The Schrödinger Equation in Complex Minkowski Space

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We develop a formalism for the Schrödinger equation in an eight dimensional (8D) complex Minkowski space and discuss its relation to the Dirac equation, properties of nonlocality, remote connectedness, Young's double slit experiment, Bell's Theorem, the EPR paradox and anticipatory parameters of spacetime; and also identify an imaginary temporal component as a small nonlinear term and find soliton or solitary wave solutions. These coherent solutions can carry information over long distances, are consistent with Lorentz invariance and appear to provide a fundamental methodology for describing the issue of quantum measurement and a new context for the basis of quantum theory. In the Copenhagen view models of reality are not desirable. However, our new approach may enable the redefinition of concepts of reality from a new nonlocal and anticipatory quantum theory. Certainly, the most desirable consequence of scientific discovery is the ability to refine our concepts of reality.

1. Remote Connectedness and Coherent Collective Phenomena

The interpretation of the extremely successful quantum theory, beyond the Copenhagen Theory, carries within it the vital need for the interpretation of what it means to make a measurement, primarily in the microdomain. The rapid and major development of the structure, content and interpretation of quantum theory in the 1920s and 1930s, as exemplified by the Heisenberg Uncertainty Principle and Schrödinger Cat Paradox and EPR Paradox [1], led to conceptual paradoxes beyond the practical application of quantum theory. The Schrödinger Cat Paradox arises over the issue of the collapse of the wave function. For two equally probable states arising from a microscopic process, only observation can determine which state exists. Heisenberg and Bohr demonstrated that the act of observation necessarily leaves the system in a new state through what Wheeler terms "participation" [2]. The Copenhagen interpretation of quantum mechanics (that is that quantum theory can only predict the probability of the outcome of a specific experiment) was an attempt to dismiss the observer's participation, but by this dismissal, we can no longer build models of reality.

The test of the universality of quantum theory's experimental validity demands that nonlocality is a fundamental property of the quantum domain. The issue of nonlocality as a fundamental property of spacetime has been thoroughly proven by experimental verification. If quantum theory is universally valid, nonlocality is necessarily true Bohm termed the nonlocal correlations [3].

Einstein's dissatisfaction with the lack of determinism of quantum theory, and its probabilistic nature, led him to write the Einstein, Podolsky, and Rosen, EPR, paper. He had hoped to find a flaw in the quantum theory that would allow a way around the Heisenberg Uncertainty Principle and the probabilistic nature of the quantum theory [1] He was not the only physicist to be discontented with the, "spooky action of a distance." Bell reformulated the EPR Paradox into a rigorous inequality that could be experimentally tested. In more recent years, the formalism of the EPR Paradox terms of Bell's theorem [4], and its extensive tests which demonstrate that quantum theory holds in all known quantum experiments which necessarily demands the properties of nonlocality on the spacetime manifold.

What are some of the possible implications from the quantum description, if we choose to pursue the development of models of reality and perhaps relax the pure objectivity constraint in physical theory? This issue is well exemplified by the Bell's theorem formulation of the Einstein, Podolsky, Rosen Paradox [1]. An indication that non-locality is a principle in Nature is contained in Bell's theorem, which asserts that no deterministic local "hidden variable" theories can give all the predictions of quantum theory [5]. However, most physicists believe that Nature is non-deterministic and that there are no hidden variables. The prevailing view is that Bell's theorem merely confirms these ideas, rather than that it is an indication of a fundamental statement of nonlocality. However, in recent years this view has changed.

Stapp demonstrates that determinism and hidden variables occupy no essential role in the proof of Bell's theorem, which Stapp has reformulated [6]. Stapp asserts that no theory which predicts the outcome of individual observations which conform to the predictions of quantum theory can be local. A less restrictive interpretation of Bell's theorem is that either locality or realism fail [7]. Realism is a philosophical view in which external reality is assumed to exist and have definite properties fundamentally independent of an observer [7,8] Stapp presents reasonable and comprehensive models of reality in which nonlocality, as implied by Bell's theorem, is inconsistent with "objective reality," in which observable attributes can become definite, independent of the observer, the so-called "collapse of the wave function".

