Implications of Multidimensional Geometries and Measurement

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Complexification of Maxwell's equations with an extension of the gauge condition to non-Abelian algebras, yields a putative metrical unification of relativity, electromagnetism and quantum theory. This unique new approach also yields a universal nonlocality with implications for Bell's Theorem and the possibility of instantaneous quantum connections because spatial separation can vanish by utilizing the complex space.

1. Introduction

In this chapter we develop non-Abelian gauge groups for real and complex amended Maxwell's equations in a complex 8-Dimensional Minkowski space in order to describe nonlocality in quantum theory and relativity which has implications for extending gravitational theory to the unitary regime. We demonstrate a mapping between the twistor algebra of the complex 8-space and the spinor calculus of 5D Kaluza-Klein geometry which attempts to unify Gravitational and EM theory. (Chap. 6) Our quantum formalism demonstrates that solving the Schrödinger equation in a complex 8D geometry yields coherent collective state phenomena with soliton wave solutions. The model shows that standard quantum theory is a linear approximation of a higher Dimensional complex space. Through this formalism we can assess that complex systems can be defined within conventional quantum theory as long as we express that theory in a hyper-geometric space. We utilize our complex dimensional geometry to formulate nonlocal correlated phenomena, including the quantum description of the 1935 EPR paradox formulated with Bell's theorem. Tests by Clauser, Aspect, and Gisin have demonstrated that particles emitted with approximate simultaneity at the speed of light, *c* remain correlated nonlocally over meter and kilometer distances. As Stapp has said, Bell's theorem and its experimental verification is one of the most profound discoveries of the 20th century. We will demonstrate the application of our formalism for complex systems and review the history of our model from 1974.

We have analyzed, calculated and extended the modification of Maxwell's equations in a complex Minkowski metric, M_4 in a C_2 space using the SU_2 gauge, SL(2,c) and other gauge groups, such as SU_n for $n > 2$ expanding the U_1 gauge theories of Weyl. This work yields additional predictions beyond the electroweak unification scheme. Some of these are: 1) modified gauge invariant conditions, 2) short range non-Abelian force terms and Abelian long-range force terms in Maxwell's Eq. 3) finite but small rest mass of the photon, and 4) a magnetic monopole like term and 5) longitudinal as well as transverse magnetic and electromagnetic field components in a complex Minkowski metric M_4 in a C_2 space.

This is an 8D complex Minkowski space, M_4+C_4 composed of 4 real and 4 imaginary dimensions consistent with Lorentz invariance and analytic continuation in the complex plane [1-6]. The unique feature of this geometry is that it admits nonlocality consistent with Bell's theorem, (EPR paradox), possibly Young's double slit experiment, the Aharonov-Bohm effect and multi mirrored interferometric experiment.

Also, expressing Maxwell's EM equations in complex 8-space, leads to some new and interesting predictions in physics, including possible detailed explanation of some of the previously mentioned nonlocality experiments [7-11]. Complexification of Maxwell's equations requires a non-Abelian gauge group which amends the usual theory, which utilizes the usual unimodular Weyl U_1 group. We have examined the modification of gauge conditions using higher symmetry groups such as SU_2, SU_n and other groups such as the SL(2,c) double cover group of the rotational group SO(3,1) related to Shipov's Ricci curvature tensor [12,13] and a possible neo-aether picture. Thus, we are led to new and interesting physics involving extended metrical space constraints, the usual transverse and also longitudinal, non Hertzian electric and magnetic field solutions to Maxwell's equations, possibly leading to new communication systems and antennae theory, non-zero solutions to $\nabla \cdot \underline{B}$, and a possible finite but small rest mass of the photon.

Comparison of our theoretical approach is made to the work on amended Maxwell's theory [14-17]. We compare our predictions such as our longitudinal field to the $B^{(3)}$ term of Vigier, and our Non-Abelian gauge groups to that of Barrett and Harmuth. This author interprets this work as leading to new and interesting physics, including a possible reinterpretation of a neo-aether with nonlocal information transmission properties.

2. Complexified EM Fields in Local & Nonlocal Minkowski Space

We expand the usual line element metric $ds^2 = g_{\nu\mu} dx^\nu dx^\mu$ in the following manner. We consider a complex 8D space, M_4 constructed so that $Z^u = x^u_{\mathrm{Re}} + i\, x^u_{\mathrm{Im}}$ and likewise for Z^ν where the indices ν and μ run 1 to 4 yielding (1, 1, 1, -1). Hence, we now have a new complex eight space metric as $ds^2 = \eta_{\nu\mu} dZ^\nu dZ^\mu$. We have developed this space and other extended complex spaces and examined their relationship with the twistor algebras and asymptotic twistor space and the spinor calculus and other implications of the theory [18-21]. The Penrose twistor SU(2,2) or U_4 is constructed from four space – time, $U_2 \otimes \widetilde{U}_2$ where U_2 is the real part of the space and $\widetilde{U}_2$ is the imaginary part of the space, this metric appears to be a fruitful area to explore.

The twistor Z can be a pair of spinors U^A and π_A which are said to represent the twistor. The condition for these representations are 1) the null infinity condition for a zero-spin field is $Z^\mu \overline{Z}_\mu = 0$, 2) conformal invariance and 3) independence of the origin. The twistor is derived from the imaginary part of the spinor field. The underlying concept of twistor theory is that of conformally invariance fields occupy a fundamental role in physics and may yield some new physics. Since the twistor algebra falls naturally out complex space.

Other researchers have examined complex dimensional Minkowski spaces. In [2], Newman demonstrates that M_4 space do not generate any major 'weird physics' or anomalous physics predictions and is consistent with an expanded or amended special and general relativity. In fact, the Kerr metric falls naturally out of this formalism as demonstrated by Newman [4,5].

As we know twistors and spinors are related by the general Lorentz conditions in such a manner that all signals are luminal in the usual four N Minkowski space but this does not preclude super or trans luminal signals in spaces where N > 4. Stapp, for example, has interpreted the Bell's theorem experimental results in terms of transluminal signals to address the nonlocality issue of the Clauser, et.al and Aspect experiments [22]. Kozameh and Newman demonstrate the role of nonlocal fields in complex 8-space.

We believe that there are some very interesting properties of the M_4 space which include the nonlocality properties of the metric applicable in the non-Abelian algebras related to the quantum theory and the conformal invariance in relativity as well as new properties of Maxwell's equations. In addition, complexification of Maxwell's equations in M_4 space yields some interesting predictions, yet we find the usual conditions on the manifold hold [23-25]. Some of these new predictions come out of the complexification of four space 2 and appear to relate to the work of Vigier, Barrett, Harmuth and others [14]. Also we fin,d that the twistor algebra of the complex eight dimensional, M_4 space is mappable 1 to 1 with the twistor algebra, C_2 space of the Kaluza-Klein five dimensional electromagnetic - gravitational metric [12,13].

Some of the predictions of the complexified form of Maxwell's equations are 1) a finite but small rest mass of the photon, 2) a possible magnetic monopole, $\nabla \cdot \underline{\beta \neq 0}$, 3) transverse as well as longitudinal $B^{(3)}$ like components of $\underline{E}$ and $\underline{B}$, 4) new extended gauge invariance conditions to include non-Abelian algebras and 5) an inherent fundamental nonlocality property on the manifold. Vigier also explores longitudinal $\underline{E}$ and $\underline{B}$ components in detail and finite rest mass of the photon [26].

Considering both the electric and magnetic fields to be complexified as $\underline{E} = \underline{E}_{\mathrm{Re}} + i\underline{E}_{\mathrm{Im}}$ and $\underline{B} = \underline{B}_{\mathrm{Re}} + i\underline{B}_{im}$ for $E_{\mathrm{Re}}, E_{\mathrm{Im}}, B_{\mathrm{Re}}$ and B_{Im} are real quantities. Then substitution of these two equations into the complex form of Maxwell's equations above yields, upon separation of real and imaginary parts, two sets of Maxwell-like equations. The first set is

$$\nabla \cdot \underline{E}_{\mathrm{Re}} = 4\pi\rho_e \,,\ \nabla \times E_{\mathrm{Re}} = -\frac{1}{c}\frac{\partial \underline{B}_{\mathrm{Re}}}{\partial t} ;\ \nabla \cdot \underline{B}_{\mathrm{Re}} = 0 \,,\ \nabla \times \underline{B}_{\mathrm{Re}} = \frac{1}{c}\frac{\partial \underline{E}_{\mathrm{Re}}}{\partial t} = \underline{J}_e \qquad (1)$$

the second set is

$$\nabla \cdot \left(i\underline{B}_{\mathrm{Im}}\right) = 4\pi i\rho_m \,, \nabla \times \left(i\underline{B}_{\mathrm{Im}}\right) = \frac{1}{c}\frac{\partial\left(i\underline{E}_{\mathrm{Im}}\right)}{\partial t} ;$$

$$\nabla \cdot \left(iE_{\mathrm{Im}}\right) = 0 \,, \nabla \times \left(i\underline{E}\right) = \frac{1}{c}\frac{\partial\left(iB_{\mathrm{Im}}\right)}{\partial t} = i\underline{J}m \qquad (2)$$

The real part of the electric and magnetic fields yield the usual Maxwell's equations and complex parts generate 'mirror' equations; for example, the divergence of the real component of the magnetic field is zero, but the divergence of the imaginary part of the electric field is zero, and so forth. The structure of the real and imaginary parts of the fields is parallel with the electric real components being substituted by the imaginary part of the magnetic fields and the real part of the magnetic field being substituted by the imaginary part of the electric field.

