Yang–Mills existence and mass gap concerned by the theory with consolidation

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Abstract

The theory with consolidation (TWC) published recently offers newer paradigm for theoretical fundamental physics. It has derived the four interactions and quantum existence in geometric manner. Outcomes of TWC address many critical problems in fundamental physics. The problem of Yang-Mills existence and mass gap needs understanding of physical mechanisms in pure mathematical sense. Appreciating Yang-Mills existence in TWC perspective takes us to a step forward towards solution of the problem. Existence of the mass gap and that of QFT is discussed in the article.

Keywords: Yang-Mills existence & mass gap, Theory with consolidation, Beyond standard model

1. Introduction

One of the seven Millennium Prize Problems defined by the Clay Mathematics Institute, of Yang–Mills existence and mass gap [1] seeks explanation of physics as pure mathematics. Often an abstract functional space is considered to define the action to lead to physical results. A recent theory [2] set a new paradigm about existence of particles, interactions and universe. This Theory with Consolidation (TWC) offers a different approach to understand the Yang-Mills fields and the mass gap problem. Objective of this paper is to explore for understanding of the problem [1] and for its solution by perspective of the TWC. In next section, TWC will be introduced in brief. Subsequently the fact of mass gap would be explicitly appreciated. Finally we will concern the TWC according to the Wightman axioms.

2. Brief introduction to the Theory with Consolidation

2.1 Configuration of real universe according to TWC

Crux of the TWC is to consider some neighboring points consolidated into 4-balls of specific radius r as the fundamental entities of universe. Thus constituents of configuration space (base space) of the universe aren't just the points but also the too tiny consolidated neighborhoods (CNs) of radius r. It is a reasonable assumption that the tremendous shock while the creation (or Big Bang) would have caused the points within tiny neighborhoods in form of the 4-balls to be consolidated as single entity. As the set of constant sized non-overlapping n-balls doesn't form cover of n-dimensional space, the CNs and the unconsolidated points constitute the space of existence of universe *M*. When the CNs are considered to have induced push, then familiar phenomena appear in *M*. Regard of the base space of

universe having CNs as fundamental constituents complementary to the unconsolidated points provides a realist model. This is a new & effective approach than (or subsidiary to) the conventional approach of considering just the abstract fiber bundles of choice to match the expectations.

Merit of TWC is that it discovers most of the physics phenomena including quantum mechanics (QM), the four interactions, dark matter, dark energy, energy of cosmological constant contained in vacuum, gravity & relativity by abandoning modern physics when provided with simple assumptions about dynamics. It assumes the space of universe to be a 4-dimensional smooth manifold *M* locally having 3+1-dimensions i.e. locally there are three spatial dimensions and a parameter of evolution- time. Further it is assumed that points in the very very small isotropic neighborhood upto certain radius r in *M* got consolidated resulting in the isotropic 4-balls. These isotropic 4-balls are nothing but the CNs. *M* is locally 3+1 dimensional while the constituent CNs are isotropically 4 dimensional. For local action, spatial projection of the CNs exist being compatible with the surrounding; then a CN evolves with same evolution parameter as that for its surrounding. That is, for agreement with surrounding of the CN and in order to general agreement, any CN exists as its projection on the local spatial 3-dimensional space making its evolution with local time possible. Thus the CNs existing as 3-balls with an evolution parameter can rotate, move and can collide in *M*. The consolidation happened once while the creation, hence number of the CNs should be constant. Set of the CNs and its unconsolidated complement in *M* being preserved, CNs can move in *M* by exchanging position with the complement points. For mathematical analysis on *M*, the CNs are equivalent to points (center of a CN is to be identified as the point representing the CN for analysis) i.e. a CN can be specified by set of coordinates in a patch, locations & relative positions of CNs can be characterized by vectors in a patch in *M*, for characteristics on manifolds like metrics & curvature the CN & point have same consideration. This formalism preserves soul calculus of geometric structures:- as two neighboring CNs do role of neighboring points defining the tangent spaces when there is a CN neighboring other CN. Otherwise unconsolidated points neighboring the CN define tangent spaces. For motion of the CNs on *M*, the unconsolidated points on *M* define tangent space. Thus key features of differential geometry are preserved here.