In Young's double slit experiment, photons from a source can go through one of two slits or openings of a slit interference arrangement. Through which slit did the photon go that blackens a photographic plate at the other end of the apparatus. The answer is not yet defined because of the Heisenberg Uncertainty Principle. One can observe interference fringes when both slits are open, but at the cost of not knowing through which slit the photon went. Or, one can know through which slit the photon went when one slit is closed, but at the cost of not having any interference fringes. Again, the choice appears to be that of the observer [9]. This experiment also brings the role of the observer into consideration and may also involve nonlocality and anticipation [10]. Certainly, one of the most desirable consequences of scientific discovery is the ability to discover and refine our concepts of reality.

2. Complex 8-Space and the Formation of Nonlocality

We have introduced a complex multi-dimensional geometry of the four real dimensions of space, X_{Re} of x_{Re}, y_{Re}, z_{Re}, and t_{Re} and four imaginary dimensions X_{Im} of ix_{Im}, iy_{Im}, iz_{Im} and it_{Im}, such that we can describe nonlocal macroscopic connections of events that do not violate causality [11]. There are several motivations for introducing such a model; one of which relates to a possible macroscopic formulation of a Bell's theorem-like nonlocal correlation function that may have macroscopic implications, leading to a new interpretation of the Bell's theorem experimental results and to a more fundamental interpretation of the quantum measurement issue. The complex Minkowski Space, M_4 is constructed so as to maintain causality and analytic continuation in the complex manifold [11-13]. The four-real dimensional space can be considered a slice though the hyperdimensional complex 8-space 13].

Events that appear remote in 4-space, M_4, are contiguous in the complex 8-space, $M_4 + \subset \mathbb{C}_4$. We have demonstrated a fundamental relationship between the complex 8-space and the topology of the Penrose twister algebra [8,14,15]. In this model, spacetime events can become contiguous in the complex 8-space, demonstrating that the remoteness of the observer and observed can become contiguous in the complex 8-space in which causality conditions are preserved and the acquisition of apparent remote information is allowed.

We have solved the Schrödinger equation in the complex 8D space and, with the inclusion of a relatively small, but significant, nonlinear term, $g^2(t_{\mathrm{Im}})$ we find soliton and solitary wave solutions. The non-linear term, which depends on the imaginary time component, overcomes dispersion giving the non-dispersive soliton waves. The coherence over remote space and time of these wave solutions relates to macroscopic-related phenomena as it does for Bell's theorem, Young's double-slit experiment and other nonlocal anticipatory phenomena. The non-linear form of the Schrödinger equation may be related formally to the non-linear gravitational phenomena [15] and also has implications for the quantum measurement problem [16]. Resolution of the observer-participant problem may be at hand as demonstrated by a new interpretation of the Schrödinger equation. In this formation, remote spacetime events are contiguous so that the observer has direct acquisition to remote observable information, in such a manner as to preserve causality.

3. Space-Like Remote Connectedness, Bell's Theorem and its Experimental Test

A most significant theorem about the nature of physical systems is Bell's formulation [4] of the Einstein, Podolsky and Rosen (EPR) "completeness" formulation of quantum mechanics [1]. The EPR paper was written in response to Bohr's proposal that the non-commuting operators (Heisenberg uncertainty principle) comprise a *complete* theory called the Copenhagen interpretation of quantum mechanics. Einstein, Podolsky and Rosen define a complete theory as one in which every element of the theory corresponds to an element of "reality". Bohm introduced additional quantum non-observable variables or "hidden variables," as we presented in the last section, in order to make the EPR quantum interpretation consistent with *causality* and *locality* [17]. In 1964, Bell "formulated" the EPR statement and showed mathematically that locality is incompatible with the statistical predictions of quantum mechanics. The locality or separability assumption states that the result of a measurement on one system is *unaffected* by operations on a *distant* system with which it may have previously interacted or had become entangled.

