Göttlers' Proof of the Collatz Conjecture

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***Abstract*—Over 80 years ago, the German mathematician Lothar Collatz formulated an interesting mathematical problem, which he was afraid to publish, for the simple reason that he could not solve it. Since then the Collatz Conjecture has been around under several names and is still unsolved, keeping people addicted. Several famous mathematicians including Richard Guy stating "Dont try to solve this problem". Paul Erdös even said "Mathematics is not yet ready for such problems" and Shizuo Kakutani joked that the problem was a Cold War invention of the Russians meant to slow the progress of mathematics in the West. We might have finally freed people from this addiction.**

I. Introduction

The Collatz Conjecture:
For $x \in \mathbb{N}$

$$\text{For } x \text{ is odd: } x \rightarrow 3x+1 \tag{1}$$
$$\text{For } x \text{ is even: } x \rightarrow \frac{x}{2} \tag{2}$$

Assumption:

$$\forall x \in \mathbb{N}\,,\, x \rightarrow^* 1 \tag{3}$$

Concept of proof
To prove the Collatz Conjecture, the following idea has been chosen: First of all, we will show that only odd numbers need to be considered. Furthermore, assuming that there are odd numbers, which are not falling onto 1, there are two possible ways how these numbers behave. Either they are increasing infinitely, moving away from 1 or there are other loops than 1. We will show that these two cases will end in contradictions.
Before starting with the actual proof, basic implications are proven and terms defined.

II. Basic definitions and terms

Lemma 1. *For all powers of two, with even k, it is*

$$2^k \pmod 3 = 1 \;\forall k \in \mathbb{N}\,|(k) \pmod 2 = 0 \tag{4}$$

Proof. For $2^2 \pmod 3 = 1$ and $2^k = 2^2 \cdot 2^2 \cdot 2^2 \cdot \ldots$
It follows:

$$2^k \pmod 3 = (2^2 \pmod 3) \cdot (2^2 \pmod 3) \cdot \ldots$$
$$2^k \pmod 3 = 1 \cdot 1 \cdot 1 \cdot \ldots$$
$$2^k \pmod 3 = 1$$

□

Lemma 2. *For all powers of two, with odd k, it is*

$$2^k \pmod 3 = 2 \;\forall k \in \mathbb{N}\,|(k) \pmod 2 = 1 \tag{5}$$

Proof. For $2^1 \pmod 3 = 2$ and $2^k = 2 \cdot 2^2 \cdot 2^2 \cdot \ldots$
It follows

$$2^k \pmod 3 = (2 \pmod 3) \cdot (2^2 \pmod 3) \cdot \ldots$$
$$2^k \pmod 3 = 2 \cdot 1 \cdot 1 \cdot \ldots$$
$$2^k \pmod 3 = 2$$

□

Theorem 1. *The statement that every number falls onto 1, is equivalent to the statement that every odd number falls onto 1.*

$$x \rightarrow^* 1 \;\forall\, x \in \mathbb{N}$$
$$\leftrightarrow u \rightarrow^* 1 \;\forall\, u \in \mathbb{N} | u \pmod 2 = 1 \tag{6}$$

Lemma 3. *All even numbers g fall onto an odd number u.*

$$g \rightarrow^* u \;\forall\, g \in \mathbb{N} | g \pmod 2 = 0 \tag{7}$$

Proof. Every even number g can be written as a product of an odd number u multiplied with a factor of 2^k. It follows according to (2), that g falls onto u:

$$\begin{aligned} g &= u \cdot 2^k \\ \frac{g}{2} &= u \cdot 2^{k-1} \\ \frac{g}{2^2} &= u \cdot 2^{k-2} \\ &\dots \\ \frac{g}{2^k} &= u \\ &\Rightarrow g \to^* u \;\forall\, g \in \mathbb{N} | g \pmod 2 = 0 \end{aligned}$$

□

Lemma 4. *All odd numbers u fall onto another odd number u'.*

$$u \to^* u' \;\forall\, u, u' \in \mathbb{N} | u \equiv u' \equiv 1 \pmod 2 \tag{8}$$

Proof. According to (1), u is mapped on $3u + 1$, which is an even number. Furthermore according to Lemma (3), every even number falls onto an odd number:

$$\begin{aligned} u &\to^* 3u + 1 = g \\ g \to^* u' \;\forall\, g \in \mathbb{N} | g &\pmod 2 = 0 \\ u &\to g \to^* u' \\ u &\to^* u' \end{aligned}$$

□

From Lemma (3) and (4), it follows that only odd numbers have to be considered.