As proved in [3], every number of dimensions of the configuration space comes with a type of ordered direction; for example, 1-dimensional with rectilinear and 2-dimensional with angular. As lower dimensional spaces can be embedded in higher dimensional one, in n-dimensional configuration space all the lower dimensional ordered directions exist. The path lengths along single type of all ordered directions in the geometric (configuration) space form a vector space; hence quantities existing along the ordered directions are vectors. In an n-dimensional configuration space, any vector entity comes in n types or versions having directions along corresponding typed ordered directions. Taking this forward, n-dimensional vector is defined as the quantity in configuration space having direction along n-dimensional ordered direction. In the configuration space having number of dimensions n, an algebraic vector space having n' dimensional vector elements has dimensions n+n'-1 i.e. therein can exist at most n+n'-1 mutually perpendicular n'-dimensional vectors. Elements of any algebraic vector space can be interpreted to have any type of ordered direction in configuration space if dimensionalities match. Alternatively, scalars along such ordered directions form exclusive vector spaces. In general, special parametric path along n-1 sphere leads to manifestation of n-dimensional ($n\neq1$) ordered direction. In this regard, continuous parametric (continuously varying

solid angle being the parameter) path on 2-sphere leads to 3-dimensional ordered direction. The 3-dimensional ordered direction is defined as 'sangular' direction. A vector quantity would exist in all the versions i.e. rectilinear, angular, sangular etc. The 1-dimensional, 2-dimensional & 3-dimensional vectors respectively called as rectilinear, angular & sangular vectors. In the case where configuration space is 4-dimensional :- rectilinear vectors space has 4 dimensions, angular vectors space has 3 dimensions, and sangular vectors space has 2 dimensions (from the n+n'-1 rule). Indeed there exist 4-dimensional vectors having direction described along 3-sphere. But dimension of such vector space is just one. As 1-dimensional vector space has least analytical value, the 4-dimensional vectors have no concern for vector analysis in 4-dimensional configuration space. All the versions of vectors have same algebra and algebraically one can't distinguish between them. The versions have geometrical difference realized when they are configured geometrically in a space or when the quantities exist along corresponding ordered directions in the space of existence.

Abandoning the 4-dimensional version, any vector quantity would exist in *M* in versions of rectilinear, angular and sangular. As TWC assumes induction of a vector push defined as 'soul vector' on the CNs while creation, it would be induced in the three versions. In simple terms the soul vector can be defined as change in local spatial position of the object point in *M* with respect to infinitesimal evolution in local time. As beginning of universe is once, the soul vector should be conserved in all the versions. Importantly, as generally, *M* is infinitesimally piecewise rectilinear; hence the rectilinear versions of vectors can exist in *M* & in the tangent spaces. Due to similar reason the angular & sangular versions can't exist on general *M*. The CNs (as 3-spheres) are infinitesimally piecewise angular & sangular, hence the angular & sangular versions of vectors can exist on the CNs but not in the tangent spaces. Let's use abbreviations for rectilinear soul vector (RSV), angular soul vector (SASV) & sangular soul vector (SSV) for the versions. Due to induction of RSV, CNs move in *M* and collide.

2.2 Interactions between the CNs

Consolidation of CNs facilitates centripetal binding for all kind of rotations of the CNs. The soul vectors induced on CN change when there is rotation; and in order to conserve the soul vectors, rotating CNs generate the unbalanced soul vector demanding packages. Soul vector is generalization of momentum; thus the demanding packages should cause forces between the CNs. As SASVs & SSVs live only on the CNs, interactions due to these versions are straightforward. But RSVs can live on tangent spaces on *M*, hence dynamics of this version is not so primary. Let's see how a CN would evolve from the creation.