Bell discusses a specific experiment, Stern-Gerlach measurements of two spin one-half particles in the singlet spin state moving freely in opposite directions. The spins are called s_1 and s_2; we make our component spin measurements remote from each other at position P_1 and P_2, such that the Stern-Gerlach magnet at P_1 does not affect one at P_2 and vice versa. Since we can predict, in advance, the result of measuring any chosen component of s_2 at P_2 by previously measuring the same component of s_1 and P_1, this implies that the result of the second measurement must actually be predetermined by the result of the first (remote from P_2) measurement. In Bell's proof, he introduces a more complete specification of the parameters of a system by introducing parameters which in essence are hidden variables. Bell's proof is most eloquent and clear. He calculates the conditions on the correlation function for measurements at P_1 and P_2, as an inequality.

Bell's precise statement made it possible for Clauser and Horne to test the predicted statistical distribution of quantum processes and demonstrate a laboratory instance of quantum connectedness and nonlocality [18,19]. Indeed, in Clauser's calcium two photon cascade system, two photodetectors remote from each other are each preceded by independent, randomly-oriented polarizers. The statistical predictions of quantum mechanics is borne out in the measurements made at the two photomultiplier tubes (PMT). In Bell's words, "there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote." Moreover, the signal involved must propagate instantaneously so that a theory could not be Lorentz invariant. Lorentz invariance, in the usual sense, implies $v \leq c$. Feinberg discusses the relationship between Lorentz invariance and superluminal signals which he found not to be incompatible. It is not completely clear that superluminal signals must be invoked to derive Bell's theorem [20] but we have demonstrated that indeed Bell's theorem may imply either $v \leq c$ in a complex 8-space [21] or an additional $t = 0$ 12D holographic-like instantaneity.

In fact, additional research should reveal that a Kantian antinomy or duality exists here that will be revealed between Newton's instantaneous and Einstein's, $v = c$ gravity wave propagation models by additional research. Recall that the issue of a putative quantum gravity is still an open question. We are in the process of redefining the graviton. It appears that there is no M_4 spin 2 field exchange quanta; but that the quadrupole structure is more complex requiring additional structural-phenomenology even beyond that hinted at by our intermediate $\hat{M}_4 + \subset \mathbb{C}_4$ complex 8-space. Some researchers have invoked the term wavicle to invoke some sort of entity beyond the either-or wave-particle duality scenario. In any case this would not be sufficient to describe a graviton from the Holographic Anthropic Multiverse (HAM) cosmological perspective where there is no phenomenological exchange as such; but rather a an energyless ontological exchange of 'topological charge' in higher dimensional (HD) M-Theoretic branes. The duality requires a form of topological switching of boundary conditions that are an ontological-phenomenological duality; where the phenomenological component is virtual in the sense that there is no measurable spin 2 quanta in 4-space. The regime of integration is not quantum but in the HD unified field [39,40].

The conclusion from Bell's theorem, then, is that any hidden variable theory that reproduces all statistical predictions of quantum mechanics must be nonlocal (implying remote connectedness). Of course, thus far, all these formulations involve microproperties only, but some recent formulations seem to imply possible macroscopic consequences of Bell's theorem as well. It is believed that the key lies in the formulation of the correlation function which represents the interconnectedness of previously correlated events. Stapp has demonstrated that hidden variable theory is not necessary to the formalism of Bell's theory [22]. Stapp has recently expanded the pragmatic view of Bell's theorem and discusses the role of the macroscopic detection apparatus as well as the possible role of superluminal signals. He explores both cases for superluminal propagation or subluminal connection issuing from the points in common to the backward lightcones coming from the two regions.

We can write a general correlation function, for example, for an angle, θ between polarization vectors in two polarizers as $C(\theta) = ½ - ½ \cos 2\theta = \cos^2\theta$ for Clauser's experiment, or for odd integers we can write $nC(\theta) - C(n\theta) - (n - 1) \leq 0$, which is Bell's inequality – specifically for n = 3; $3C(\theta) - C(3\theta) - 2 \leq 0$. We can write in general $C(\theta) = ½ + g\cos 2\theta$ where g is determined by the particular experiment under consideration. The magnitude correlation function constant, g relates to the type of nonlocal correlation experiment. For $g = ½$, we have the Bell's theorem photon-photon correlation.

