and $\dot{Z}$ are continuous functions, with argument $\left(A^X + B^Y\right)$, having non-constant rates of infinite ascent.

Expressing $\left(A^X + B^Y\right)$ as the value Gamma ($\Gamma$), we see that $\dot{C}$ as a $f(\Gamma)$ achieves all positive integers $-$emphasis on positive odd prime integers$-$ during its continuous non-constant rate of infinite ascent. While $\dot{Z}$, also a $f(\Gamma)$, achieves all positive integers during its own continuous non-constant rate of infinite ascent.

Possibilities for each base and exponent in the reformulated general equation $A^X + B^Y = \dot{C}^{\dot{Z}}$ are examined using a binary table (1's representing integer and 0's representing non-integer), along with analyzing user input restrictions and $\dot{C}$'s value relative to $A$ and $B$. Lastly, an indirect proof is made, where the continuity theorem is shown to hold over the conjecture.

Beal Conjecture general equation:

$$A^X + B^Y = C^Z \tag{1}$$

Beal Conjecture reformulated general equation:

$$A^X + B^Y = \left(e^{\frac{ln\left(\Gamma^2\right)}{2\sqrt{ln(\Gamma)}}}\right)^{\sqrt{ln(\Gamma)}} \tag{2}$$

*where,*

$$\dot{C} = C = \left(e^{\frac{ln\left(\Gamma^2\right)}{2\sqrt{ln(\Gamma)}}}\right) \tag{3}$$

*and,*

$$\dot{Z} = Z = \sqrt{ln\,(\Gamma)} \tag{4}$$

*then,*

$$A^X + B^Y = \dot{C}^{\dot{Z}} \tag{5}$$

## 2. Current State of Knowledge

Proposed in 1994, by Andrew Beal, in correspondence letters and early on referred to as a 'generalization of Fermat's Last Theorem' (Edwards 1994), the Beal Conjecture has evolved into a remarkable problem for mathematicians. Since this time, many contributions have been made.

Andrew Wiles (1994) investigated that Fermat's Last Theorem where equal exponents $X$, $Y$, and $Z$ had no solutions. Darmon and Merel (1995) investigated exponents $X$, $Y$, and $Z$ of $(2, n, n)$ and $(3, n, n)$. Bjorn Poonen et al (2005) investigated exponents $(2, 3, 7)$. David Brown (2009) investigated exponents $(2, 3, 10)$. Bennet et al (2009) investigated exponents $(2, 4, n)$. Samir Siksek and Michael Stoll (2013) investigated exponents $(2, 3, 15)$.

Each of these works have contributed towards a better understanding of the Beal Conjecture and where we stand today.

## 3. Background on Author's Formation of these Equations and Functions

In deriving our equations and functions we began our analysis by focusing on minimizing the total number of variables associated with the Beal Conjecture's general equation. With this, we then found our functions, and our function arguments to which were comprised of the remaining variables held within the general equation.

Next, we used these functions and their arguments to arrive at our conclusions through binary analysis, user restricted inputs, calculated results, probability analysis, cycle analysis, and indirect proof.

There were many peaks and valleys during the formation of these equations and functions, but in the end we found something extremely beautiful.

## 4. Reformulation & Substitution

We provide our reformulation for base $C$ and exponent $Z$ below, followed by graphing the reformulations on a primary and secondary $Y$ axis.

Reformulation for $C$:

$$A^X + B^Y = C^Z \tag{6}$$

$$e^{(X)ln(A)} + e^{(Y)ln(B)} = e^{(Z)ln(C)} \tag{7}$$

$$\left(e^{(X)ln(A)} + e^{(Y)ln(B)}\right)^2 = \left(e^{(Z)ln(C)}\right)^2 \tag{8}$$

$$\left(e^{(X)ln(A)} + e^{(Y)ln(B)}\right)^2 = e^{2(Z)ln(C)} \tag{9}$$

$$e^{2(X)ln(A)} + 2e^{(X)ln(A)+(Y)ln(B)} + e^{2(Y)ln(B)} = e^{2(Z)ln(C)} \tag{10}$$

$$2e^{(X)ln(A)+(Y)ln(B)} = e^{2(Z)ln(C)} - e^{2(X)ln(A)} - e^{2(Y)ln(B)} \tag{11}$$

$$e^{(X)ln(A)+(Y)ln(B)} = \frac{e^{2(Z)ln(C)} - e^{2(X)ln(A)} - e^{2(Y)ln(B)}}{2} \tag{12}$$

$$A^X B^Y = e^{2(Z)ln(C)-ln(2)} - e^{2(X)ln(A)-ln(2)} - e^{2(Y)ln(B)-ln(2)} \tag{13}$$