In the second set of equations, (2), the i's, 'go out' so that the quantities in the equations are real, hence $\nabla \cdot \underline{B}_{\mathrm{Im}} = 4\pi\rho_m$, and not zero, yielding a term that may be associated with some classes of monopole theories. See references in [16,17]. We express the charge density and current density as complex quantities based on the separation of Maxwell's equations above. Then, in generalized form $\rho = \rho_e = i\rho_m$ and $J=J_e+iJ_m$ where it may be possible to associate the imaginary complex charge with the magnetic monopole and conversely the electric current has an associated imaginary magnetic current.

The alternate of defining and using, which Evans does $\underline{E} = \underline{E}_{\mathrm{Re}} + i\underline{B}_{\mathrm{Im}}$ and $\underline{B} = \underline{B}_{\mathrm{Re}} + i\underline{E}_{\mathrm{Im}}$ would not yield a description of the magnetic monopole in terms of complex quantities but would yield, for example $\nabla \cdot \left(i\underline{B}_{\mathrm{Im}}\right) = 0$ in the second set of equations. Using the invariance of the line element $s^2 = x^2 - c^2t^2$ for $r = ct = \sqrt{x}^2$ and for $x^2 = x^2 + y^2 + z^2$ for the distance from an electron charge, we can write the relation, $\frac{1}{c}\frac{\partial\left(iB_{im}\right)}{\partial t} = iJm$ or $\frac{1}{c}\frac{\partial B_{im}}{\partial t} = J_m$; $\nabla \times \left(iE_{\mathrm{Im}}\right) = 0$ for $\underline{E}_{\mathrm{Im}} = 0$ or

$$\frac{1}{c}\frac{\partial\left(iB_{\mathrm{Im}}\right)}{\partial t} = iJm \qquad (3)$$

3. Complex Minkowski Space: Implications for Physics

In a series of papers, Barrett, Harmuth and Rauscher have examined the modification of gauge conditions in modified or amended Maxwell theory. The Rauscher approach, as briefly explained in the preceding section is to write complexified Maxwell's equation in consistent form to complex Minkowski space [17].

The Barrett amended Maxwell theory utilizes non-Abelian algebras and leads to some very interesting predictions which have interested me for some years. He utilizes the non-commutative SU_2 gauge symmetry rather than the U_1 symmetry. Although the Glashow electroweak theory utilizes U_1 and SU_2, but in a different manner, but his theory does not lead to the interesting and unique predictions of the Barrett theory.

Barrett, in his amended Maxwell theory, predicts that the velocity of the propagation of signals is not the velocity of light. He presents the magnetic monopole concept resulting from the amended Maxwell picture. His motive goes beyond standard Maxwell formalism and generate new physics utilizing a non-Abelian gauge theory.

The SU_2 group gives us symmetry breaking to the U_1 group which can act to create a mass splitting symmetry that yield a photon of finite (but necessarily small) rest mass which may be created as self energy produced by the existence of the vacuum. This finite rest mass photon can constitute a propagation signal carrier less than the velocity of light.

We can construct the generators of the SU2 algebra in terms of the fields $\underline{E}$, $\underline{B}$, and $\underline{A}$. The usual potentials, A_μ is the important four vector quality $A_\mu = (\underline{A}, \phi)$ where the index runs 1 to 4. One of the major purposes of introducing the vector and scalar potentials and also to subscribe to their physicality is the desire by physicists to avoid action at a distance. In fact in, gauge theories A_μ is all there is! Yet, it appears that, in fact, these potentials yield a basis for a fundamental nonlocality!

Let us address the specific case of the SU_2 group and consider the elements of a non-Abelian algebra such as the fields with SU_2 (or even SU_n) symmetry then we have the commutation relations where XY-YX$\neq 0$ or [X,Y] $\neq 0$. Which is reminiscent of the Heisenberg uncertainty principle non-Abelian gauge. Barrett does explain that SU_2 fields can be transformed into U_1 fields by symmetry breaking. For the SU_2 gauge amended Maxwell theory additional terms appear in term of operations such $A \cdot E, A \cdot B$ and $A \times B$ and their non-Abelian converses. For example $\nabla \cdot B$ no longer equals zero but is given as $\nabla \cdot B = -jg(A \cdot B - B \cdot A) \neq 0$ where $[A,B] \neq 0$ for the dot product of A and B and hence we have a magnetic monopole term and *j* is the current and *g* is a constant. Also, Barrett gives references to the Dirac, Schwinger and t' Hooft monopole work. Further commentary on the SU_2 gauge conjecture of Mamuth that under symmetry breaking, electric charge is considered but magnetic charges are not. Barrett further states that the symmetry breaking conditions chosen are to be determined by the physics of the problem. These non-Abelian algebras have consistence to quantum theory.

In our analysis, using the SU_2 group there is the automatic introduction of short range forces in addition to the long-range force of the U_1 group. U_1 is one dimensional and Abelian and SU_2 is three dimensional and is non-Abelian. U_1 is also a subgroup of SU_2. The U_1 group is associated with the long range $1/r^2$ force and SU_2, such as for its application to the weak force yields short range associated fields. Also SU_2 is a subgroup of the useful SL(2,c) group of non compact operations on the manifold. SL(2,c) is a semi simple four dimensional Lie group and is a spinor group relevant to the relativistic formalism and is isomorphic to the connected Lorentz group associated with the Lorentz transformations. It is a conjugate group to the SU_2 group and contains an inverse. The double cover group of SU_2 is SL(2,c) where SL(2,c) is a complexification of SU_2. Also SL(2,c) is the double cover group of SU_3 related to the set of rotations in three dimensional space. Topologically, SU_2 is associated with isomorphic to the 3D spherical, O_3^+ (or 3D rotations) and U_1 is associated with the O_2 group of rotations in two dimensions. The ratio of Abelian to non-Abelian components, moving from U_1 to SU_2, gauge is 1 to 2 so that the short-range components are twice as many as the long-range components.

Instead of using the SU_2 gauge condition we use SL (2,c) we have a non-Abelian gauge and hence quantum theory and since this group is a spinor and is the double cover group of the Lorentz group (for spin ½) we have the conditions for a relativistic formalism. The Barrett formalism is non-relativistic. SL (2,c) is the double cover group of SU_2 but utilizing a similar approach using twistor algebras yields relativistic physics.

It appears that complex geometry can yield a new complementary unification of quantum theory, relativity and allow a domain of action for nonlocality phenomena, such as displayed in the results of the Bell's theorem tests of the EPR paradox, and in which the principles of the quantum theory hold to be universally. The properties of the nonlocal connections in complex 4-space may be mediated by non -or low dispersive loss solutions. We solved Schrödinger equation in complex Minkowski space [27,28].

In progress is research involving other extended gauge theory models, with particular interest in the nonlocality properties on the S pact-time manifold, quantum properties such as expressed in the EPR paradox and coherent states in matter.

Utilizing Coxeter graphs or Dynkin diagrams, Sirag lays out a comprehensive program in terms of the A_n, D_n and E_6, E_7 and E_8 Lie algebras constructing a hyper dimensional geometry for as a classification scheme for elementary particles. Inherently, this theory utilizes complexified spaces involving twistors and Kaluza-Klein geometries. This space incorporates string theory and GUT models [29].

4. Complex Vector and Advanced Potentials & Bell's Inequality

The issue of whether Bell's theorem and other remote connectedness phenomena, such as Young's double slit experiment, demands superluminal or space-like signals or prior luminal signals is an area of hot debate. Also, the issue of advanced vs. retarded potentials is of interest in this regard.

Using the complex model of A^{μ} we will examine the issue of the non-locality of Bell's theorem as quantum mechanical 'transactions' providing a microscopic communication path between detectors across space-like intervals, which violate the EPR locality postulate. This picture appears to be consistent with the remote connectedness properties of complex Minkowski space. Also, there are implications for macroscopic communications channels; another area of hot debate. Detailed discussions of Bell's theorem are given in [30].