An exciting and extremely important test of Bell's inequality was designed and implemented by Clauser et al. in the early 1970s at the UC Berkeley, which author (EAR) had the privilege to observe [7,18], as well as the work of Aspect, et. al. at the University of Orsay [23]. These extremely well designed and implemented experiments demonstrate a fundamental and unique remote causal connections and nonlocality on the spacetime manifold. Photon correlations have been observed over meter distances in the Aspect experiment. More recently, Gisin et al. has tested Bell's inequality over kilometer distances [24,25]. Rauscher and Targ apply the complex 8-space and its description of nonlocality, such as exemplified in the Bell's theorem tests, to the nonlocal aspects of consciousness [26,27]. Precognition and retrocognition comprise an anticipatory system. Clauser expressed his impression of these nonlocality experiments to the above authors. He stated that quantum experiments have been carried out with photons, electrons, atoms, and even 60-carbon-atom Buckyballs. He said that, "it may be impossible to keep anything in a box anymore." Bell emphasizes, "no theory of reality compatible with quantum theory can require spatially separate events to be independent." That is to say, the measurement of the polarization of one photon determines the polarization of the other photon at their respective measurement sites. Stapp also stated to one of us (EAR) that these quantum connections could be the, "most profound discovery in all of science" [26].

Bohm has conducted research on the concept of the undivided nonlocal whole, and Bohm and Hiley [3], having extensive discussions with one of us (EAR). Also, Wheeler's fundamental explanations on the concept of nonlocal interactions and the foundations of the quantum theory in publications and discussion with author (EAR) are fundamental to anticipatory systems [2]. Wheeler's design of his delayed choice experiment demonstrates that, according to quantum theory, the choice to measure one or another pair of complimentary variables at a given time can apparently affect the physical state of events for considerable periods of time *before* such a decision is made. Such complimentary variables are typically momentum and distance, or in Wheeler's experiment refer to the dual wave and particle nature of light, as observed in a two-slit interference apparatus.

Wigner attempted to formulate a nonlinear quantum theory and stated support of the complex Minkowski 8-space which has macroscopic nonlocal consequences [28]. The fundamental issue he addressed is when are where does the measurement observation occur for a stochastic causal system. Earlier, von Neumann had suggested a sequence of observations, or von Neumann chain. Wigner also addresses the issue of multiple observers of a quantum generated event [28].

4. Complex 8-Space and Nonlocality

Within the context of a fundamental observation and theoretical formalism of nonlocality and anticipation, such a theory must be consistent with the main body of the principles of physics. The major universal principles are used to determine the structure and nature of physical laws and act as constraints on physical phenomena. These are *Poincairé invariance* and its corollary, *Lorentz invariance* (which expresses the spacetime independence of scientific laws in different frames of reference), *analyticity* (which is a general statement of causality conditions in the complex space), and *unitarity* (which can be related to the conservation of physical quantities such as energy and momentum).

These principles apply to microscopic as well as macroscopic phenomena. The quantum description of elementary particles has led to the formulation of the analyticity principle in the complex momentum plane [29]. Complex geometries occupy a vital role in many areas of physics and engineering. Analyticity relates to the manner in which events are correlated with each other in the spacetime metric (that is, causality). When we apply this critical principle to the complex eight-dimensional space we can reconcile nonlocality and anticipatory systems with physics, without violating causality. It has been mathematically demonstrated that the equations of Newton, Maxwell, Einstein, and Schrödinger are consistent with the eight-dimensional complex space described here [12-14,20,30-33]. In addition, nondispersive solitary wave solutions are obtained for the complex 8-space Schrödinger equation [21].

The least number of dimensions that has the property of nonlocality and that is consistent with Poincairé invariance or Lorentz invariance is eight dimensions. In this space, each physical *spatial* distance has an imaginary temporal counterpart, such that there is a zero-spatial separation in the higher dimensional space. Likewise, for every real physically temporal separation, there is a counterpart imaginary *spatial* separation that subtracts to zero on the metric, allowing access to future information and bringing it into the present, which acts as an anticipatory system.

We have also demonstrated the properties of nonlocality with the formalism of Maxwell's equations in complex 8-space [29-31]. In the next section, we present a brief description of the complex Minkowski 8-space and its properties and implications. Then we present in section 5 the solution to the Schrödinger equation in complex 8-space and nonlinear recursive solutions which are consistent with and explanatory of Bell's nonlocality and the general principles of nonlocality and anticipatory phenomena in the quantum domain.