$$\frac{2A^XB^Y}{B^{2Y}} = \frac{e^{2(Z)ln(C)}}{B^{2Y}} - \frac{A^{2X}}{B^{2Y}} - 1 \tag{14}$$

$$2A^XB^{Y-2Y} = \frac{e^{2(Z)ln(C)}}{B^{2Y}} - \frac{A^{2X}}{B^{2Y}} - 1 \tag{15}$$

$$2A^XB^{Y-2Y} + \frac{A^{2X}}{B^{2Y}} = \frac{e^{2(Z)ln(C)}}{B^{2Y}} - 1 \tag{16}$$

$$\frac{2A^XB^Y + A^{2X}}{B^{2Y}} = \frac{e^{2(Z)ln(C)}}{B^{2Y}} - 1 \tag{17}$$

$$\frac{2A^XB^Y + A^{2X}}{B^{2Y}} + 1 = \frac{e^{2(Z)ln(C)}}{B^{2Y}} \tag{18}$$

$$\frac{2A^XB^Y + A^{2X} + B^{2Y}}{B^{2Y}} = \frac{e^{2(Z)ln(C)}}{B^{2Y}} \tag{19}$$

$$e^{2(Z)ln(C)} = 2A^XB^Y + A^{2X} + B^{2Y} \tag{20}$$

$$2\left(Z\right)ln\left(C\right) = ln\left(2A^XB^Y + A^{2X} + B^{2Y}\right) \tag{21}$$

$$C = e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2(Z)}} \tag{22}$$

$$\dot{C} = C = e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2(Z)}} \tag{23}$$

*substituting $\dot{C}$ back into the general equation results in:*

$$A^X + B^Y = \left(e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2(Z)}}\right)^Z \tag{24}$$

Reformulation for $Z$:

$$A^X + B^Y = \left(e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2(Z)}}\right)^Z \tag{25}$$

$$\sqrt[Z]{A^X + B^Y} = e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2(Z)}} \tag{26}$$

$$\left(A^X + B^Y\right)^{\frac{1}{Z}} = e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2(Z)}} \tag{27}$$

$$e^{\frac{1}{Z}ln\left(A^X + B^Y\right)} = e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2(Z)}} \tag{28}$$

$$e^{Z}e^{\frac{1}{Z}ln\left(A^{X}+B^{Y}\right)}=e^{\frac{ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)}{2(Z)}}e^{Z} \tag{29}$$

$$e^{\frac{1}{Z}ln\left(A^{X}+B^{Y}\right)+Z}=e^{\frac{ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)}{2(Z)}+Z} \tag{30}$$

$$e^{\frac{1}{Z}ln\left(A^{X}+B^{Y}\right)+Z}=e^{\frac{ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)}{2(Z)}+\frac{2Z^{2}}{2Z}} \tag{31}$$

$$e^{\frac{1}{Z}ln\left(A^{X}+B^{Y}\right)+Z}=e^{\frac{ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)+2Z^{2}}{2(Z)}} \tag{32}$$

$$\frac{1}{Z}ln\left(A^{X}+B^{Y}\right)+Z=\frac{ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)+2Z^{2}}{2\left(Z\right)} \tag{33}$$

$$\frac{ln\left(A^{X}+B^{Y}\right)}{Z}+Z=\frac{ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)+2Z^{2}}{2\left(Z\right)} \tag{34}$$

$$\frac{ln\left(A^{X}+B^{Y}\right)}{Z}+\frac{Z^{2}}{Z}=\frac{ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)+2Z^{2}}{2\left(Z\right)} \tag{35}$$

$$\frac{ln\left(A^{X}+B^{Y}\right)+Z^{2}}{Z}=\frac{ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)+2Z^{2}}{2\left(Z\right)} \tag{36}$$

$$2ln\left(A^{X}+B^{Y}\right)+2Z^{2}=ln\left(2A^{X}B^{Y}+A^{2X}+B^{2Y}\right)+2Z^{2} \tag{37}$$

$$2Z^2 + 2ln\left(A^X + B^Y\right) = 2Z^2 + ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right) \tag{38}$$

*Applying the quadratic formula to both sides,*

$$Z = \frac{-K \pm \sqrt{K^2 - 4JL}}{2J} \tag{39}$$

Table 1: Quadratic Formula Inputs

| **Left-Hand Side** | **Right-Hand Side** |
|---|---|
| $J = 2$ | $J = 2$ |
| $K = 0$ | $K = 0$ |
| $L = 2ln\left(A^X + B^Y\right)$ | $L = ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)$ |
| $Z = \frac{-0 \pm \sqrt{0^2 - 4(2)2ln(A^X + B^Y)}}{2(2)}$ | $Z = \frac{-0 \pm \sqrt{0^2 - 4(2)ln(2A^X B^Y + A^{2X} + B^{2Y})}}{2(2)}$ |
| $Z = \pm i\sqrt{ln\left(A^X + B^Y\right)}$ | $Z = \frac{\pm i\sqrt{(2)ln(2A^X B^Y + A^{2X} + B^{2Y})}}{(2)}$ |
| $Z = \sqrt{ln\left(A^X + B^Y\right)}$ | |