We will formulate fields in terms of A or $\underset{\sim}{A} = (A^j, \phi)$ where A^j is $\underline{A}$ rather than the tensor $F_{\mu\nu}$ or $\underline{E}$ or $\underline{B}$. We can proceed from the continuity equation $\nabla \cdot J + \partial\rho / \partial t = 0$ and the expression $F_{\mu\nu} = \partial A_\nu / \partial X_\mu - \partial A_\mu / \partial X_\nu$. For the usual restored potentials then, we have the Lorentz condition

$$\nabla \cdot A + \mu\varepsilon \frac{\partial \phi}{\partial t} = 0 \text{ and also } \nabla^2 A - \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu J \qquad (4)$$

We can also derive

$$\nabla^2 \phi - \mu\varepsilon \frac{\partial^2 \phi}{\partial t^2} = -\frac{1}{\varepsilon}\rho \qquad (5)$$

These equations possess a restored potential solution. The radiation field in quantum electrodynamics is usually quantized in terms of (A, ϕ) .[We can also convert back to the $\bar{E}$ and $\bar{B}$ .fields using $E = -\nabla\phi - \partial A / \partial t$ and $B = \nabla \times A$]. Quantization of the field consists of regarding the coordinates (x, k) or (q, p) as quantum mechanical coordinates of a set of equivalent harmonic oscillators. In the second quantized method treating k_r, q_r and A_r as quantum numbers then we have quantized allowable energy levels such as $W = \sum_r (n_r + \eta_2)\hbar\omega_r$. Solutions are given in the form

$$\Psi \propto \sum_{n_r} n_r e \exp - \frac{iW(n_r)}{\hbar} \tag{6}$$

and we have a Hamiltonian equation of motion $\dot{p}_{ab} + (ck)^2 q_{ab} = 0$ or $\dot{q}_{ab} = p_{ab}$ and

$$\mathcal{H} = \tfrac{1}{2}\sum [p_{ab}^2 + (ck)^2 q^2 q_{ab}^2]. \tag{7}$$

The electromagnetic field energy of the volume integral $(E^2 + B^2)/8\pi$ is just equal to the Hamiltonian.
We can examine such things as absorption and polarization in terms of the complexification of $\underline{E}$ and $\underline{B}$ or $\underline{A}$ and ϕ. We define the usual $D = \varepsilon E$ (or displacement field) and $B = \mu H$ for a homogeneous isotopic media. If we introduce p_0 and m_0 as independent of $\underline{E}$ and $\underline{H}$ where the induced polarizations of the media are absorbed into the parameters ε and μ, we have

$$D = \varepsilon E + p_0 \text{ and } H = \tfrac{1}{\mu} B - m_0 \tag{8}$$

Then we define a complex field as

$$\underline{\underline{Q}} \equiv \underline{B} + i\sqrt{\varepsilon\mu}\,\underline{E} \tag{9}$$

so that we have Maxwell's equations now written as

$$\nabla \times Q + i\sqrt{\varepsilon\mu}\, {\partial Q}/{\partial t} = \mu J \text{ and } \nabla \cdot Q = i\sqrt{{\mu}/{\varepsilon}}\quad \rho \tag{10}$$

Using vector identities [23-25] and resolving into real and imaginary parts, we have

$$\nabla^2 H - \varepsilon\mu \frac{\partial^2 H}{\partial t^2} = -\nabla \times J \text{ and } \nabla^2 E - \varepsilon\mu \frac{\partial E}{\partial t^2} = \mu \frac{\partial J}{\partial t} + \frac{1}{\varepsilon}\nabla\rho \tag{11}$$

We define Q in terms of the complex vector potential that $A_{\mathrm{Re}} \rightarrow L_{complex}$ and $\phi_{\mathrm{Re}} \rightarrow \phi_{complex}$.
Then

$$Q = \nabla \times L - i\sqrt{\varepsilon\mu}\frac{\partial L}{\partial t} - i\sqrt{\varepsilon\mu}\nabla\phi \tag{12}$$

subject to the condition similar to before, $\nabla \cdot \underline{L} + \varepsilon\mu\, {\partial\phi}/{\partial t} = 0$. Then we have

$$\nabla^2 L - \varepsilon\mu \,{\partial^2 L}\Big/{\partial t^2} = -\mu J \text{ and } \nabla^2 \phi - \varepsilon\mu \,{\partial^2 \phi}\Big/{\partial t^2} = -{1}\big/{\varepsilon}\ \ \rho \qquad (13)$$

Separation into real and imaginary parts of these potentials, L and ϕ can be written as

$$L = A_{\mathrm{Re}} - i\sqrt{{\mu}\big/{\varepsilon}}\ A_{\mathrm{Im}} \text{ and } \phi = \phi_{\mathrm{Re}} - i\sqrt{{\mu}\big/{\varepsilon}}\ \phi_{\mathrm{Im}} \qquad (14)$$

Upon substitution into the equation for Q and separation into real and imaginary parts we have

$$\underline{B}_{\mathrm{Re}} = \nabla \times \underline{A}_{\mathrm{Re}} - \frac{\mu \partial \underline{A}_{\mathrm{Im}}}{\partial t} - \mu \nabla \phi_{\mathrm{Im}}\,; \quad \underline{E}_{\mathrm{Re}} = -\nabla \phi_{\mathrm{Re}} - \frac{\partial \underline{A}_{\mathrm{Re}}}{\partial t} - \frac{1}{c}\nabla \times \underline{A}_{\mathrm{Im}} \qquad (15)$$

The usual equations are allowed when A_{Im} and ϕ_{Im} are taken as zero.
If free currents and charges are everywhere zero in the region under consideration, then we have

$$\nabla \times Q + i\sqrt{\varepsilon\mu}\,{\partial Q}\big/{\partial t} = 0\,; \quad \nabla Q = 0 \qquad (16)$$

and we can express the field in terms of a single complex Hertzian vector $\underline{\Gamma}$ as the solution of

$$\nabla^2 \underline{\Gamma} - \varepsilon\mu \ \ {\partial^2 \underline{\Gamma}}\Big/{\partial t^2} = 0 \qquad (17)$$

We can define Γ by

$$\Gamma \equiv \pi_{\mathrm{Re}} - i\sqrt{{\mu}\big/{\varepsilon}}\ \ \pi_{\mathrm{Im}} \qquad (18)$$

where $\phi_{\mathrm{Re}} = -\nabla \cdot \underline{\pi}$ and we can write such expressions as

$$A_{\mathrm{Im}} = \mu\varepsilon \,{\partial \pi_{\mathrm{Im}}}\big/{\partial t} \text{ and } \quad \phi_{\mathrm{Im}} = \nabla \cdot \underline{\pi}_{\mathrm{Im}} \qquad (19)$$

This formalism works for a dielectric media but if the media is conducting the field equations is no longer symmetric then the method fails. Symmetry can be maintained by introducing a complex induced capacity $\varepsilon' = \varepsilon_{\mathrm{Re}} \pm i\ \ {\sigma_{\mathrm{Im}}}\big/{\omega}$. The vector B is in a solenoid charge-free region; this method works. Calculation of states of polarization by the complex method demonstrates its usefulness and validity. Also, absorption can be considered in terms of complex fields. We will apply this method to solutions that can be described as restored and advanced and may explain Bell's theorem of non-locality. Linear and circular polarization can be expressed in terms of complex vectors $A = A_{\mathrm{Re}} + iA_{\mathrm{Im}}$. The light quanta undergoing this polarization is given as $\hbar\omega\hat{n} = \hbar\sigma = \hbar k$. Complex unit vectors are introduced so that real and imaginary components are considered orthogonal. We have a form such as $A = (A \cdot \hat{\ell}_{\mathrm{Im}})\hat{\ell}_{\mathrm{Re}} + (A \cdot \hat{j}_{\mathrm{Im}})\hat{j}_{\mathrm{Re}}$. The linearly polarized wave at angle θ is

$$A = \frac{A}{\sqrt{2}}(\ell_{\mathrm{Re}} \quad e^{-i\theta} - ij_{\mathrm{Re}} \quad e^{i\theta}) . \qquad (20)$$

Now let us consider use of this polarization formalism to describe the polarization-detection process in the calcium source photon experiment of J. Clauser et al [31]. Let us first look at solutions to the field equations for time-like and space-like events. The non-locality of Bell's theorem appears to be related to the remote connected-ness of the complex geometry and the stability of the soliton over space and time.

We will consider periodically varying fields which move along the x-axis. For source-free space, we can write

$$c^2 \nabla^2 \underline{F} = \phi^2 \underline{F} \big/ \phi t^2 \qquad (21)$$

where $\underline{F}$ represents either $\underline{E}$ or $\underline{B}$. The two independent solutions for this equation are [32] $\underline{E}_{\pm}(x,t) = E_0 \sin(2\pi kx \pm \nu t)$ and

$$\underline{B}_{\pm}(x,t) = B_0 \sin 2\pi(kx \pm \nu t) \qquad (22)$$

and k is the wave number and ν the frequency of the wave. The $\forall$ sign refers to the two independent solutions to the above second order equation in space and time. The wave corresponding to E_+ and B_+ will exist only when t < 0 (past lightcone) and the wave corresponding to *E_* and *B_* will exist for $t > 0$ (future lightcone). Then the *E_* wave arrives at a point x in a time t after emission, while E_+ wave arrive at x in time, t before emission (like a tachyon).