Both special and general relativistic forms of the complex 8-space have been formulated and examined in applications [11,13,15]. We present a brief description of the formalism which we utilize to solve the Schrödinger equation. We express the solution of the Schrödinger equation in complex 8-space. In the usual 4D Minkowski space, where Einstein considered time as the fourth dimension of three space, this formulated as a 4D lightcone diagram displayed in two dimensions, in which the ordinate is the time coordinate and the abscissa is the space coordinate, representing the three dimensions of space as $X = x,y,z$. the sides of the forward and backward lightcone form signal connections at the velocity of light, c, and the apex of the cone represents "now" spacetime. Inside the forward, future time, and backward, past time, lightcone event connections are represented by signaling for $v < c$ called time-like signaling. The space-like signaling outside of the lightcone represents greater than light speed, or space-like signaling, or $v >$ c.

Bell's nonlocality test implies spacetime signaling and hence, even though experimentally well-verified, some physicists find nonlocality unsatisfactory. However, as we know, the truth is in what Nature shows us, not in our particular biased beliefs. The complex 8-space formalism not only yields a mathematical description of nonlocality, but the complexified Schrödinger formalism gives a detailed picture of the quantum nonlocality that is consistent with the statistical nature of the quantum theory, but is also consistent with the formalism of relativity. Apparent superluminal signaling can occur for the connection of correlated past time events that remain correlated for present measurement and are related by luminal velocity of light signaling in the complex 8-space. Also, this formalism allows anticipatory measurements such as in the Aspect, Gisin experiments and Wheeler's delayed choice experimental proposal.

The conditions for causality in the usual 4-space, distance ds^2 is invariant and given as $ds^2 = g_{ab}dx^a dx^b$ where the indices a and b run 1 to 4. We use the metrical signature (+,+,+,-) for the three spatial and one temporal component in the metric g_{ab}. This metric is expressed as a 16-element 4 x 4 matrix which represents a measure of the form and shape of space. This is the metric defined on the lightcone, connecting time-like events. A second four imaginary dimensional space lightcone can be constructed, which intersects with the usual 4D Minkowski space, can be constructed. These two lightcones coincide in there "now" spacetime realities. The complexified 8-space metric is denoted as M_4 because it represents the complexification of four spacetime dimensions. The complex space is expressed in terms of the complex 8-space variable Z^μ, where $Z^\mu = X^\mu_{\mathrm{Re}} + iX^\mu_{\mathrm{Im}}$, and $Z^{*\nu}$ is the complex conjugate of Z^μ so that $Z^\nu = X^\nu_{\mathrm{Re}} - iX^\nu_{\mathrm{Im}}$. We now form the complex 8-space differential line element $dS^2 = \eta_{\mu\nu} dZ^\mu dZ^{*\nu}$ where the indices run 1 to 8 and $\eta_{\mu\nu}$ is the complex metric of 8-space. The generalized complex metric in the previous equation is analogous to the usual Einsteinian 4-space metric. In our formalism, we proceed by extending the usual 4D Minkowski space into a 4D complex spacetime. This new manifold (or spacetime structure) is analytically expressed in the complexified 8-space.

As stated before we represent X_{Re} by x_{Re}, y_{Re}, z_{Re} and t_{Re} i.e. the dimensions of our usual four space. Likewise, X_{Im} represent the four additional imaginary dimensions of x_{Im}, y_{Im}, z_{Im}, and t_{Im}. Hence, we represent the dimensions of our complex space as Z^μ or x_{Re}, y_{Re}, z_{Re}, t_{Re}, x_{Im}, y_{Im}, z_{Im}, and t_{Im}. These are all real quantities. It is the i before the x_{Im}, etc. that complexifies the space. We write the expression showing the separation of the real and imaginary parts of the differential form of the metric: $dZ^\mu dZ^{*\mu} = \left(dX^\mu_{\mathrm{Re}}\right)^2 + \left(dX^\mu_{\mathrm{Im}}\right)^2$. We can write in general for real and imaginary space and time components in the special relativistic formalism.