Definition 1. *The number of (2)-steps (dividing an even number by 2) until u reaches u' is equal to k and defined as the rank of u (R_u).*

Definition 2. *The mapping $u \to u'$ is defined as $C(u)$.*
Whereby $u' = \frac{3u+1}{2^k} = \frac{3u+1}{2^{R_u}}$

Lemma 5. *The number 1 maps to itself:*

$$1 \to^* 1 \tag{9}$$

Proof. It is $1 \pmod 2 = 1$ and $1 \to 3 \cdot 1 + 1 = 4$
It follows:

$$\begin{aligned} 4 \pmod 2 &= 0 \\ 4 \to \frac{4}{2} &= 2 \\ 2 \pmod 2 &= 0 \\ 2 \to \frac{2}{2} &= 1 \\ \Rightarrow 1 &\to^* 1 \end{aligned}$$

□

Lemma 6. *Only the number 1 falls onto itself after exactly one $C(u)$ (mapping from one odd number to the following odd number). The following equation has to be fullfilled:*

$$\text{For } u \in \mathbb{N}, \text{ it is } \frac{3u+1}{2^{R_u}} = u \tag{10}$$

Proof.

$$\begin{aligned} \frac{3u+1}{2^{R_u}} &= u \\ u \cdot 3 + 1 &= u \cdot 2^{R_u} \\ u \cdot 3 + 1 - u \cdot 2^{R_u} &= 0 \\ u \cdot (3 - 2^{R_u}) + 1 &= 0 \\ u \cdot (3 - 2^{R_u}) &= -1 \end{aligned} \tag{11}$$

For $R_u = 1 \Rightarrow u = -1 \not\in u \in \mathbb{N}$
For $R_u > 2 \Rightarrow 1 > u > 0 \not\in u \in \mathbb{N}$
It follows that equation (11) is only true for $R_u = 2$ and $u = 1$ as shown in Lemma (5). □

III. Proof against infinite increase

In the following, we are going to show, that it is impossible to have a set of numbers which are not falling onto 1.

Reminder: Only odd numbers are considered in the following, according to the definitions above.

Concept of proof

Under the condition, that there are no other loops than the number 1, it follows that if there is an odd number A which is not mapping onto 1, then there must be an uncountably infinite amount of odd numbers that is not falling onto 1, which is a contradiction, since the natural number space only holds a countably infinite amount of numbers.

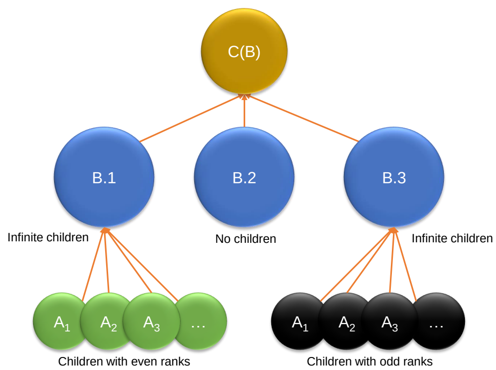

Fig. 1. There are three different types of parents. Both, the B.1 and B.3 type parents have an infinite number of children. B.1 has children with even ranks, while B.3 has children with odd ranks. B.2 type parents have no children.

For the following, we define the terms, children, siblings and parents:

Definition 3. *A child (A) is an odd number that is directly mapping onto one specific odd number (B). Every other odd number (A_n) that is mapping onto the same specific odd number (B) is defined as a sibling.*
It is $A_n = \frac{(B)\cdot 2^{(R_{A_n})}-1}{3}$, with R_{A_n} being defined as the rank of A_n.

The parent (B) is an odd number, that has exactly one parent (C(B)) itself. Furthermore it has siblings. The parent can have children or not as it will be shown later.

Theorem 2. *Either an infinite number of odd numbers u are falling on one specific number u' or none.*
To show this, we studied, which odd numbers A_n (children) are mapping onto one specific number B (parent). We found that there are three different groups (Fig. 1).

For $B \in \mathbb{N}, B$ can have three different forms:

$$B = \begin{cases} 2(3i)+1 & B.1 \\ 2(3i+1)+1 & B.2 \\ 2(3i+2)+1 & B.3 \end{cases} \tag{12}$$

B is therefore divided into three groups which fall into the three different residue classes of 3.