Using Maxwell's equations for one spatial dimension, x, and the Poynting vector which indicates the direction of energy and momentum flow of the electromagnetic wave, we find that E_+ and B_+ correspond to a wave emitted in the $+x$ direction but with energy flowing in the $-x$ direction. For example, $E_+(x, t)$ is a negatives-energy and negative-frequency solution. The wave signal will arrive $t = x/c$ before it is emitted, and is termed an advanced wave. The solution *E_(x,t)* is the normal positive-energy solution and arrives at x in time, $t = x/c$, after the instant of emission and is called the retarded potential, which is the usual potential.

The negative energy solutions can be interpreted in the quantum picture in quantum electrodynamics as virtual quantum states such as vacuum states in the Fermi-sea model. 'These virtual states are not fully realizable as a single real state but can definitely effect real physical processes to a significant testable extent'. The causality conditions in S-matrix theory, as expressed by analytic continuation in the complex plane, relate real and virtual states [28,29]. Virtual states can operate as a polarizable media leading to modification of real physical states. In fact, coherent collective excitations of a real media can be explained through the operations in a underlying virtual media.

Four solutions emerge: Two <u>retarded</u> (F_1 and F_2) connecting processes in the forward light cone and two <u>advanced</u>, (F_3 and F_4) connecting processes in the backward slight cone [33]. These four solutions are

$$\begin{aligned} F_1 &= F_0 e^{-i(-kx-\omega t)}, \quad F_2 = F_0 e^{i(kx-\omega t)}; \\ F_3 &= F_0 e^{i(-kx+\omega t)}, \quad F_4 = e^{i(kx+\omega t)} \end{aligned} \qquad (23)$$

where F_1 is for a wave moving in the (-x, + t) direction, F_2 is for a (+x, +t) moving wave, F_3 is for a (-x, -t) moving wave, and F_4 is a (+x, -t) moving wave. F_1 and F_4 are complex conjugates of each other and F_2 and F_3, are complex conjugates of each other, so that $F_1^+ = F_4$ and $F_2^+ = F_3$. Then the usual solutions to Maxwell's equations are retarded plane wave solutions.

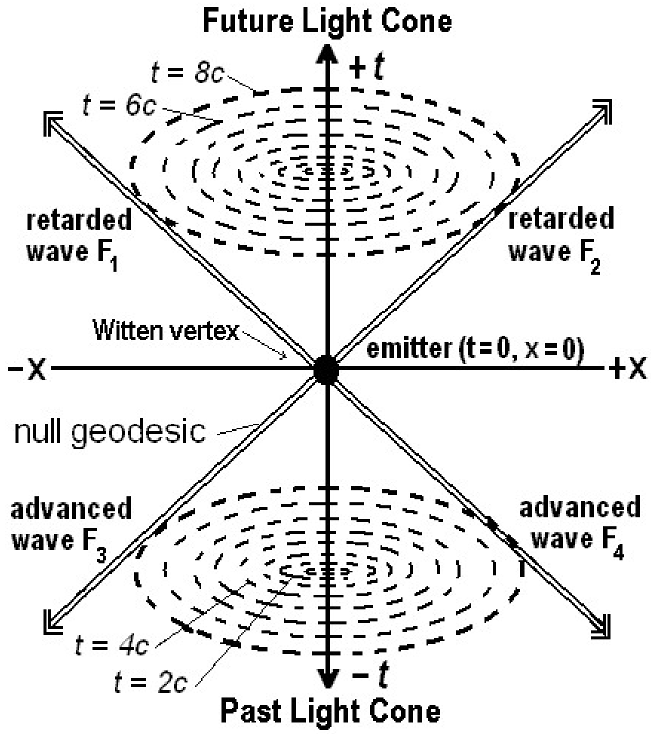

Figure 1. Adaptation of a complex Minkowski light-cone showing advanced-retarded future-past Cramer wavefront transactions with a central Witten Ising lattice string vertex able to undergo symmetry transformations.

The proper formulation of nonlocal correlations, which appear to come out of complex geometries may provide a conceptual framework for a number of quantum mechanical paradoxes and appear to be explained by Bell's nonlocality, Young's double slit experiment, the Schrödinger cat paradox, superconductivity, superfluidity, and plasma 'instabilities' including Wheeler's 'delayed choice experiment'. Interpretation of these phenomena is made in terms of their implications about the lack of locality and the decomposition of the wave function which arises from the action of advanced waves which 'verify' the quantum-mechanical transactions or communications.

Cramer [33] has demonstrated that the communication path between detectors in the Bell inequality experiments can be represented by space-like intervals and produce the quantum mechanical result. By the addition of two time-like four vectors having time components of opposite signs which demonstrate the locality violations of Bell's theorem and is consistent with the Clauser, Fry and Aspect experimental results. This model essentially is an 'action-at-a-distance' formalism.

One can think of the emitter (in Bell's or Young's quantum condition) as sending out a pilot or probe 'wave' in various allowed directions to seek a 'transaction' or collapse of the wave function. A receiver or absorber detects or senses one of these probe waves, 'sets its state' and sends a 'verifying wave' back to the emitter confirming the transaction and arranging for the transfer of actual energy and momentum. This process comprises the non-local collapse of the wave function. The question now becomes: does such a principle have macroscopic effects? Bell's non-locality theorem cancan be effective over a matter of distance.

An attempt to examine such a possible macroscopic effect over large distances has been made by Partridge [34]. Using 9.7 GHz microwave transmitted by a conical horn antenna so that waves were beamed in various directions. Partridge found that there was little evidence for decreased emission intensities in any direction for an accuracy of a few parts per 10^{9th}. Interpretation of such a process is made in terms of advanced potentials. Previously mentioned complex dimensional geometries give rise to solutions of equations that form subluminal and superluminal signal propagations or solitons.

The possibility of a remote transmitter-absorber communicator now appears to be a possibility. The key to this end is an experiment by Pflelgov and Mandel [35]. Interference effects have been demonstrated, according to the authors, in the superposition of two light beams from two independent lasers. Intensity is kept so low that, to high probability, one photon is absorbed before the next one is emitted. The analogy to Young's double slit experiment is enormous.

In Wheeler's recent paper, he presents a detailed discussion of the physics of delayed choice proton interference and the double slit experiment (from the Solvay conference, Bohr-Einstein dialogue). Wheeler discusses the so-called Bohm 'hidden variables' as a possible determinant that nonlocality collapses the wave function [36]. Further theoretical and experimental investigation is indicated; but there appears to be a vast potential for remote non-local communication and perhaps even energy transfer (See Chaps 5,12). In the next section we detail the forms of transformations of the vector and scalar potentials at rest and in moving frames, continuing our formulation in terms of $(\underline{A,\phi})$. The issues of sub and superluminal transformations of $\underline{A}$ and ϕ are given in a complex Minkowski space. Both damped and oscillatory solutions are found and conditions for advanced and restored potentials are given.

5. Superluminal Vector & Scalar Potential Transformation Laws

For simplicity we will consider superluminal boost $v_x = \infty$ along the positive x direction. The space and time vectors in the real 4D Minkowski space transform as follows [37]

$$x' = +t,\ y' = -iy,\ z' = iz,\ t' = x \tag{24}$$

for real and imaginary parts separately, where x, y, z, t are real quantities in the laboratory (S) frame, and x', y', z', t' are the real quantities in the moving (S') frame. Now in the 6D (M^6) complex Minkowski space, the above transformation laws for a superluminal boost $(v_x = +\infty)$ in the positive x direction become [38]

$$\begin{aligned} &x'_{\mathrm{Re}} + ix'_{\mathrm{Im}} = t_{x,\mathrm{Re}} + it_{x,\mathrm{Im}}, \quad y'_{\mathrm{Re}} + iy'_{\mathrm{Im}} = y_{\mathrm{Im}} - iy_{\mathrm{Re}}, \\ &z'_{\mathrm{Re}} + iz'_{\mathrm{Im}} = z_{\mathrm{Im}} - iz_{\mathrm{Re}}; \quad t'_{x,\mathrm{Re}} + it'_{x,\mathrm{Im}} = x_{\mathrm{Re}} + ix_{\mathrm{Im}}, \\ &t'_{y,\mathrm{Re}} + it'_{y,\mathrm{Im}} = t_{y,\mathrm{Im}} - it_{y,\mathrm{Re}}, \quad t'_{z,\mathrm{Re}} + it'_{z,\mathrm{Im}} = t_{z,\mathrm{Im}} - it_{z,\mathrm{Re}} \end{aligned} \tag{25}$$

The transformation laws given by (25) preserve the magnitude of the line element but not the sign as in:

$$-x'^{\mu}x'^{\nu} = x^{\mu}x^{\nu} \tag{26}$$

where index μ and ν run over 1,2,3,4 representing 1 as time vector and 2,3,4 as spatial vectors. Therefore, we have the signature (+++-). Similar to the transformation laws for space and time vectors as given by (25) we can write the transformation laws for the vector and scalar potential. For a superluminal boost in positive x direction, the transformation laws for $(\underline{A,\phi})$ are:

$$A_x^{'} = \gamma\left(A_x - \frac{v_x^2}{c^2}\phi\right), \quad A_y^{'} = A_y, \quad A_z^{'} = A_z, \quad \phi^{'} = \gamma(\phi - v_x A_x) \quad (27)$$

where ϕ is the scalar potential and γ is the usual Lorentz term

$$\gamma^{'} \equiv 1 \Big/ \left(\frac{v_x^2}{c^2} - 1\right)^{\frac{1}{2}} \quad (28)$$

We consider $A_x^{'}$, etc., transforming as a gauge. In Eq. (27), the vector potential A is considered to be a four-vector real quantity, A_μ or $\underset{\sim}{A} = (A_x, A_y, A_z, \frac{i\phi}{c}$, which preserves the length of the line element but not the sign, i.e. we have

$$A_\mu A_\mu = -A_\mu^{'} A_\mu^{'} \quad (29)$$

Eq. (27) then simplifies to the following relationships for the velocities approaching infinity, $v_x = \infty$.