By reformulation of $C$ to $\dot{C}$, our $Z$ may now be chosen selectively, see $Z$ within (24), so we choose the positive real part of the simplest term that fulfills the function requirement for a $\left(A^X + B^Y\right)$ as the argument.

$$Z = \sqrt{ln\left(A^X + B^Y\right)} \tag{40}$$

*and now,*

$$\dot{Z} = Z = \sqrt{ln\left(A^X + B^Y\right)} \tag{41}$$

We see that our final reformulated general equation is now (shown in three equivalent forms):

$$A^X + B^Y = \left( e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2\sqrt{ln(A^X + B^Y)}}} \right)^{\sqrt{ln(A^X + B^Y)}} \tag{42}$$

$$A^X + B^Y = \left( e^{\frac{ln\left(\left(A^X + B^Y\right)^2\right)}{2\sqrt{ln(A^X + B^Y)}}} \right)^{\sqrt{ln(A^X + B^Y)}} \tag{43}$$

$$A^X + B^Y = \left( e^{\frac{ln\left(\Gamma^2\right)}{2\sqrt{ln(\Gamma)}}} \right)^{\sqrt{ln(\Gamma)}} \tag{44}$$

Where when graphed, with $\Gamma = X$ (where the $X$ here is different from the $X$ exponent in the general equation, as the $X$ here is zero to infinity representing all values of $\Gamma$) we see Figure 1:

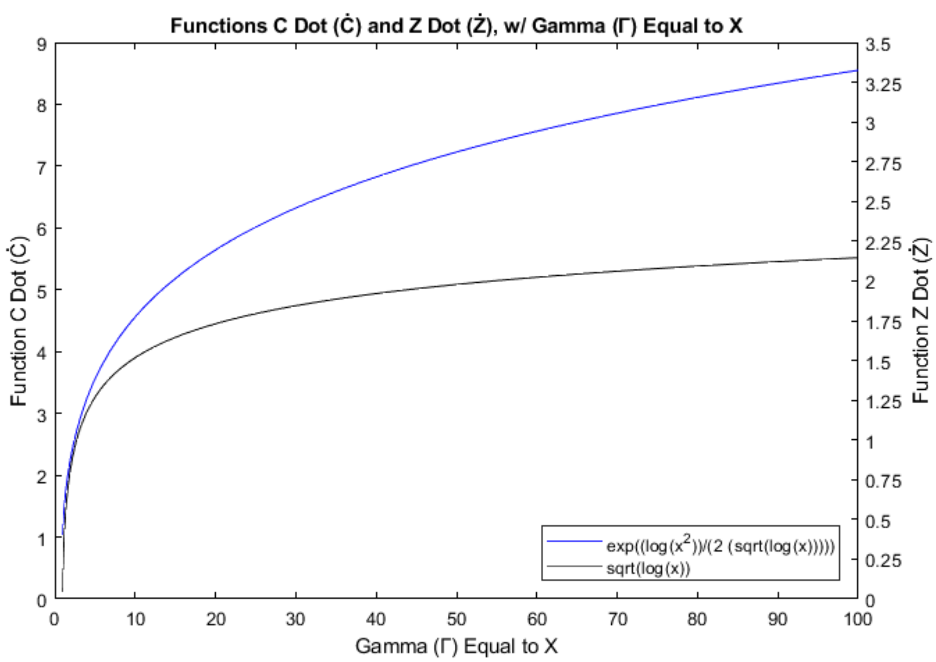


Figure 1: Functions '$C$ dot' ($\dot{C}$) and '$Z$ dot' ($\dot{Z}$), w/ Gamma ($\Gamma$) Equal to $X$

## 5. Continuity Theorem

Referencing the well-known 'continuity theorem', if "$g$" is a continuous function at all values of argument "$a$", and function "$f$" is continuous with "$x$" argument at "$g(a)$", then the composition $f \circ g$ is continuous at "$x = a$".