Lemma 7. *The parent only has children with even ranks. In other words A_n has an even rank. The parent B would look like:*

$$B.1 = 2(3i)+1 \tag{13}$$

Accordingly, the children A_n mapping onto this parent would look like:

$$A_n = \frac{(2(3i)+1)\cdot 2^k - 1}{3} \tag{14}$$
$$\text{with } k = R_{A_n}$$

Proof. It has to be shown that for the parent $B = 2(3i)+1$, the children A_n have an even rank. Meaning that $(k) \pmod 2 = 0$ and that there are no A_n with $k \pmod 2 = 1$. It is:

$$[(2(3i)+1)\cdot 2^k - 1] \pmod 3 = 0$$
$$\Rightarrow [2\cdot 2^k\cdot(3i) + 2^k - 1] \pmod 3 = 0$$
$$\Rightarrow [(2\cdot 2^k\cdot(3i)) \pmod 3 +$$
$$(2^k) \pmod 3 - 1 \pmod 3] \pmod 3 = 0$$

Case 1: For $(k) \pmod 2 = 0$ and according to Lemma (1):

$$\Rightarrow [0+1-1] \pmod 3 = 0$$
$$\Rightarrow 0 \pmod 3 = 0$$

Case 2: For $(k) \pmod 2 = 1$ and according to Lemma (2):

$$\Rightarrow [0+2-1] \pmod 3 = 0$$
$$\Rightarrow 1 \pmod 3 = 0 \ \lightning$$

$\square$

Lemma 8. *The parent only has children with odd ranks. In other words A_n has an odd rank. The parent B would look like:*

$$B.3 = 2(3i+2)+1 \tag{15}$$

Accordingly, the children A_n mapping onto this parent would look like:

$$A_n = \frac{(2(3i+2)+1)\cdot 2^k - 1}{3} \textit{with } k = R_{A_n}$$

Proof. It has to be shown that for the parent $B = 2(3i+2)+1$, the children A_n have an odd rank. Meaning that $(k) \pmod 2 = 1$ and that there are no A_n with $k \pmod 2 = 0$. It is:

$$\begin{aligned}
&[(2(3i+2)+1)\cdot 2^k - 1] \pmod 3 = 0\\
&\Rightarrow [(2\cdot(3i)+5)\cdot 2^k - 1] \pmod 3 = 0\\
&\Rightarrow [(2\cdot(3i) \pmod 3 + 5 \pmod 3)\cdot\\
&2^k \pmod 3 - 1 \pmod 3] \pmod 3 = 0
\end{aligned}$$

Case 1: For $(k) \pmod 2 = 1$ and according to Lemma (2):

$$\begin{aligned}
&\Rightarrow [(0+2)\cdot 2 - 1] \pmod 3 = 0\\
&\Rightarrow 3 \pmod 3 = 0
\end{aligned}$$

Case 2: For $(k) \pmod 2 = 0$ and according to Lemma (1):

$$\begin{aligned}
&\Rightarrow [(0+2)\cdot 1 - 1] \pmod 3 = 0\\
&\Rightarrow 1 \pmod 3 = 0 \lightning
\end{aligned}$$

$\square$

Lemma 9. *The parent has no children. The parent B would look like:*

$$B.2 = 2(3i+1)+1 \tag{16}$$

Accordingly, the children A_n mapping onto this parent would look like:

$$A_n = \frac{(2(3i+1)+1)\cdot 2^k - 1}{3} \textit{with } k = R_{A_n}$$

Proof. It has to be shown that for the parent $B = 2(3i+1)+1$, there are no children A_n. Meaning that $(2(3i+1)+1)\cdot 2^k - 1$ is not divisible by 3. It is:

$$\begin{aligned}
&[(2(3i+1)+1)\cdot 2^k - 1] \pmod 3 = 0\\
&\Rightarrow [(2\cdot(3i)+3)\cdot 2^k - 1] \pmod 3 = 0\\
&\Rightarrow [(2\cdot(3i) \pmod 3 + 3 \pmod 3)\cdot\\
&2^k \pmod 3 - 1 \pmod 3] \pmod 3 = 0\\
&\Rightarrow [(0)\cdot(2^k \pmod 3) - 1] \pmod 3 = 0\\
&\Rightarrow [0-1] \pmod 3 = 0\\
&\Rightarrow -1 \pmod 3 = 0 \lightning
\end{aligned}$$

$\square$

In the next step, we show that if the parent B has an infinite amount of children A_n, that among these children, there is an infinite amount of children having infinite amount of children themselves.