We can write the transformation laws for scalar and vector potentials under the superluminal boost in the positive x direction for $v_x = +\infty$. From the rest frame, S, to the moving frame, S', for unaccelerated vector and scalar potentials, we have

$$A_x = -\phi^{'}, \quad A_y = A_y^{'}, \quad A_z = A_z^{'}, \quad \phi = -A_x^{'} \quad (30)$$

From the moving frame, S', to the rest frame, S, for the unaccelerated vector and scalar potentials we obtain

$$A_x^{'} = -\phi, \quad A_y^{'} = A_y, \quad A_z^{'} = A_z, \quad \phi^{'} = -A_x \quad (31)$$

Eq. (31) is valid for real or complex vector and scalar potentials. Real and imaginary parts are easily separable in a complex quantity and they will transform according to Eq. (31) under the influence of a superluminal boost in the positive x direction. Now if these are the retarded (or accelerated or advanced) vector and scalar potentials then the transformation laws under the superluminal boosts will be different from the ones given by Eq. (31). These will be given by the combination of Eq. (31) and the transformation laws of the complex space and time vectors as given by Eq. (25).

These conditions are illustrated in Fig. 2. In 2a we represent a generalized point P(x_{Re},t_{Re},t_{Im}), displaced from the origin which is denoted as P_1. This point can be projected on each dimension x_{Re}, t_{Re} and t_{Im} as points P_2, P_3, and P_4 respectively. In Fig. 2b, we denote the case where a real-time *spatial* separation exists between points, P_1 and P_2 on the x_{Re} axis, so that $\Delta x_{\mathrm{Re}} \neq 0$, and there is no anticipation, so that $t_{\mathrm{Re}} = 0$, and access to imaginary time t_{Im}, nonlocality can occur between the P_1 to P_4 interval, so that $\Delta t_{\mathrm{Im}} \neq 0$. Then, our metric gives us $\Delta s^2 = 0$, where nonlocality is the contiguity between P_1 and P_2 by its access to the path to P_4. By using this complex path, the physical spatial separation between P_1 and P_2 becomes equal to zero, allowing direct nonlocal connectedness of distant spatial locations

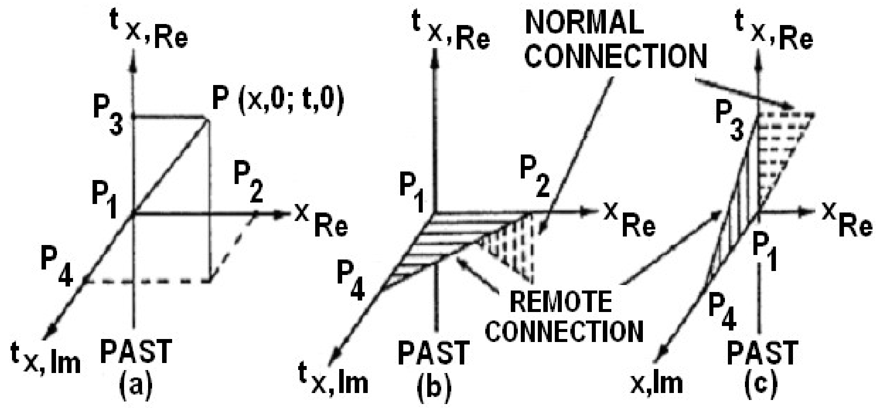

Figure 2. We represent the location of four points in the complex manifold. In figure 1a, point P_1 is the origin, and P is a generalized point which is spatially and temporally separated from P_1. In figure 5.2b, the Points P_1 and P_2 are separated in space but synchronous in time. This could be a representation of real-time nonlocal spatial separation. In figure 5.2c, points P_1 and P_3 are separated temporally and spatially contiguous. This represents an anticipatory temporal connection.

, observed as a fundamental nonlocality of remote connectedness on the spacetime manifold.

Figure 2c represents the case where anticipation occurs between P_1 and an apparent future anticipatory accessed event, P_3 on the t_{Re} axis. In this case, no physical spatial separation between observer and event is represented in the figure. Often such separation on the x_{Re} exists. In the case where $x_{Re} = 0$, then access to anticipatory information, along t_{Re} can be achieved by access to the imaginary temporal component, t_{Im}. Hence, remote, nonlocal events in four space or the usual Minkowski space, appear contiguous in the complex eight space and nonlocal temporal events in the four space appear as anticipatory in the complex eight space metric. Both nonlocality and anticipatory systems occur in experimental tests of Bell's Theorem and perhaps in all quantum measurement processes.

The propagation constant is considered to be isotropic in vacuum and defined as $d_x = \omega / v_\phi$, where v_ϕ, is the phase velocity and ω is the radian frequency of the propagating signal. Usually in most cases the phase velocity of propagation in vacuum is a constant $v_\phi = c$, where c is the velocity of light in vacuum. For the purpose of this paper, we will consider a tachyon traveling faster than light emitting an electromagnetic signal at frequency ω which propagates at the velocity of light. This assumption will simplify the subject matter of this paper. Later on, in a separate paper, we will examine the faster than light electromagnetic signals emitted by a traveling tachyon which might lead into a Doppler effect at velocities faster than light.

Considering only the advanced potential solution from (24). Eq. (24) can now be rewritten as two separate terms, so that in the *S* frame,

$$\begin{aligned} A_x &= (A_{0x,\mathrm{Re}} + iA_{0x,\mathrm{Im}})\{e\exp i[\omega t_{x,\mathrm{Re}} - kx_{\mathrm{Re}}]\times \\ & e\exp-[\omega t_{x,\mathrm{Im}} - kx_{\mathrm{Im}}]\} \end{aligned} \quad (32)$$

where the first exponent represents the usual type of oscillatory terms and the second exponent represents a decaying component which is not present in the usual 4D spacetime model. Note also that we have used the isotropy of the vector *k* in Eq. (32) as examined in the previous section.

Now let us examine the complex exponential of Eq. (32) using the transformations of Eq. (24) as follows so that we have for the exponents

$$e\exp i[\omega x'_{\mathrm{Re}} - kt'_{x,\mathrm{Re}}]; \quad e\exp -[\omega x'_{\mathrm{Im}} - kt'_{x,\mathrm{Im}}] \tag{33}$$

We regroup terms in ω and k so that we have

$$e\exp i[\omega(x'_{\mathrm{Re}} + ix'_{\mathrm{Im}}) - k(t'_{x,\mathrm{Re}} - it'_{x,\mathrm{Im}})] \tag{34}$$

Now using equations from for $\underline{x}' = x'_{\mathrm{Re}} + ix'_{\mathrm{Im}}$ we have

$$e\exp i[\omega x' - k(t'_{z,\mathrm{Re}} - it'_{x,\mathrm{Im}})] \tag{35}$$

Note that the second part of the exponent for the k term does not reduce to t' since there is a minus before $it'_{x,\mathrm{Im}}$. Thus, for the boost $v_x \to \infty$ or $v > c$, we obtain for $e\exp i[\omega t + kx]$ from Eq. (24) under this transformation going to

$$e\exp i[\omega x']; \quad e\exp - k[t'_{x,\mathrm{Re}} - it'_{x,\mathrm{Im}}] \tag{36}$$

Let us look at the example of the transformation from A'_x (in the moving frame *S*') to its form in the restframe, *S*. We find a mixing vector and scalar potential. In the SLT from the restframe *S* to the moving S' frames we have a change of length of the time component vector in Eq. (36). The vector potential term A_{0x} transforms as

$$A'_x = \gamma\left(A_x - \frac{v_x^2}{c^2}\phi\right) \tag{37}$$

which is the same as Eq. (28), so that for the superluminal boost $v_x \to \infty$, implies that

$$\gamma \equiv \frac{1}{\sqrt{\frac{v_x^2}{c^2} - 1}} = \frac{1}{\frac{v_x}{c}\sqrt{1 - \frac{c^2}{v_x^2}}} \cong \frac{c}{v_x} \tag{38}$$

where the $\sqrt{1 - c^2 / v_x^2}$ term approaches unity as $v_x \to \infty$. Then we rewrite the transformed vector potential as

$$A'_x = \frac{1}{\sqrt{\frac{v_x^2}{c^2} - 1}}, \qquad A_x - \frac{\frac{v_x}{c}}{\sqrt{\frac{v_x^2}{c^2} - 1}}\phi \tag{39}$$

Then for $v_x \to \infty$ and from Eqs. (38) and (39),

$$A'_x = \frac{cA_x}{v_x} - \frac{v_x}{c^2}\frac{c}{v_x}\phi = 0 - \frac{1}{c\phi} \equiv -\phi \tag{40}$$

for units in which $c = 1$. Therefore $A_x^{'} = -\phi$ for a superluminal boost, $v_x \to \infty$.