*with,*

$$f(x) = e^x \tag{45}$$

*and,*

$$g(a) = a \tag{46}$$

*then also continuous everywhere at,*

$$f(g(a)) = e^a \tag{47}$$

Where then we can state our reformulated general equation (shown in three equivalent forms) is continuous everywhere,

$$A^X + B^Y = \left( e^{\frac{ln\left(2A^X B^Y + A^{2X} + B^{2Y}\right)}{2\sqrt{ln(A^X+B^Y)}}} \right)^{\sqrt{ln(A^X+B^Y)}} \tag{48}$$

$$A^X + B^Y = \left( e^{\frac{ln\left(\left(A^X+B^Y\right)^2\right)}{2\sqrt{ln(A^X+B^Y)}}} \right)^{\sqrt{ln(A^X+B^Y)}} \tag{49}$$

$$A^X + B^Y = \left( e^{\frac{ln\left(\Gamma^2\right)}{2\sqrt{ln(\Gamma)}}} \right)^{\sqrt{ln(\Gamma)}} \tag{50}$$

## 6. Binary Table, Restrictions, and Equality Proof

With our bases $A$, $B$, and $\dot{C}$ and exponents of $X$, $Y$, and $\dot{Z}$ we assign all possibilities for integer and non-integer at any moment in time. Where a 1 represents an integer and a 0 represents a non-integer:

Table 2: Reformulated General Equation Integer & Non-Integer Possibilities

| | Bases | | | Exponents | | |
|---|---|---|---|---|---|---|
| **Rows** | $A$ | $B$ | $\dot{C}$ | $X$ | $Y$ | $\dot{Z}$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 1 |
| 3 | 0 | 0 | 0 | 0 | 1 | 0 |
| 4 | 0 | 0 | 0 | 0 | 1 | 1 |
| 5 | 0 | 0 | 0 | 1 | 0 | 0 |
| 6 | 0 | 0 | 0 | 1 | 0 | 1 |
| 7 | 0 | 0 | 0 | 1 | 1 | 0 |
| 8 | 0 | 0 | 0 | 1 | 1 | 1 |
| 9 | 0 | 0 | 1 | 0 | 0 | 0 |
| 10 | 0 | 0 | 1 | 0 | 0 | 1 |
| 11 | 0 | 0 | 1 | 0 | 1 | 0 |
| 12 | 0 | 0 | 1 | 0 | 1 | 1 |
| 13 | 0 | 0 | 1 | 1 | 0 | 0 |
| 14 | 0 | 0 | 1 | 1 | 0 | 1 |
| 15 | 0 | 0 | 1 | 1 | 1 | 0 |
| 16 | 0 | 0 | 1 | 1 | 1 | 1 |
| 17 | 0 | 1 | 0 | 0 | 0 | 0 |
| 18 | 0 | 1 | 0 | 0 | 0 | 1 |
| 19 | 0 | 1 | 0 | 0 | 1 | 0 |
| 20 | 0 | 1 | 0 | 0 | 1 | 1 |
| 21 | 0 | 1 | 0 | 1 | 0 | 0 |
| 22 | 0 | 1 | 0 | 1 | 0 | 1 |
| 23 | 0 | 1 | 0 | 1 | 1 | 0 |
| 24 | 0 | 1 | 0 | 1 | 1 | 1 |
| 25 | 0 | 1 | 1 | 0 | 0 | 0 |
| 26 | 0 | 1 | 1 | 0 | 0 | 1 |

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Table 2 – *Continued from previous page*

| | Bases | | | Exponents | | |
|---|---|---|---|---|---|---|
| **Rows** | $A$ | $B$ | $\dot{C}$ | $X$ | $Y$ | $\dot{Z}$ |
| 27 | 0 | 1 | 1 | 0 | 1 | 0 |
| 28 | 0 | 1 | 1 | 0 | 1 | 1 |
| 29 | 0 | 1 | 1 | 1 | 0 | 0 |
| 30 | 0 | 1 | 1 | 1 | 0 | 1 |
| 31 | 0 | 1 | 1 | 1 | 1 | 0 |
| 32 | 0 | 1 | 1 | 1 | 1 | 1 |
| 33 | 1 | 0 | 0 | 0 | 0 | 0 |
| 34 | 1 | 0 | 0 | 0 | 0 | 1 |
| 35 | 1 | 0 | 0 | 0 | 1 | 0 |
| 36 | 1 | 0 | 0 | 0 | 1 | 1 |
| 37 | 1 | 0 | 0 | 1 | 0 | 0 |
| 38 | 1 | 0 | 0 | 1 | 0 | 1 |
| 39 | 1 | 0 | 0 | 1 | 1 | 0 |
| 40 | 1 | 0 | 0 | 1 | 1 | 1 |
| 41 | 1 | 0 | 1 | 0 | 0 | 0 |
| 42 | 1 | 0 | 1 | 0 | 0 | 1 |
| 43 | 1 | 0 | 1 | 0 | 1 | 0 |
| 44 | 1 | 0 | 1 | 0 | 1 | 1 |
| 45 | 1 | 0 | 1 | 1 | 0 | 0 |
| 46 | 1 | 0 | 1 | 1 | 0 | 1 |
| 47 | 1 | 0 | 1 | 1 | 1 | 0 |
| 48 | 1 | 0 | 1 | 1 | 1 | 1 |
| 49 | 1 | 1 | 0 | 0 | 0 | 0 |
| 50 | 1 | 1 | 0 | 0 | 0 | 1 |
| 51 | 1 | 1 | 0 | 0 | 1 | 0 |
| 52 | 1 | 1 | 0 | 0 | 1 | 1 |
| 53 | 1 | 1 | 0 | 1 | 0 | 0 |
| 54 | 1 | 1 | 0 | 1 | 0 | 1 |
| 55 | 1 | 1 | 0 | 1 | 1 | 0 |
| 56 | 1 | 1 | 0 | 1 | 1 | 1 |
| 57 | 1 | 1 | 1 | 0 | 0 | 0 |
| 58 | 1 | 1 | 1 | 0 | 0 | 1 |
| 59 | 1 | 1 | 1 | 0 | 1 | 0 |
| 60 | 1 | 1 | 1 | 0 | 1 | 1 |