$$\begin{aligned} ds^2 &= \left(dx^2{}_{\mathrm{Re}} + dx^2{}_{\mathrm{Im}}\right) + \left(dy^2{}_{\mathrm{Re}} + dy^2{}_{\mathrm{Im}}\right) + \\ &\left(dz^2{}_{\mathrm{Re}} + dz^2{}_{\mathrm{Im}}\right) - c^2\left(dt^2{}_{\mathrm{Re}} + dt^2{}_{\mathrm{Im}}\right) \end{aligned} . \tag{1}$$

We now use lower case x and t for the three dimensions of space and of time. We represent the three real spatial components dx_{Re}, dy_{Re}, dz_{Re} as dx_{Re} and the three imaginary spatial components dx_{Im}, dy_{Im}, dz_{Im} as dx_{Im} and similarly for the real-time component $dt_{\mathrm{Re}} = dt$, the ordinary time and imaginary time component dt_{Im} remains dt_{Im}. We then introduce complex space -time coordinates as a space-like part x_{Im} and time-like part t_{Im} as imaginary parts of x and t. Now we have the invariant line elements as,

$$s^2 = |x'|^2 - c^2|t'|^2 = |x'|^2 - |t'|^2 \tag{2}$$

again where we take the units $c^2 = c = 1$ which is made for convenience

$$x' = x_{\mathrm{Re}} + \mathrm{i}x_{\mathrm{Im}} \tag{3a}$$

and

$$t' = t_{\mathrm{Re}} + \mathrm{i}t_{\mathrm{Im}} \tag{3b}$$

as our complex dimensional components so that [11,26,27]

$$x'^2 = |x'|^2 = x^2_{\mathrm{Re}} + x^2_{\mathrm{Im}} \tag{4a}$$

and

$$t^2 = |t|^2 = t^2_{\mathrm{Re}} + t^2_{\mathrm{Im}} . \tag{4b}$$

Recalling that the square of a complex number is given as,

$$|x'|^2 = x' x'^* = (x_{Re} + ix_{Im})(x_{Re} - ix_{Im}) \tag{5}$$

where the modulus of a complex number squared is $|x'|^2 = x_{\mathrm{Re}}^2 + x_{\mathrm{Im}}^2$ so that x_{Re} and x_{Im} are real numbers. This is a very important point, as we can only measure events described in terms of the mathematics of real numbers. Therefore, we have the eight-space line element where spatial and temporal distances are taken from the origin.

$$s^2 = x_{\mathrm{Re}}^2 - c^2 t_{\mathrm{Re}}^2 + x_{\mathrm{Im}}^2 - c^2 t_{\mathrm{Im}}^2 \tag{6a}$$

$$\mathrm{s}^2 = x_{\mathrm{Re}}^2 - t_{\mathrm{Re}}^2 + x_{\mathrm{Im}}^2 - t_{\mathrm{Im}}^2 \tag{6b}$$

Causality is defined by remaining on the right cone, in real spacetime as,

$$s^2 = x_{\mathrm{Re}}^2 - c^2 t_{\mathrm{Re}}^2 = x_{\mathrm{Re}}^2 - t_{\mathrm{Re}}^2 \tag{7}$$

using the units of $c = 1$. Then the generalized causality in complex spacetime is defined by

$$s^2 = x_{\mathrm{Re}}^2 - t_{\mathrm{Re}}^2 + x_{\mathrm{Im}}^2 - t_{\mathrm{Im}}^2 \tag{8}$$

where the coordinates in complex 8-space can be represented by $x_{\mathrm{Re}}, t_{\mathrm{Re}}, x_{\mathrm{Im}}, t_{\mathrm{Im}}$ on two generalized lightcones in 8D space [11,12,31].

We calculate the interval separation between two events or occurrences, Z_1 and Z_2 with real separation $\Delta x_{\mathrm{Re}} = x_{\mathrm{Re},2} - x_{\mathrm{Re},1}$ and imaginary separation $\Delta x_{\mathrm{Im}} = x_{\mathrm{Im},2} - x_{\mathrm{Im},1}$. Then the distance along the line element is $\Delta s^2 = \Delta\left(x_{\mathrm{Re}}^2 + x_{\mathrm{Im}}^2 - t_{\mathrm{Re}}^2 - t_{\mathrm{Im}}^2\right)$ and it must be true that the line interval is a real separation. The spatial and temporal distances that are generalized are not taken only from the origin, but from any two points in space and time. Then,

$$\begin{aligned}\Delta s^2 = &\left(x_{\mathrm{Re},2} - x_{\mathrm{Re},1}\right)^2 + \left(x_{\mathrm{Im},2} - x_{\mathrm{Im},1}\right)^2 - \\ &\left(t_{\mathrm{Re},2} - t_{\mathrm{Re},1}\right)^2 - \left(t_{\mathrm{Im},2} - t_{\mathrm{Im},2}\right)^2\end{aligned} \tag{9a}$$