A CN moves in *M* due to induced RSV. Thus two CNs can collide. While colliding, fraction of each CN's RSV acts on other CN. The external RSV resolves in tangential & radial components w.r.t. the CN. The radial component gets arithmetically added to RSV of the CN to constitute motion on *M*. While the tangential component gets trapped tangentially on the CN; and causes rotation due to the centripetal binding. Denoting magnitude of the tangentially trapped RSV **s** on CN by s, as proved in section IV of [2] we get instantaneous energy associated with the rotating CN E as

$$E = k\mu v''^{2} \qquad (1)$$

Where k is a universal constant, μ is ratio of r & s, and v" is rotation velocity of RSV **s**. s is special property of a rotating CN and so the μ as r is constant. General covariance demands that v" is to be regarded as largest constant velocity, as discussed in section IV-B of [2] (this is basically due to need of a constant reference for time measurement, like the r for spatial measurement). It suggests that (1) is mass-energy equivalence if kμ is mass of the particle (i.e. of rotating CN) & v" is c.

It is obvious that CNs represent fundamental particles. We get that mass of a particle is direct measure of tangential RSV **s** associated with the particle. Due to the continuous rotation, the **s** changes continuously in any frame; thus a massive particle should demand the unbalanced RSV from its local surrounding. That is, induction of mass (or μ) leads to interaction; mass leads to gravity. As further discussed in [2], this theory explains relativity explicitly.

A rotating CN can further undergo collision. The external RSV due to second collision again resolves as radial & tangential components. The radial component constitutes the RSV arithmetic on *M*, while tangential component wouldn't matter for just arithmetic of the tangentially trapped RSVs. This is because of the local geometry/dynamics at the CN we would see, and also because of the mass due to first collision should bring inertia into the picture. To be strict, let's resolve the tangential external RSV from second collision in components coplanar with the mass (i.e. first) rotation and transverse to the mass rotation. First consider the coplanar component; pretend that this soul vector would change **s** associated with the particle and so the angular velocity ω of the mass rotation. In such case there would be simultaneous increase or simultaneous decrease in ω & s both. But this isn't possible due to constancy of their product c. Hence this component should remain as just an induced soul vector on the CN and would live in a different analytical space than of RSVs. We regard that the mass rotation should convert this component in angular nature. This form of soul vector associated with CNs is defined as spatial angular soul vector, abbreviated as ASV. Due to geometric constraint of ω, s & c, ASV should be regarded as an angular version of soul vector induced from second collision. While, the transverse component from the second collision should naturally rotate the massive CN in the transverse direction. This rotation is associated with inertia due to prior existence of mass, thus it has to be accounted to lead angular momentum of the particle. TWC provides sufficient reasons to accept this transverse component from second collision be same as the spin. Such spin causes rotation of the particle in terms of that of the mass circle and of the ASV. Rotation of ASV should cause change in it; and in order to conserve this type of angular soul vector, it would generate an ASV demanding package in surrounding. Conclusively, second collision should induce ASV interaction by the CN; this is completely different from the RSV interaction induced due to first collision. Discussion in [2] claims that ASV defines electrical charge on the CN that causes ASV interaction. ASV interaction has all the characteristics of electromagnetic interaction.

The two collisions cause induction of two types of soul vectors on the CN and of two perpendicular rotations (of mass & spin). Further collisions wouldn't cause a newer mechanism as there seems no evolutionary constraint other than inertia & the ω-s constraint. Hence further collisions, if happen can be configured by arithmetic of so explored quantities like RSV or mass, ASV or electrical charge, and spin.

So far we considered the RSV induced on the CNs and abandoned the SASV & SSV. This is due to infinitesimally piecewise rectilinear nature of *M*. RSVs drive dynamics on *M*. The two perpendicular rotations induced are sufficient to change SASV & SSV associated with the CN. Hence such CNs would generate SASV & SSV demanding packages in surrounding leading to the two separate interactions. Thus in total there are four types of interactions caused by different versions of soul vectors associated with the CNs. TWC excellently explains interactions of gravity, electromagnetism, strong & weak respectively through conservation of RSV, ASV, SASV & SSV. Strengths of all the interactions are different, and of RSV is far weakest among all, see section XIV-A of [2]. RSV is r times weaker than equivalent of other versions, while kr has magnitude of order 10^{-43}. Additionally, TWC explicitly explored mechanisms of dark matter & dark energy.