*Continued on next page*

Table 2 – *Continued from previous page*

| | Bases | | | Exponents | | |
|---|---|---|---|---|---|---|
| **Rows** | $A$ | $B$ | $\dot{C}$ | $X$ | $Y$ | $\dot{Z}$ |
| 61 | 1 | 1 | 1 | 1 | 0 | 0 |
| 62 | 1 | 1 | 1 | 1 | 0 | 1 |
| 63 | 1 | 1 | 1 | 1 | 1 | 0 |
| 64 | 1 | 1 | 1 | 1 | 1 | 1 |

As shown in Table 2, the reformulated general equation is not restricted to any particular row or rows, but in fact accommodates all rows therein.

When the user restricts the reformulated general equation to certain inputs (e.g., $A$, $B$, $X$, and $Y$ being only positive integers) then focus on $\dot{C}$ and $\dot{Z}$ is essential in witnessing fluctuations between positive non-integers and integers.

All solutions reside on the $\dot{C}$ and $\dot{Z}$ lines, therefore we analyze important intersections ('intersections' being where two avenues meet) and some user restriction scenarios relative to the conjecture:

i) when the user restricts $A$, $B$, $X$, and $Y$ to positive integers greater than 2;

ii) when the user restricts $A$, $B$, $X$, and $Y$ to positive integers greater than 2, where $A$ and $B$ are of non-common prime factor; and

iii) when the user restricts $A$ and $B$ to positive odd prime integers and $X$ and $Y$ to positive integers greater than 2.

Under these restriction scenarios, vertical avenues occur at every value of the $\Gamma$ argument, which we assign to the horizontal axis, whereas the remaining horizontal avenues occur from either positive integers (for $\dot{C}$ and $\dot{Z}$) or positive integers (for $\dot{Z}$) and positive odd prime integers (for $\dot{C}$), which we assign to the vertical axis. The restricted scenarios, and their infinite number of avenues and intersections are essential to understanding the journey of $\dot{C}$ and $\dot{Z}$ during their non-constant rates of infinite ascent.

Visualizing a sliver of this journey, we present Figure 2 showing only the $\dot{C}$ path under a sample portion of one set of $\Gamma$ arguments from an infinite set of $\Gamma$ arguments available.

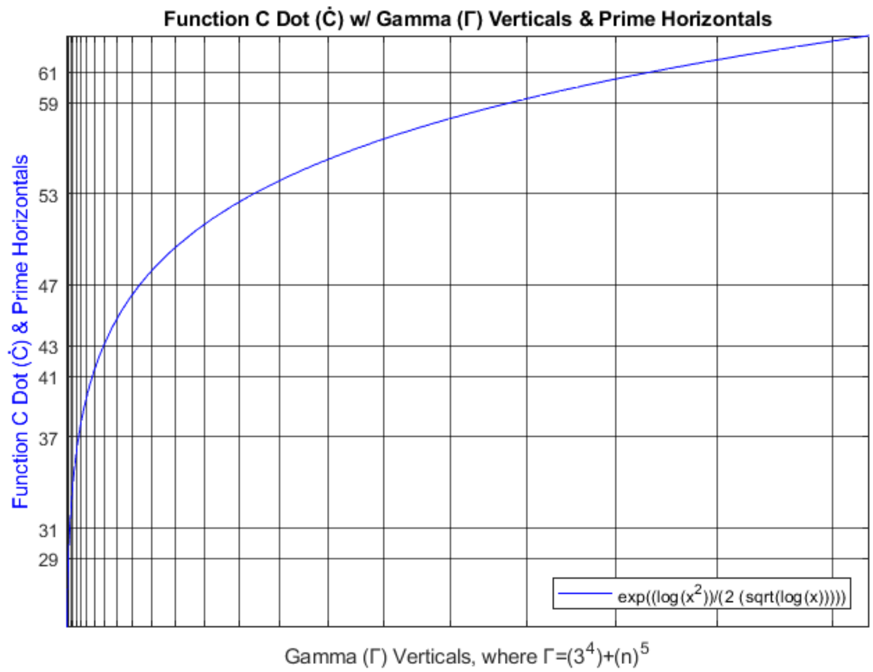


Figure 2: Function '$C$ dot' ($\dot{C}$) w/Gamma ($\Gamma$) Verticals & Prime Horizontals