For the transformation of the scalar potential, in analogy to Eq. (28), we have

$$\phi' = \gamma(\phi - v_x A_x) \tag{41}$$

and for $v_x \to \infty$, we have $\gamma \cong c / v_x$ so that in the limit of the SLT,

$$\phi' \lim_{v\to\infty} = \frac{c}{v_x}\phi - cA_x = -cA_x \tag{42}$$

and for the units of $c = 1$, then $\phi' - A_x$. Compare this equation to Eq. (40). Also for $A_y^{'} = A_y$ and $A_z^{'} = A_z$ we can now write

$$\begin{aligned} A_x &= [A_{0x,\mathrm{Re}} + iA_{0x,\mathrm{Im}}] e \exp i[\omega t + kx] = \\ &[-\phi_{\mathrm{Re}}^{'} - i\phi_{\mathrm{Im}}^{'}] e \exp i\omega x' \quad e \exp \pm k_x [t_{x,\mathrm{Re}} - it_{x,\mathrm{Im}}^{'}] \end{aligned} \tag{43}$$

where $x' = x_{\mathrm{Re}}^{'} + ix_{\mathrm{Im}}^{'}$ and using the result of Eq. (40) and (42) for the non-exponent part and the exponential term which is given in Eq. (35), Eq. (43) gives us the vector and scalar form in the moving S' frame.

If we consider only the accelerated potential, then we consider only the plus sign in Eq. (43). By use of the definition of complex quantities, Eq. (43) can be rewritten in a compact, simplified form:

$$A_x = -\phi_{0x}^{'} \exp(i\omega x') \cdot \exp(ik_x t_x^{'}) . \tag{44}$$

Then by use of Eq. (44) we can describe the x component of the complex vector potential in moving frame S' after a superluminal boost in the positive x direction. The same vector potential in the rest frame is defined.

The transformation of the A_y and A_z components of the complex vector potential under a superluminal boost in the positive x direction can similarly be written as

$$\begin{aligned} A_y &= A_{0y}^{'} \exp[-\omega(t_{y,\mathrm{Re}}^{'} + it_{y,\mathrm{Im}}^{'})] \cdot \exp[-ky(z_{\mathrm{Re}}^{'} + iy_{\mathrm{Im}}^{'})] \\ &= A_{0z}^{'} \exp[-\omega(t_{z,\mathrm{Re}}^{'} + it_{z,\mathrm{Im}}^{'})] \cdot \exp[-ky(z_{\mathrm{Re}}^{'} + iz_{\mathrm{Im}}^{'})] \end{aligned} \tag{45}$$

We will now consider the scalar potential as defined by a complex quantity, so that

$$\phi' = \phi_{\mathrm{Re}}^{'} + i\phi_{\mathrm{Im}}^{'} \tag{46}$$

which we use for the non-exponential term of Eq. (45) which then becomes

$$A_x = -\phi' e \exp i\omega x' \quad e \exp k[t_{x,\mathrm{Re}} - it_{x',\mathrm{Im}}] \tag{47}$$

Let us now compare the vector potential forms of A_x in Eq. (42) in the S or laboratory frame, and A_x of Eq. (47) in the S' frame or moving frame. (See Table 5.1)

TABLE 1 Comparison of The Exponential Part of the Vector Potential A_x In The *S* and *S'* Frames Of Reference

	OSCILLATORY	DAMPED
S Frame	$A_{0x} \propto e \exp i[\omega t_{x,\mathrm{Re}} - kx_{\mathrm{Re}}]$	$e \exp - [\omega t_{x,\mathrm{Im}} - kx_{\mathrm{Im}}]$
S' Frame	$\phi' \propto \exp i[\omega x']$	$e \exp k[t'_{x,\mathrm{Re}} - it'_{x,\mathrm{Im}}]$

In the oscillatory solution of the *S*' frame for ϕ', we find no dependence on the wave number factor *k* and hence we have apparent media independence, recalling $x' = x_{\mathrm{Re}} + ix_{\mathrm{Im}}$, whereas in the *S* frame for A_{ox}, we have dependence on ω and *k*.

For the damped solution, we have ω and *k* dependence in the *S* frame for A_{ox}, which is a pure real exponential and hence not oscillatory. In the *S*' frame then, ϕ' sometimes has a damped solution dependent on *k* which has a real and imaginary component. The exponential factor can be written a

$$t'_{x,\mathrm{Re}} - it'_{x,\mathrm{Im}} = x_{\mathrm{Re}} - ix_{\mathrm{Im}} \tag{48}$$

Time dilation and vector length are modified in the complex 12D space [38]. We find that a superluminal, unidimensional (*x*-dimensional) boost in complex Minkowski space not only modifies space and time (as well as mass) by the γ factor, it also modifies $\underset{\sim}{A} = (\underline{A}, \phi)$ and we find a mixing of $\underline{A}$ and ϕ for $\underline{A} = A_j$ where *j* runs 1 to 3 (or spacelike quantities) and ϕ transforms as a temporal quantity for subluminal transformations.

6. Insights Into Dirac and Penrose Spinor Calculus

The spinor calculus of the Kaluza-Klein geometry [11,12] mappable one to one with the twistor space of the complex 8-space. The Dirac equation is based on the fundamental properties of spinors. The complexification of four space by the Rauscher [39] and Newman [2-5] method yields a manner to relate Maxwell's equations to the relativistic space time metric, as shown above. In this section we detail the Dirac spinor formalism with the twistor topology.

The Penrose and other twistor approaches have been in an attempt to quantize gravity in order to unify the physics of the micro-cosmos and macro-cosmos. Such an approach has been taken by Penrose, et al. and is based on the concept of a more general theory that has limits in the quantum theory and the relativistic theory [40]. In addition, there have been approaches to the underlying structure of space-time in the quantum and structural regime [40-43]. A structured and/or quantized space-time may allow a formalism that unequally relates the electromagnetic fields with the gravitational metric. Feynman and Penrose graphs were developed in an attempt to overcome the divergences of such an approach. In order to translate the equations of motion and Lagrangians from spinors to twistors, one can use the eigenfunctions of the Casimir operators of the Lie algebra of $U(2,2)$.

The simplest case of a zero-rest mass field is the simplest and can represent the photon for $n/2$ spin where $n \neq 0$, and we can write

$$\nabla_{AA'} \varphi^{A \ldots N} = 0 \tag{49}$$

for $A,....,N$ written in terms of n indices, and for $n=1$, we have the Dirac equation for massless particles. For a spin zero field, we have the Klein-Gordon equation

$$\nabla^{AA'}\nabla_{AA'}\varphi = 0 \tag{50}$$

and for $n=2$, we have the source-free Maxwell equation $\Box F^{\mu\nu}=0$ for spin 1 or U_1 for the electromagnetic fields, and for $n=4$, we have the spin Einstein free field equations, $R_{\mu\nu}=0$. The indices μ and ν run 0 to 3. For a system with charge, then $\Box F^{\mu\nu}=J_{\mu\nu}-J_{\mu\nu}$, or this can be written as $\frac{F_{\mu\nu}}{\partial x_\nu}=J_\mu$ and then we can write

$$\gamma_{\mu\nu}\frac{\partial F_{\mu\nu}}{\partial x_\nu}=J_\mu \tag{51}$$

We present an approach to relate the twistor topology to the spinor space and specifically to the Dirac spinors. Both Fermi-Dirac and Bose-Einstein statistics are considered. The twistor theory and Dirac models can be related to electrodynamics, and gravitation. The Penrose spin approach is designed to facilitate the calculation of angular momentum states for SL(2,2). The spinor formalism, in the Dirac equation, utilize spinors within the quantum theory. The twistor formalisms are related to the structure of space-time and the relation of the spinors and twistors is also of interest because it may yield a relationship between quantum mechanics and relativity. The twistor theory has been related to conformal field theory and the string theory [44]. Also, twistor theory has been related to quaternions and complex quaterionic manifolds [45]. The projective twistor space, PT, corresponds to two copies of the associated complex projective space of CP^3 or $CP^3\times CP^3$. It is through the conformal geometry of surfaces in S^4, utilizing the fact that CP^3 is an S^2 bundle over S^4, that can be related to quaternions [44].