Theorem 3. *$B.1$ and $B.3$ type parents have an infinite number of children A_n each, which themselves are equally divided into the three different parent groups $B.1$, $B.2$ and $B.3$.*
To prove this, we study how the children A_n look like. For the children from the parent $B.1$, the relationship to their siblings is as followed: Since k has to be even for $B.1$ children (Lemma (7)), the sibling has to have an even rank too (multplication with 2^2).

$$\begin{aligned}
A_{n+1} &= \frac{(2(3i)+1)\cdot 2^k\cdot 2^2 - 1}{3}\\
A_{n+1} &= \frac{(2(3i)+1)\cdot 2^k\cdot 2^2 - 4 + 3}{3}\\
A_{n+1} &= \frac{(2(3i)+1)\cdot 2^k\cdot 2^2 - 4}{3} + 1\\
A_{n+1} &= \frac{((2(3i)+1)\cdot 2^k - 1)}{3}\cdot 4 + 1\\
A_{n+1} &= A_n \cdot 4 + 1
\end{aligned} \tag{17}$$

Same behaviour accounts for the children from the parent $B.3$. Since k has to be odd for $B.3$ children

(Lemma (8)), the sibling has to have an odd rank too (multplication with 2^2).

$$A_{n+1} = \frac{(2(3i+2)+1)\cdot 2^k \cdot 2^2 - 1}{3}$$
$$\ldots$$
$$A_{n+1} = A_n \cdot 4 + 1 \tag{18}$$

Comment: *The multiplication with 2^2 means nothing else than:*

$$R_{A_{n+1}} = R_{A_n} + 2; \tag{19}$$

with R_u being the rank of u.

In the next step, it will be shown, that the children A behave similiar to their parents B, meaning that they can have infinite children or none, since they can also be divided into the three different residue classes of 3.

Lemma 10. *For $A_n \in \mathbb{N}$,*
it is $A_n \equiv (A_{n+1}+1) \pmod 3$

Proof. In the following the residue class of A_{n+1} is shown depending on the residue class of A_n.

For $A_n \pmod 3 = 0$
$A_{n+1} \pmod 3$
$= (A_n \cdot 4 + 1) \pmod 3$
$= ((A_n \cdot 4) \pmod 3 + (1) \pmod 3) \pmod 3$
$= (0+1) \pmod 3$
$\Rightarrow A_{n+1} \pmod 3 = 1$

For $A_n \pmod 3 = 1$
$A_{n+1} \pmod 3$
$= (A_n \cdot 4 + 1) \pmod 3$
$= ((A_n \cdot 4) \pmod 3 + (1) \pmod 3) \pmod 3$
$= (4+1) \pmod 3$
$\Rightarrow A_{n+1} \pmod 3 = 2$

For $A_n \pmod 3 = 2$
$A_{n+1} \pmod 3$
$= (A_n \cdot 4 + 1) \pmod 3$
$= ((A_n \cdot 4) \pmod 3 + (1) \pmod 3) \pmod 3$
$= (8+1) \pmod 3$
$\Rightarrow A_{n+1} \pmod 3 = 0$

□

Theorem 4. *Under the condition, that there are no other loops than the number 1, it follows that every odd number B is mapping onto 1.*

1) *We know, that there are odd numbers which fall onto 1 after a finite number of steps.*
2) *Assuming, there is an odd number B, which does not fall onto 1, then we know that there must be another number (it's parent) C(B), which is also not falling onto 1.*
3) *Under the condition that there are no other loops than 1, we can follow that there is an infinite amount of numbers $C(B), C(C(B)), \ldots$ not falling onto 1.*
4) *Since C(B) has one child, we can follow, that it has an infinite amount of children. This accounts also for $C(C(B)), C(C(C(B))), \cdots$.*
5) *We know, that the set of children of C(B) has an infinite subset of the kind B.1 and B.3 children (Fig. 1).*
6) *Each of them, again have an infinite subset of B.1 and B.3 children. This process repeats recursively.*
7) *This results in a directional tree graph in which every element is connected.*

Lemma 11. *Under the condition that there is no loop, every element in this directional tree graph is disjoint.*

Proof. Assuming that there are two elements E_1 and E_2, which are not disjoint. Since every element in this directional tree graph is connected, E_1 and E_2 are connected in exactly one point X.