As we can see in Figure 2, $\dot{C}$ is on an endless journey of a non-constant rate of infinite ascent where any crossing of an intersection results in $\dot{C}$ having a positive odd prime value driven from argument $\Gamma$. (Keeping in mind that Figure 2 is only a small sample of possible $\Gamma$ values under the three proposed user restriction scenarios.)

Where $\dot{C}$'s path is shown in Figure 2, a similar but different path is also being taken by its exponent $\dot{Z}$. When $\dot{C}$ and $\dot{Z}$ cross different intersections simultaneously the reformulated general equation under the user's restrictions unlocks its delight with respect to the Beal Conjecture.

If regularity in $\Gamma$ or odd prime numbers existed there could have been a cyclic miss of intersection crossings. Given such regularity does not exist, we counter intuitively now can count on the irregularity of these two values.

The non-constant rate of ascent found in the path of $\dot{C}$, combined with the irregularity of the odd primes, coupled with the irregularity of the $\Gamma$ arguments guarantees no cycle could take place that would cause continuously missed intersection crossings.

To further analyze the situation, we look at probabilities. Where we can comfortably say[1] that the probability of $\dot{C}$ being a positive integer or positive odd prime integer at future $\Gamma$ arguments is 1, and the probability that $\dot{Z}$ is a positive integer at future $\Gamma$ arguments is also 1. Therefore, as the 'mutually exclusive' rule does not apply here, the probability that $\dot{C}$ and $\dot{Z}$ occur simultaneously is the product of the individual probabilities,

*where,*

$$P\left(\dot{C}\right) = 1 \tag{51}$$

*and,*

$$P\left(\dot{Z}\right) = 1 \tag{52}$$

*then,*

$$P\left(\dot{C} \cap \dot{Z}\right) = P\left(\dot{C}\right) \times P\left(\dot{Z}\right) = 1 \tag{53}$$

Nothing in our investigations, have risen to say that these two events will not occur simultaneously at some future $\Gamma$ argument, and based off of our analysis here we can say that in fact these events will occur simultaneously. Much contemplation has been given to these two events, and accordingly, the remaining part of the analysis involves analyzing the relationship between values of $A$, $B$, and $\dot{C}$ with respect to when they are all positive odd prime integers.

We look at when $A$ and $B$ are different positive odd prime integers and the three possible positive odd prime scenarios for $\dot{C}$.

But first, for completeness, we show by an equality proof that $\dot{C}$ may be equal to $A$ or $B$, keeping in mind this does not mean $\dot{C}$ is restricted to only these two possible positive odd prime values, as in the $(A^{n_1} + B^{n_2})$ case $\dot{C}$ may achieve many different positive odd prime integer values during its non-constant rate of ascent.

[1]See $\dot{C}$ at odd prime integer 269 given arbitrary argument of 39243047905669 (one of many findings by us while experimenting with $\dot{C}$).

We assume the following is true and show it can be true.

*where if,*

$$A = \mathbb{A} \tag{54}$$

*then if,*

$$\mathbb{A} = e^{\frac{ln\left(\left(A^X+B^Y\right)^2\right)}{2\sqrt{ln\left(A^X+B^Y\right)}}} \tag{55}$$

*we see that,*

$$ln\left(\mathbb{A}\right) = \frac{ln\left(\left(A^X+B^Y\right)^2\right)}{2\sqrt{ln\left(A^X+B^Y\right)}} \tag{56}$$

$$2ln\left(\mathbb{A}\right) = \frac{ln\left(\left(A^X+B^Y\right)^2\right)\sqrt{ln\left(A^X+B^Y\right)}}{ln\left(A^X+B^Y\right)} \tag{57}$$

$$2ln\left(\mathbb{A}\right) = \frac{ln\left(\left(A^X+B^Y\right)^2\right)\left(ln\left(A^X+B^Y\right)\right)^{\frac{1}{2}}}{ln\left(A^X+B^Y\right)} \tag{58}$$

$$\frac{2ln\left(A^X+B^Y\right)}{\left(ln\left(A^X+B^Y\right)\right)^{\frac{1}{2}}} = \frac{ln\left(\left(A^X+B^Y\right)^2\right)}{ln\left(\mathbb{A}\right)} \tag{59}$$

$$2\left(ln\left(A^X+B^Y\right)\right)^{\frac{1}{2}}ln\left(\mathbb{A}\right) = ln\left(\left(A^X+B^Y\right)^2\right) \tag{60}$$

$$2\sqrt{ln\left(A^X+B^Y\right)}ln\left(\mathbb{A}\right) = ln\left(\left(A^X+B^Y\right)^2\right) \tag{61}$$

$$e^{2\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})} = \left(A^X + B^Y\right)^2 \tag{62}$$

$$\sqrt{e^{2\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})}} = \left(A^X + B^Y\right) \tag{63}$$

$$\sqrt[X]{\sqrt{e^{2\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})}} - B^Y} = A \tag{64}$$