The complex 8-space and the Penrose twistor topology are fundamentally related since the twistor is derived from the imaginary part of the spinor field. The Kerr Theorem results naturally from this approach in which twisting is shear free in the limit of asymptotic flat space. The twistor is described as a two-plane in complex Minkowski space, M^4. Twistors define the conformal invariance of the tensor field, which can be identified with spin or spinless particles. For particles with a specific intrinsic spin, s, we have $Z^\alpha\overline{Z}_\alpha=2s$, and for zero spin, such as the photon, $Z^\alpha\overline{Z}_\alpha=0$ where $\overline{Z}_\alpha$ is the Hermitian conjugate of Z^α, and Z^α and Z_α can be regarded as canonical variables such as $\underline{x}$, $\underline{p}$ in the quantum theory phase space analysis. Note that these fields are independent of the origin [59]. The twist free conditions, $Z^\alpha\overline{Z}_\alpha$, hold precisely when Z^α is a null twistor. The upper case Latin indices are used for spinors, and the Greek indices for twistors. The spinor field of a twistor is conformally invariant and independent of the choice of origin [45]. For the spinor, the indexes A and A' take on values 1, 2 [44]. We briefly follow along the lines of Hanson and Newman in the formalism relating the complex Minkowski space to the twistor algebra. Spinors and twistors are related by the general Lorentz conditions in such a manner as to retain the fact that all signals are luminal in the real four-space, which does not preclude superluminal signals in an $N>4$ dimensional space. The twistor Z^α can be expressed in terms of a pair of spinors, ω^A and π_A, which are said to represent the twistor. We write

$$Z^\alpha=\left(\omega^A,\pi_{A'}\right) \tag{52}$$

where $\omega^{A} = i\, r^{AA'} \pi_{A'}$

Every twistor Z^{α} is associated with a point in complex Minkowski space, which yields an associated spinor, ω^{A}, $\pi_{A'}$. The spinor is associated with a tensor which can be Hermitian, but is not necessarily Hermitian. The spinor can be written equivalently as a bivector forming antisymmetry. In terms of spinors ω^{A} and $\pi_{A'}$, they are said to represent the twistor Z^{α} as $Z^{\alpha} = \left(\omega^{A}, \pi_{A'}\right)$. In terms of components of the twistor space in Hermitian form, φ for $\varphi_{AA'} = \varphi_{A'A}$, we have,

$$\varphi\left(Z^{\alpha} Z^{\beta}\right) = \overline{Z^{0} Z^{2}} + \overline{Z^{1} Z^{3}} + \overline{Z^{2} Z^{0}} + \overline{Z^{3} Z^{1}} \tag{53}$$

where the α index runs 0 to 3. The components of Z^{α} are $Z^{0}, Z^{1}, Z^{2}, Z^{3}$ and are identifiable with a pair of spinors, ω^{A} and $\pi_{A'}$, so that

$$\omega' = Z^{1}, \quad \pi_{0'} = Z^{2}, \quad \pi_{1'} = Z^{3} \tag{54}$$

so that we have

$$Z\overline{Z}_{\mu} = \mu^{\circ}\overline{\pi}_{\circ} + \mu'\overline{\pi}_{1} + \pi_{\circ}{}' \mu^{\circ'} + \pi_{1'}\overline{\mu}^{1'} \tag{55}$$

Note that the spinor ω^{A} is the more general case of μ^{A}. This approach ensures that the transformations on the spin space preserve the linear transformations on twistor space, which preserves the Hermitian form, φ.

The underlying concept of twistor theory is that of conformal invariance or the invariance of certain fields under different scalings of the metric under the general relativistic space-time metric, $g_{\mu\nu}$. Related to the Kerr theorem, for asymptotic shear-free null flat space, the analytic functions in the complex space of twistors may be considered a twisting of shear-free geodesics. In certain specific cases, shear inclusive geodesics can be accommodated. Twistors are formally connected to the topology of certain surfaces in complex Minkowski space M^{4}. This space, the complex space C^{4}, is the cover space of R^{4}, the 4D Riemannian space. On the Riemann surface, one can interpret spinors as roots of the conformal tangent plane of a Riemann surface into R^{3}. This approach is significant because it ensures the diffeomorphism of the manifold. Complexification is formulated as $Z^{\mu} = X^{\mu}_{\mathrm{Re}} + X^{\mu}_{\mathrm{Im}}$, which constitutes the complexification of the Minkowski space, M^{4}. The usual form Minkowski space is a submanifold of complex Minkowski space. Twistors are space-time structures in Minkowski space, which is based upon the representation of twistors in terms of a pair of spinors. Twistors provide a unique formulation of complexification. The interpretation of twistors in terms of asymptotic continuation accommodate curved space-time.

The spinor representation of a twistor makes it possible to interpret a twistor as a two-plane in complex Minkowski space, M^{4}. Then we can relate ω^{A} and $\pi_{B'}$ so that $\xi^{AA'}$ is a solution as

$$\omega^{A} = i\xi^{AB'}\pi_{B'} \tag{56}$$

for the position vector $\xi^{AB'}$ in the complex Minkowski space. We can also consider the relationship of $Z^{AA'}$ and $\pi^{A'}$ to a complex position vector as

$$Z^{AA'} = \xi^{AA'} + \omega^A \pi^{A'} \qquad (57)$$

where ω^A is a variable spinor. Just as in the conformal group on Minkowski space, spin space forms a two-valued representation of the Lorentz group. Note that SU_2 is the four-value covering group of C $(1, 2)$, the conformal group of Minkowski space. The element of a 4D space can be carried over to the complex eight space. The Dirac spinor space for spin, n is a covering group of SO_n where this cohomology theory will allow us to admit spin structure and can be related to the SU_2 Lie group. Now let us consider the spin conditions associated with the Dirac equation and formulate the Dirac 'string trick' that describes the electron spin path. The requirement for a 720° twist or rotation results from the electron spin and chirality where the spin is aligned or anti-aligned along the particle's direction of motion.

For a spin, $s = \frac{1}{2}$ particle, the spin vector $u(p)$ is written as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for spin up and spin down and p is momentum. For a particle with mass we have for $c \neq 1$,

$$\left(-i\hbar c \alpha_\mu \frac{\partial}{\partial x_\mu} + \beta m c^2 \right) \psi = 0 \qquad (58)$$

for the time independent equation, and we can divide Eq. (58) by $i\hbar c$ and have,

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + \frac{mc}{\hbar} \right) \psi = 0 \qquad (59)$$

where $k_\circ = mc/\hbar$ and $\gamma_\mu = ic\hbar\alpha\mu$ where indices μ run 0 to 3. The dependent Dirac equation is given as,

$$\left(-i\hbar c \alpha_\mu \frac{\partial}{\partial x_\mu} + \beta m c^2 \right) \psi + \frac{i}{\hbar} \frac{\partial \psi}{\partial t} = 0 \qquad (60)$$

The solution to the Dirac equation is in terms of spin $u(p)$ as

$$\psi = u(p) e \frac{i}{\hbar} (p \cdot \underline{x} - Et) \qquad (61)$$

the Dirac spin matrices $\gamma_\mu = ic\hbar\alpha_\mu$. The spinor calculus is related to the twistor algebra, which relates a 2-space to an associated complex 8-space.

The Dirac equation and spinors are fundamentally connected. For example, we have the Dirac spin matrices, $\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} = -i\,\beta\alpha_\kappa$ where terms such as $\gamma_\mu(1 - \gamma_5)$ come into the electroweak vector - axial vector formalism. The three Dirac spinors, which are also related to the Pauli spin matrices, are given as

$$\sigma_x = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \quad \sigma_y = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \text{and } \sigma_z = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \tag{62}$$

and $\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3 = i\gamma^0\gamma^1\gamma^2\gamma^3$ for $\gamma_0 = \beta$ is given as,

$$\gamma_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{63}$$

for trace $tr\beta = 0$, that is, Eq. (63) can be written as,

$$\gamma_0 = \beta = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix} \tag{64}$$

where we have the 2x2 spin matrix as $I_2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$. Note that the Dirac spinors are the standard generators of the Lie algebra of SU_2.

The commutation relations of the Dirac spin matrices is given as

$$\left\{ \gamma^\mu, \gamma^\nu \right\}_+ = \gamma^\mu\gamma^\nu + \gamma^\mu\gamma^\nu = ig^{\mu\nu} \underset{\sim}{I} \tag{65}$$

and $\det\left|\gamma_{\mu\nu} = \det\right\| g_{\mu\nu}|$ where $g_{\mu\nu}$ is the metric tensor. The Dirac spin matrices come into use in the electroweak vector-axial vector model as $\gamma_\mu(1-\gamma_5)$ for γ_5 as,

$$\gamma_5 = i\,\gamma_0\gamma_1\gamma_2\gamma_3 = i\,\gamma^0\gamma^1\gamma^2\gamma^3 \tag{66}$$

where indices run 0 to 3. We can also write,

$$\gamma_{\mu\nu}\left(x^5, x^\mu\right) = \sum_{n=-\infty}^{\infty} \gamma_{\mu\nu}^{(n)}\left(x^\nu\right)\, e^{inx^5} \tag{67}$$

which expresses some of the properties of a 5D space having $\gamma_0, \gamma_{1,} \gamma_2, \gamma_3$ and γ_5. Note that γ_5 is associated with a 5D metric tensor. This five-dimensional space passes exactly one geodesic curve which returns to the same point with a continuous direction which is similar to the formation of the Dirac string trick which requires a 720°path of an electron to return to its exact original quantum state.