$$E_1 \rightarrow^{k_1} X \rightarrow \ldots$$
$$E_2 \rightarrow^{k_2} X \rightarrow \ldots$$

If $k_1 = k_2$, it is trivial, since then the elements E_1 and E_2 have to be the same node. This derives simply from the fact, that a function cannot have two different outputs for the same input.
If $k_1 < k_2$, it results in:

$$E_1 \to^{k_1} X \to \dots$$
$$E_2 \to^{k_1} X \to^{k_2-k_1} X \to \dots \lightning$$

This is a contradiction, since X would loop. Same accounts for $k_1 > k_2$ $\square$

Finally, this means that if there is one single odd number B that is not falling onto 1, under the condition that there is no other loop than 1, there is a directional tree graph with infinite disjoint elements. By enumerating the directional tree graph with every real number (Fig. 2), it can be shown that the cardinality of the real numbers equals that of the directional tree graph. As proven by Cantor [1] real numbers are uncountably infinite, it can be followed that the directional tree graph has an uncountably infinite amount of elements. Meaning that an uncountably infinite amount of odd numbers would not fall onto 1, which is a contradiction, since there is only a countably infinite amount of numbers in the natural number space. Therefore we can conclude that under the given condition (no other loop than 1), every number has to fall onto 1.

IV. Proof against loops

In the final step, we are proving that the condition, only the number 1 loops, is true. Therefore we introduce a corollary:

Corollary 1. *For u is an odd number and $u \in \mathbb{N}$, it is:*

$$C(u) = \frac{u \cdot 3 + 1}{2^{R_u}} = u_1$$
$$C(C(u)) = \frac{C(u) \cdot 3 + 1}{2^{R_{u_1}}} = \frac{\frac{u \cdot 3+1}{2^{R_u}} \cdot 3 + 1}{2^{R_{u_1}}} = u_2$$
$$C(...C(C(u))...) = \frac{(\frac{\frac{u\cdot 3+1}{2^{R_u}} \cdot 3+1}{2^{R_{u_1}}} \dots) \cdot 3 + 1}{2^{R_{u_n}}} = u_{n+1}$$

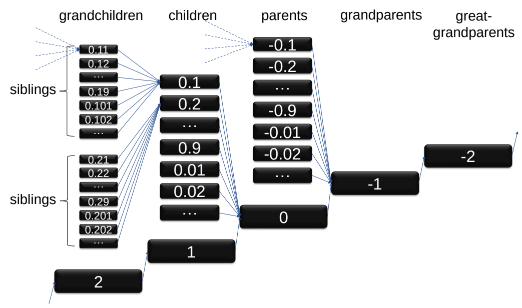

Fig. 2. If there is one odd number of type B.1 or B.3, which is not falling onto 1, this number is labeled with zero. In case the odd number is of type B.2, meaning it has no children itself, the parent is labeled with zero. The zero-labeled number has an infinite amount of disjoint ancestors (negative numbers), which are not falling onto 1, too. Furthermore the zero-labeled number, has an infinite amount of disjoint siblings and disjoint descendants. The directional tree graph can be enumerated one-to-one with all real numbers, making it uncountably infinite.

$$= \frac{u \cdot 3 \cdot 3^n + 3^n + 3^{n-1} \cdot 2^{R_u} + 3^{n-2} \cdot 2^{R_u} \cdot 2^{R_{u_1}} + \dots + 2^{(R_u + R_{u_1} + \dots + R_{u_{n-1}})}}{2^{(R_u + R_{u_1} + \dots + R_{u_n})}}$$

In the following we will summarize this term into:

$$u_{n+1} = \frac{u \cdot x + y}{2^{(R_u + z)}}$$

We have already proven that there is exactly one number which falls directly onto itself after one step ($B = C(B)$), which is 1 (as shown in Lemma (5) and (6)). So the only possible way, for a number looping is as follows:
$A_1 \to B \to^n A_1$, whereas $B \neq A_1$ and the loop has a length of $n+1$, with $n > 0$.