*setting (55) and (64) equal,*

$$e^{\frac{ln\left(\left(A^X+B^Y\right)^2\right)}{2\sqrt{ln(A^X+B^Y)}}} = \sqrt[X]{\sqrt{e^{2\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})}} - B^Y} \tag{65}$$

$$e^{\frac{(X)ln\left(\left(A^X+B^Y\right)^2\right)}{2\sqrt{ln(A^X+B^Y)}}} + e^{(Y)ln(B)} = \sqrt{e^{2\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})}} \tag{66}$$

$$e^{\frac{(X)ln\left(\left(A^X+B^Y\right)^2\right)}{2\sqrt{ln(A^X+B^Y)}}} + e^{(Y)ln(B)} = e^{\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})} \tag{67}$$

$$e^{\frac{(X)}{2\sqrt{ln(A^X+B^Y)}}ln\left(\left(A^X+B^Y\right)^2\right)} + e^{(Y)ln(B)} = e^{\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})} \tag{68}$$

$$\left(\left(A^X + B^Y\right)^2\right)^{\frac{(X)}{2\sqrt{ln(A^X+B^Y)}}} + e^{(Y)ln(B)} = e^{\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})} \tag{69}$$

$$\left(\left(A^X + B^Y\right)^2\right)^{\frac{(X)}{2\sqrt{ln(A^X+B^Y)}}} + B^Y = e^{\sqrt{ln(A^X+B^Y)}ln(\mathbb{A})} \tag{70}$$

$$\left(\left(A^X + B^Y\right)^2\right)^{\frac{(X)}{2\sqrt{ln(A^X+B^Y)}}} = \mathbb{A}^{\sqrt{ln(A^X+B^Y)}} - B^Y \tag{71}$$

$$\left(A^X + B^Y\right)^2 = \sqrt[\frac{(X)}{2\sqrt{ln(A^X+B^Y)}}]{\mathbb{A}^{\sqrt{ln(A^X+B^Y)}} - B^Y} \tag{72}$$

$$\left(A^X + B^Y\right) = \sqrt[\frac{(X)}{\sqrt{ln(A^X+B^Y)}}]{\mathbb{A}^{\sqrt{ln(A^X+B^Y)}} - B^Y} \tag{73}$$

$$\left(A^X + B^Y\right)^{\frac{(X)}{\sqrt{ln(A^X+B^Y)}}} = \mathbb{A}^{\sqrt{ln(A^X+B^Y)}} - B^Y \tag{74}$$

*where then,*

$$B^Y = \mathbb{A}^{\sqrt{ln(A^X+B^Y)}} - \left(A^X + B^Y\right)^{\frac{(X)}{\sqrt{ln(A^X+B^Y)}}} \tag{75}$$

*and now substituting for $B^Y$ in (74) yields,*

$$\left(A^X + B^Y\right)^{\frac{(X)}{\sqrt{ln(A^X+B^Y)}}} = \mathbb{A}^{\sqrt{ln(A^X+B^Y)}} - \mathbb{A}^{\sqrt{ln(A^X+B^Y)}} + \left(A^X + B^Y\right)^{\frac{(X)}{\sqrt{ln(A^X+B^Y)}}} \tag{76}$$

*which then shows,*

$$\left(A^X + B^Y\right)^{\frac{(X)}{\sqrt{ln(A^X+B^Y)}}} = \left(A^X + B^Y\right)^{\frac{(X)}{\sqrt{ln(A^X+B^Y)}}} \tag{77}$$

*where we see,*

$$1 = 1 \tag{78}$$

*Therefore (55) can be true,*

$$\mathbb{A} = e^{\frac{ln\left(\left(A^X+B^Y\right)^2\right)}{2\sqrt{ln(A^X+B^Y)}}} \tag{79}$$

With this proof (and the same result if one were to solve for $\mathbb{B}$), we now consider when $A$ and $B$ are different positive odd prime integers that $\dot{C}$ could be one of the same positive odd prime integers.