Epistemic gauge principle for the three interactions of standard model is also discovered independently by TWC [2].

2.3 Abstract universe according to TWC

A massive particle moving in *M* when seen by virtue of CN is simultaneously rotating & translating. That is a specific wave is associated with a moving particle having periodicity implied by the mass rotation. We call such real wave be 'ontic' wave. TWC provides correspondence between the real universe composed of CNs and the abstract sample space for inference of the real universe. This stems from a proposition that a probabilistic epistemic wave can be considered amenable to be used for inferring properties of corresponding ontic wave having same periodicity and energy.

When we concern for periodicity of the wave due to motion of massive particle, we get generalized form of the known De-Broglie hypothesis. We also get relation between the ontic parameters and the known plank constant. Epistemic functions dependent of the ontic parameters can be constructed for facilitating probabilistic inference about the ontic system. That is, one can assume an epistemic wave corresponding to the ontic wave of a CN. The epistemic wave can be exploited statistically to infer properties of the CN (i.e. particle). The epistemic study will result for true result only if there is reasonable correspondence between the epistemic & ontic system. For this TWC has two constraints while the correspondence: If epistemic function is to be used to contain information about an ontic wave, it should have same periodicity. And, expected energy of the wave should be same as mechanical energy of ontic wave. With these two constraints TWC explores equations of Schrödinger and Dirac for the epistemic functions accountable for ontic systems of the CNs. It also gets the operator mechanism, uncertainties, plank energy, collapse of wave function, entanglement, and importantly the gauge principle & gauge covariant derivatives for the interactions. Most importantly, it further explains massive mediators for the SSV interactions.

The mechanisms are revealed by concerning the ontic models, and their concern in the epistemic models lead to what we know as Standard Model. Particles fundamentally exist in forms of CNs in the universe, we should call such fundamental & explicit model be ontic or real universe. On other hand, statistics offers construction of complex probabilities wherein the dependent functions can be exploited to understand the ontic universe on which the statistic is based. The secondary & statistical model is defined as epistemic or abstract universe. The abstract universe is nothing

but the probabilistic sample space constructed on properties of real universe. The abstract universe is general functional space with inner product as demanded by the dynamics explored. That is, real universe is the manifold *M* while abstract universe is Hilbert space **S** based on *M*. Mathematics for the correspondence between real universe & abstract universe is already reported by Kryukov [4]. TWC makes its use for the rigid formalism as discussed in section IX of [2]. Key output is: an observer attains frame in *M* and also in **S** simultaneously. On the fundamental scale where direct observation of CNs isn't possible, inferences from **S** are effective.

TWC reveals the mechanisms by concerning evolutions of the CNs; when the mechanisms are put in epistemic study, we get the Standard Model. Some essential mechanisms are described in 2.1 & 2.2. Now we would discuss the counterpart in *M* of Yang-Mills fields in **S**.

3. Yang-Mills fields and mass gap

The four interactions are caused in order to conserve different versions of soul vector. When the soul vector associated with CN changes, it should generate the unbalanced soul vector demanding package. Ontic soul vector demanding package is epistemically same as gauge boson. The package holds the unbalanced quantities with respect to the CN from which it is generated, normally just the soul vector.

As discussed in 2.1, the vector space formed in *M* of SASVs is 3-dimensional, and of SSVs is 2-dimensional. Thus these versions of soul vector being exclusive to spatial dynamics, fibers to represent SASV in **S** are 3-dimensional vector spaces while those to represent SSV are 2-dimensional vector spaces. RSV & ASV don't have similar representation as they have share in representation of the CNs in **S** (see VIII-E & XII of [2]). Mass (due to RSV) and electrical charge (due to ASV) of the CN or particle is precondition for representing it in **S**. This isn't the case for SASV & SSV associated with the CN, thus separate fiber bundles are required to represent those on the CNs. Dimensions of the fibers should be such that they span the 4-ball (or its neighborhood); hence respectively 3-dimensional & 2-dimensional fibers are needed to represent SASVs & SSVs, in addition to the basic bundle.