According to Corollary (1), the following has to be valid if A_1 loops:

$$\frac{A_1 \cdot x + y}{2^{(R_{A_1} + z)}} = A_1$$

Furthermore, as shown in the section before, B has to have an infinite number of children, since it has already one child (A_1), (Theorem (2)).

$$A_1 \to B \to \cdots \to A_1$$
$$A_2 \to B \to \cdots \to A_1$$
$$A_3 \to B \to \cdots \to A_1$$
$$\ldots$$

Moreover, the siblings are related in the following way (Theorem (3)):

$$A_{n+1} = 4 \cdot A_n + 1, \text{with } R_{A_{n+1}} = R_{A_n} + 2;$$

It follows that:

$$\frac{A_2 \cdot x + y}{2^{(R_{A_2}+z)}} = A_1$$
$$\frac{(4 \cdot A_1 + 1) \cdot x + y}{2^{(R_{A_1}+2+z)}} = A_1$$

From the formular given above it follows:

$$\Rightarrow \frac{(4 \cdot A_1 + 1) \cdot x + y}{2^{(R_{A_1}+2+z)}} = \frac{A_1 \cdot x + y}{2^{(R_{A_1}+z)}}$$
$$\Rightarrow \frac{(4 \cdot A_1 + 1) \cdot x + y}{2^2} = \frac{A_1 \cdot x + y}{1}$$
$$\Rightarrow (4 \cdot A_1 + 1) \cdot x + y = 4 \cdot (A_1 \cdot x + y)$$
$$\Rightarrow 4 \cdot A_1 \cdot x + x + y = 4 \cdot A_1 \cdot x + 4 \cdot y$$
$$\Rightarrow x + y = 4 \cdot y$$
$$\Rightarrow x = 3y$$

In general:

$$\Rightarrow \frac{A_{n+1} \cdot x + y}{2^{(R_{A_{n+1}}+z)}} = A_1$$
$$\Rightarrow \frac{(4 \cdot A_n + 1) \cdot x + y}{2^{(R_{A_n}+2+z)}} = A_1$$
$$\Rightarrow \frac{(4 \cdot A_n + 1) \cdot x + y}{2^{(R_{A_1}+2n+z)}} = \frac{A_1 \cdot x + y}{2^{(R_{A_1}+z)}}$$
$$\Rightarrow \frac{(4 \cdot A_n + 1) \cdot x + y}{2^{2n}} = \frac{A_1 \cdot x + y}{1}$$
$$\Rightarrow (4 \cdot A_n + 1) \cdot x + y = 4^n \cdot (A_1 \cdot x + y)$$
$$\Rightarrow 4 \cdot A_n \cdot x + x + y = 4^n \cdot A_1 \cdot x + 4^n \cdot y$$
$$\Rightarrow x(4 \cdot A_n + 1 - 4^n \cdot A_1) = y(4^n - 1)$$

It is

$$A_n = 4^{n-1} \cdot A_1 + 4^{n-2} + 4^{n-3} + \cdots + 4^0$$
$$A_n = 4^{n-1} \cdot A_1 + \sum_{i=0}^{n-2} 4^i$$

$$\Rightarrow x \cdot (4 \cdot A_n + 1 - 4^n \cdot A_1) = y \cdot (4^n - 1)$$
$$\Rightarrow x \cdot (4 \cdot (4^{n-1} \cdot A_1 + \sum_{i=0}^{n-2} 4^i) + 1 - 4^n \cdot A_1) = y \cdot (4^n - 1)$$
$$\Rightarrow x \cdot (4^n \cdot A_1 + \sum_{i=0}^{n-1} 4^i - 4^n \cdot A_1) = y \cdot (4^n - 1)$$
$$\Rightarrow x \cdot \sum_{i=0}^{n-1} 4^i = y \cdot (4^n - 1)$$

It is $\sum_{i=0}^{n-1} 4^i = \frac{1-4^n}{1-4} = \frac{4^n-1}{3}$. (Geometric series)

$$\Rightarrow x = \frac{y \cdot (4^n - 1)}{\sum_{i=0}^{n-1} 4^i} \qquad \Rightarrow x = \frac{y \cdot (4^n - 1)}{\frac{4^n-1}{3}}$$
$$\Rightarrow x = 3y$$

For a loop with $n+1$-steps ($n > 0$):

$$x = 3 \cdot 3^n$$
$$y = 3^n + 3^{n-1} \cdot 2^{R_u} + 3^{n-2} \cdot 2^{R_u} \cdot 2^{R_{u_1}} + \ldots + 2^{(R_u + R_{u_1} + \cdots + R_{u_{n-1}})}$$
$$y = 3^n + c, \text{with } c > 0$$

As we derived from above $x = 3y$. After insertion, we get:

$$3 \cdot 3^n = 3 \cdot (3^n + c)$$
$$0 = 3c$$
$$c = 0 \lightning$$

Q.E.D

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