Therefore, we attend to this by looking at when $\dot{C}$ is a positive odd prime integer and looking at a scenario comparison under the three possible positive odd prime integer scenarios for $\dot{C}$. Those being:

i) where $\dot{C}$ is equal to that positive odd prime integer $A$;
ii) where $\dot{C}$ is equal to that positive odd prime integer $B$; and
iii) where $\dot{C}$ is equal to some other positive odd prime integer (which there are an infinite number of)

Under these three positive odd prime integer scenarios for $\dot{C}$, we examine a user restriction input where the $\Gamma$ argument is comprised of $A$ and $B$, being two different positive odd prime integers, with $A$'s exponent set to $n_1$ and $B$'s exponent set to $n_2$, were each $n$ exponent goes to infinity at +1 integer intervals (keeping in mind the infinite permutation abilities of this arrangement).

$$(A^{n_1} + B^{n_2}) = \Gamma \tag{80}$$

$$n_1 \to \infty \text{ at } +1 \text{ integer intervals} \tag{81}$$

$$n_2 \to \infty \text{ at } +1 \text{ integer intervals} \tag{82}$$

Under this particular user restriction input $\dot{C}$ could achieve positive odd prime integer values of $A$, $B$, and/or any $\dot{C}_{PRIME}$ along its journey under varying $\Gamma$ arguments.

Most importantly though, any achievement by $\dot{C}$ of any $A$ or $B$ positive odd prime integer value does not preclude $\dot{C}$ from also achieving $\dot{C}_{PRIME}$. While similarly, any achievement by $\dot{C}$ of a $\dot{C}_{PRIME}$ positive odd prime integer value does not preclude $\dot{C}$ from also achieving a positive odd prime value of $A$ or $B$. Accordingly, $\dot{C}$ is not restricted by, nor limited, during its infinite non-constant rate of ascent.

With this, we conclude:

1) $\dot{C}$ will achieve many different positive odd prime integer values during its infinite non-constant rate of ascent; and
2) $\dot{C}$ will eventually be that positive odd prime integer $\dot{C}_{PRIME}$ having a $\Gamma$ argument comprised of different positive odd prime integers $A$ and $B$ (where $\dot{C}_{PRIME} \neq A$ and $B$).

Next, we turn to the remaining aspects of the Beal Conjecture.

## 7. Beal Conjecture Indirect Proof

We can rely on our reformulated general equation that when $A$ and $B$ are different positive odd prime integers, and $X$ and $Y$ are integers greater than 2, that $\dot{C}$ and $\dot{Z}$ fluctuate from non-integer to integer status during their infinite non-constant rate of ascent. It is this non-constant rate of ascent to infinity that allows us to claim that eventually $\dot{C}$ and $\dot{Z}$ will simultaneously cross intersections and be positive integers, while $\dot{C}$'s integer $-$in particular$-$ will also be a positive odd prime one not equal to $A$ and $B$.

From this, we now consider an indirect proof of the Beal Conjecture:

> Beal Conjecture states[2] that the sequence of events outlined above will not occur. So, we assume it will not occur, and where it will naturally occur during that infinite non-constant rate of ascent to which $\dot{C}$ and $\dot{Z}$ simultaneously achieve positive integers (and $\dot{C}$ is also odd prime not equal to those different positive odd prime integers $A$ and $B$) the functions $\dot{C}$ and $\dot{Z}$ would then not be continuous. But by the continuity theorem functions $\dot{C}$ and $\dot{Z}$ are continuous at all times. Contradiction. Therefore, the Beal Conjecture does not hold by the continuity theorem.

[2] "...and then $A$, $B$, and $C$ must have a common prime factor"

## 8. Summary and Conclusions

We presented the reformulated general equation of the Beal Conjecture, two separate function equations $\dot{C}$ and $\dot{Z}$, relationship with the continuity theorem, a binary table for integer and non-integer within the reformulated general equation, scenario analysis, and an indirect proof.

We relied on our reformulations of $C$ to $\dot{C}$ and $Z$ to $\dot{Z}$, the continuity theorem, user restricted scenarios, experiment results, probability of non-mutually exclusive, irregularity of the odd primes coupled with the irregularity of the $\Gamma$ arguments guaranteeing no cycle could take place that would cause continuously missed intersection crossings, a three scenario analysis for $\dot{C}$'s value with respect to $A$, $B$, and $\dot{C}_{PRIME}$, and an indirect proof of the Beal Conjecture to validate our claims.

The irregularity qualities of positive odd primes, $C^Z$, and $A^X + B^Y$ values that once was thought as the barrier in solving this conjecture have now been used in reverse to solve it. Without the ability to reformulate $C$ to $\dot{C}$ and $Z$ to $\dot{Z}$ (having the sole argument of $\Gamma$) it would have been difficult to see things from a new vantage point. Yet, from this new vantage point, the non-constant rates of infinite ascent by $\dot{C}$ and $\dot{Z}$ showed us the way.

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