It is proved in section XII-A of [2] that consideration of SSV demanding packages causing the interaction leads to a connection (or gauge) which can be used to mathematically introduce a covariant derivative. The dynamics is unchanged if we use the gauge covariant derivatives instead of usual derivatives while considering the SSV interaction. There the dynamics is unchanged by arbitrary choice of SSV representation in the gauge covariant derivative, as long as it has SU(2) symmetry in **S**. Similarly section XII-B of [2] concludes SASV field to have SU(3) symmetry causing the strong interaction. It makes clear the existence of quantum Yang-Mills fields on $\mathbf{R}^4$ with groups SU(2) & SU(3) those satisfy the standard model rigor.

The soul vector demanding packages in *M* which are equivalent to gauge bosons in **S** propagate from the particle in surrounding to get balanced. Let's consider this ontically. The soul vector associated with the CN changes due to rotations, and the differential soul vector opposite to the change is demanded for balancing. For the balancing to happen, it would propagate in the surrounding i.e. in *M*. *M* is locally 3+1-dimensional while CNs are isotropically 4-dimensional; this fact is pivoting for higher dimensional versions of vectors. Locally CNs exist as 3-dimensional

projection of the 4-balls undergoing time evolution in *M* (so that it gets embedded in the 3+1-dimensional surrounding). If change in soul vector (& hence the demand) lies in the spatial surrounding, then generation of the demanding package is straightforward. Tangential RSV (that causes mass) and ASV (causing electromagnetism) essentially exist in spatial three dimensions with the coherent time evolution. This is because these soul vectors are associated with CNs due to collisions where time is exclusively & coherently different dimension. But SSV & SASV are associated with CNs from the creation (where the 4 dimensions of 4-balls are isotropic). Locally they exist as projections on the spatial 3-ball; rotations due to the collisions should cause change in them.

Change in any soul vector with respect to time should essentially lie within the spatial 3 dimensions for its local existence. As the spaces of SSV & SASV span 4-ball, change in them isn't essentially within the spatial dimensions. The rotations induced due to RSV should cause changes in them. SSV & SASV lie to span 4-ball, thus they can vary in the 4-dimensions. Single SSV exists in three dimensions while single SASV in two. Thus though any such vector lies along the extraspatial dimension on the 4-ball, it always has definite projection on the spatial 3-ball (& so on the *M*). Hence general changes in directions of these kinds of soul vector can be through their projections. Further, the cause of the changes (i.e. rotation due to mass or spin) also exists in two dimensions, and this cause should lie along the change it has caused. Hence projection of mass circle will also vary in case of going along extraspatial dimension while causing variation in SSV & SASV associated with the CN. Electromagnetism inducing ASV also lies along same mass circle, and spin is transverse to mass circle, thus there should be change in projection of mass, electrical charge & spin while causing general changes in SSV & SASV. As a result, the SSV or SASV demanding packages would come with mass and electrical charge in order to balance those properties.

Single SSV spans three dimensions; thus change in its direction should essentially go along the extraspatial dimension. Hence SSV demanding package should essentially come with mass (&/or electrical charge) as some fraction of the mass circle should lie along the extraspatial dimension to cause SSV change. This concern is discussed in XIII of [2]. On other hand, single SASV spans two dimensions leaving a dimension out of the spatial three to change it along. That is, change in SASV direction is possible within the spatial dimensions. Therefore the mass & spin circles need not to go along the extraspatial dimension. Hence SASV demand wouldn't essentially come with mass.

Mass of the demanding packages is nothing but the change in projection of mass circles on spatial 3-balls while causing the changes. At the moment mass circles goes extraspatial, the instantaneous change in its projection is the mass that package should carry. It is beauty of TWC that it has exclusive meaning for instantaneous variation; the infinitesimal time in *M* is not about zero, but it has the lower bound

$$\mathrm{dt} = \frac{2r \sin}{ic} \qquad (2)$$

Where β is complementary of angle between minimum separation 2r between neighboring CNs and time axis on spacetime diagram (i.e. β depends on velocities of the CNs in the chart). More discussion about this is done in section VII of [2].