The electromagnetic potential; and the metric of the Kaluza-Klein geometry are related where we express $\gamma_{\mu 5}$ in terms of a potential φ_μ so that we have

$$\gamma_{\mu 5} = \sqrt{2\kappa}\phi_\mu \tag{68}$$

where $\kappa \equiv {}^{8\pi}\!/_{F}$ and where $F = {}^{c^4}\!/_{G}$ or the Rauscher quantized cosmological force. Then we have a five space vector as,

$$\gamma_{\nu 5} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{69}$$

Through this approach, we can relate covariance and gauge invariance.

For the Poisson's equation we have,

$$\nabla \varphi_{\mu} = \frac{1}{2} \kappa c^4 \mu_{\circ} \tag{70}$$

where again $\kappa \equiv {}^{8\pi}\!/_{F}$ as above. The electromagnetic field, $F_{\mu\nu}$, can be expressed as,

$$F_{\mu\nu} = \frac{\partial \varphi_{\mu}}{\partial x^{\nu}} - \frac{\partial \varphi_{\nu}}{\partial x^{\mu}} \tag{71}$$

which yields an interesting relation of the gravitational metric to the electromagnetic field. Also, the Lagrangian is given as $L = \frac{1}{2} F^{\mu\nu} F_{\mu\nu}$ so that $\mathsf{L} = L\sqrt{-g}$ for the space-time metric g. Note that we have $L = \int \sqrt{g}\ d\tau$, where $d\tau$ represents a four space. Now let us return to our discussion of the twistor algebra and relate it to the spinor calculus. The Penrose twistor space also yields a 5D formalism similar to that formulated by the Kaluza-Klein theory.

The quanta are associated with a quantum field of particles that carry both momentum and energy. The total energy Hamiltonian can be defined in terms of a number of simple phonon states which can be expressed in terms of a_n^+ creation and a_n destruction operator states. Since all creation operators commute, these states are completely symmetric and satisfy Bose-Einstein statistics. Such phonon states, having a definite number of phonons, are called Fock states, which is the vector sum of the momentum of each of the photons in the state. The ground state $|0\rangle$ can be considered the photon vacuum state or Fock state where the photon is taken as a phonon state. The creation and destruction operators commute as $\{a_n, a_n^+\} = \delta_{nn'}$ for the delta function $\delta_{nn'}$. Both projective and non-projective twistors are considered as images in a complex Riemannian manifold in its strong conformal field condition. In analogy to the Hartree-Fock spaces, or Fock space, using the appropriate spin statistics, Bose-Einstein or Fermi-Dirac; duality, analytic continuation, unitary and other symmetry principles. Particles can be considered as states as the Fock space elements or the 'end' of each disconnected portion of the boundary of the manifold [47].

We can consider an n-function as a 'twistor wave' function for a state of n-particles. In the first order consideration, Penrose considers a set of n-massless particles as a first order approximation. We form a series on a complex manifold as elements of the space C_n as

$$f_0,\ f_1(z^{\alpha}),\ f_2(z^{\alpha}, y^{\alpha}),\ f_3(z^{\alpha}, y^{\alpha}, x^{\alpha}), \ldots \tag{72}$$

which are, respectively, the 0th function, 1st function, 2nd function, and 3rd function, etc. of the twistor space, which are also elements of C_n. We can also consider f_0, f_1, f_2, f_3, ... as the functions of several nested twistors in which f_0 is the central term of the wave of the twistor space. We can say that these nested tori can act as a recursive sequence. Penrose relates the twistor to particle physics by suggesting that, to a first approximation, f_1 corresponds to the amplitude of a massless, spin 1 particle, f_2 to a lepton spin ½ particle, and f_3 to Hadron particle states, and f_4 to higher energy and exotic Hadron particle states. Mass results from the breaking of conformal invariance for f_n for $n=2$ or greater, similar to the $S-$matrix approach [48]. The harmonic functions, f_n, form a harmonic sequence, where f_n for $n=2$ form the Fermion states, and f_n for $n=3$ form the Hadron twistor states. Essentially, in the twistor space, we have a center state f_0 around which f_1, f_2, ... occur. Each of these sequences waves forms a torus-like topology, hence, f_1 and f_2 form a double nested tori set consistent with both spin 1 and spin ½ particle states where all n states are elements of the twistor, z, as $n \in z$. In the specific case of a massless particle with spin for f_1, the two-surface in complex Minkowski space corresponding to the twistor represents the center of mass of the system so that the surface does not intersect the real Minkowski space. This reflects the system's intrinsic spin. We see an analogy to the 3-tori Calabi-Yau string theory. The higher order f_n may describe higher order string modes or oscillations of $Z^\alpha \overline{Z}_\alpha = 0$ or f_0. This occurs for the case using f_1, f_2, and f_3 and, hence, all known particle states.

The topology of the first three Penrose projective twistor states are PT, PT^+, and PT^-. The PT^+, and PT^- are the domain of the projective twistor space, PT, where we denote these two states in which $(-1,1)$ are elements of t where ε is small. We denote two line elements which are denoted in terms of twistors as a surface on the sphere S^3 as $PT^\pm$ which corresponds to $\underset{-t}{Z^\alpha}\,\overline{\underset{-t}{Z_\alpha}} = 0$ and $\underset{t}{Z^\alpha}\,\overline{\underset{t}{Z_\alpha}} = 0$ for $t = 1-\varepsilon$ for PT^+, and PT^- gives $t = 1-\varepsilon = \varepsilon - 1$. These two branches correspond to a transformation matrix,

$$\begin{pmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{pmatrix} \tag{73}$$

This gives us a translation formulation for vectors into the states of spinors in terms of t, in terms of the spinors

$$\begin{pmatrix} \underset{t}{\omega}^0_{\;1} \\ \underset{t}{\omega}^1{}' \\ \underset{t}{\pi}_0{}' \\ \underset{t}{\pi}_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{pmatrix} \begin{pmatrix} \omega^0 \\ \omega^1{}' \\ \pi_0{}' \\ \pi_1 \end{pmatrix} \tag{74}$$

which is Z^α_t and $t \sim \pm 1$ since ε is small. Then in terms of twistors,

$$\hat{\omega}^A = \omega^A + \varepsilon\xi^{AB}\frac{\partial f}{\partial \omega^B} \tag{75}$$

for $\hat{\pi}_{A'} = \pi_{A'}$ where ω and π are orthogonal spinors. The term $\varepsilon\xi^{AB}\frac{\partial f}{\partial \omega^B}$ is small compared to ω^A and π_A since ε is small. The unit spinors or vectors are $\hat{\omega}^A$ and $\hat{\pi}_{A'}$ for both $A, B = 1,2$

A 5D surface of projective twistors in a spin free state, which can have genus $g = 0$ for a spherical, no 'hole' surface to $g \neq 0$ for $S^3 \times R$. Penrose has formulated the relations between the conformal geometry of Minkowski space, complex analysis, and hence, analytic continuation, and the solutions to certain conformally invariant differential equations such as Maxwell's equations. Gauge theory in this context also allows the formalism of the Yang-Mills equations, which have become a major tool in four-dimensional differential topology. The Yang-Mills theory is a non-Abelian gauge group theory, which is the basis of modern quantum particle field theory. Invariance under the local gauge group SU_2 can be extended to larger groups SU_n for $n > 2$. A theory which is invariant under the local gauge group SU_2 is referred to as a Yang-Mills theory. For example, chromodynamics is a Yang-Mills theory with the gauge group SU_3. The exploration of conformally invariant conditions on Minkowski space is formulated for contour integral formulation process solutions to the Dirac equation. The contour integral methods allow integrability and are used to deal with the 'holes' or singularities in real and complex manifolds [49,50].

Work is in progress to complete the complexification of the Dirac equation [51] in the complex-8 space.

7. Conclusions

It appears that utilizing a complexification of Maxwell's equations with the extension of the gauge condition to non-Abelian algebras, yields a possible metrical unification of relativity, electromagnetism and quantum theory. This unique new approach yields a universal nonlocality. No radical spurious predictions result from the theory, but some new predictions are made which can be experimentally examined. Also, this unique approach in terms of the twistor algebras may lead to a broader understanding of macro and micro nonlocality and possible transverse electromagnetic fields observed as nonlocality in collective plasma state and other media. In the next chapter we demonstrate application of the model to complex 12-space and develop correspondence to M-Theory and F-Theory.

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