If we regard Θ be angular velocity (or so) transverse to spatial subspace while causing the soul vector variation, then change in mass projection at the instant of extraspatial change can be calculated as

$$\Delta = \cos\left(\frac{2r\sin \quad .\Theta}{ic}\right) \tag{3}$$

This Δ is mass of single package. For relative time concerns, β can't be zero; and it can't proceed π i.e. $\pi>\beta>0$. Also $\Theta>0$. Further, Θ is parameterized for variation within the moment dt; it would be about infinitesimal and not proceed even significant angle. In general, $\pi/2 > \Theta > 0$. Hence $\Delta > 0$. This implies the essential positive mass gap for SSV (& SASV) interactions. In epistemic picture, the lowest excitations of a pure SSV (& SASV) fields have a finite mass-gap with regard to the vacuum state. Thus TWC explains positive mass gap for gauge fields of SU(2) & SU(3) symmetry.

4. The problem perspective

As per TWC we have inputs from ontic universe such as the interactions due to soul vector demanding packages, dimensionality of the space of existence of them, mass gap (i.e. single SSV & SASV demanding package has positive mass), mechanism of relativity etc. We also have a tool for construction of epistemic space based on the ontic universe discussed in above sections and in [2]. When the inputs from ontic universe *M* are concerned as constraints, the desired Hilbert space can be constructed satisfying the Wightman axioms. Thus one can appreciate mass gap mechanism in the Hilbert space, because just like we regard principle of relativity be constraint for choice of operator representation, mass gap too is an ontic constraint.

Here I will try to elaborate this specifically. A correspondence between ontic universe *M* and epistemic sample space for its inference **S** is discussed in [2]. Delta function at a point in *M* implies definite existence of the soul vector (or so field property) at the point. A concern that common QM space (of Lebesgue square integrable functions) doesn't contain delta functions & their derivatives is resolved by Kryukov's 2nd & 3rd theorems [4]. Importantly, delta functions at all the points in *M* provide basis for **S** [4]. As deltas represent pure states and the basis for **S**, the pure states are given by the one-dimensional subspaces of the separable complex Hilbert space. **S** is probabilistic sample space for inference of *M* and we regard elements of **S** as probable outcomes; there is relevant discussion in VIII & IX of [2]. The operator mechanism for obtaining expectation values is discovered in VIII-C of [2] for analysis in **S**. As all the operators are linear, they have eigenvalues and eigenfunctions; thus the operator specific basis can be constructed for the probabilistic operator space. Any nonlinear operator in *M* becomes linear when extended to **S** and the Poincaré group acts unitarily on **S** [4]. To span the CNs, SSVs live in 2-dimensional vector space while SASVs in 3-dimensional vector space; i.e. they are to be considered doublets and triplets for probabilistic analysis. This is concerned in XII of [2], the epistemic study of SSV and SASV field results to the conclusion that these are gauge fields with the groups being SU(2) and SU(3) respectively. Also, TWC in geometric manner has discovered mechanism of relativity (see IV-B & VI of [2]). An observer attains simultaneous realization of *M* & **S** both as implied in fig.3 of [2]. Thus as a result of all this, constraints of the Wightman axioms are satisfied. Rest is just the mathematical refinements for obtaining real outputs from analysis on **S** for agreement with *M* (as an observer get frames in both these general spaces simultaneously.

5. Conclusion

TWC discovers the physical mechanisms like field interactions, relativity, ontic-epistemic (*M*-**S**) correspondence, mass gap for SU(2) & SU(3) gauge interactions. This offers configuration of SU(2) & SU(3) gauge interactions according to Wightman axioms while explaining the mass gap. SSV fields essentially have mass gap while for SASV fields, there is scope for $\Theta = 0$.

This is potential solution to the problem [1].

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