Or we can write equation 9a as:

$$\begin{aligned}\Delta s^2 = &\left(x_{\mathrm{Re},2} - x_{\mathrm{Re},1}\right)^2 + \left(x_{\mathrm{Im},2} - x_{\mathrm{Im},1}\right)^2 - \\ &\left(t_{\mathrm{Re},2} - t_{\mathrm{Re}.1}\right)^2 - \left(t_{\mathrm{Im},2} - t_{\mathrm{Im},1}\right)^2\end{aligned} \tag{9b}$$

In equation (9b), the upper left diagonal term $\left(x_{\mathrm{Re},2} - x_{\mathrm{Re},1}\right)^2$ is be offset or "cancelled" by the lower right diagonal term $-\left(t_{\mathrm{Im},2} - t_{\mathrm{Im},1}\right)^2$, and the lower left diagonal term $-\left(t_{\mathrm{Re},1} - t_{\mathrm{Re},1}\right)^2$ is off set by the upper right diagonal term $\left(x_{\mathrm{Im},2} - x_{\mathrm{Im},1}\right)^2$. Because of the relative signs of the real and imaginary space and time components, and in order to achieve the causality connectedness condition between the two events, or $\Delta s^2 = 0$, we must "mix" space and time. That is, we use the imaginary time component to effect a zero space separation. We identify $\left(x_{\mathrm{Re},1}, t_{\mathrm{Re},1}\right)$ with a subject receiver remotely perceiving information from an even target $\left(x_{\mathrm{Re},2}, t_{\mathrm{Re},1}\right)$.

The nonlocality of Bell's theorem and its experimental test involves a real physical separation $\Delta x_{\mathrm{Re}} = x_{\mathrm{Re},2} - x_{\mathrm{Re},1} \neq 0$ and can either involve a current time observation such that $\Delta t_{\mathrm{Re}} = t_{\mathrm{Re},2} - t_{\mathrm{Re},1} = 0$ or a anticipatory time interval $\Delta t_{\mathrm{Re}} = t_{\mathrm{Re},2} - t_{\mathrm{Re},1} > 0$. The case where there is no anticipatory time element $\Delta t_{\mathrm{Re}} = 0$. The simplest causal connection then is one in which $\Delta x_{\mathrm{Im}} = 0$, and we have,

$$\Delta s^2 = 0 = (x_{Re,2} - x_{Re,1})^2 - (t_{Im,2} - t_{Im,1})^2 . \qquad (10)$$

These conditions are illustrated in fig. 1 In fig. 1a we represent a generalized point $P(x_{\mathrm{Re}}, t_{\mathrm{Re}}, t_{\mathrm{Im}})$, displaced from the origin which is denoted as P_1. This point can be projected on each dimension x_{Re}, t_{Re} and t_{Im} as points P_2, P_3, and P_4 respectively. In Figure 1b, we denote the case where a real-time *spatial* separation exists between points, P_1 and P_2 on the x_{Re} axis, so that $\Delta x_{\mathrm{Re}} \neq 0$, and there is no anticipation, so that $t_{\mathrm{Re}} = 0$, and access to imaginary time t_{Im}, nonlocality can occur between the P_1 to P_4 interval, so that $\Delta t_{\mathrm{Im}} \neq 0$. Then, our metric gives us $\Delta s^2 = 0$, where nonlocality is the contiguity between P_1 and P_2 by its access to the path to P_4. By using this complex path, the physical spatial separation between P_1 and P_2 becomes equal to zero, allowing direct nonlocal connectedness of distant spatial locations, observed as a fundamental nonlocality of remote connectedness on the spacetime manifold.

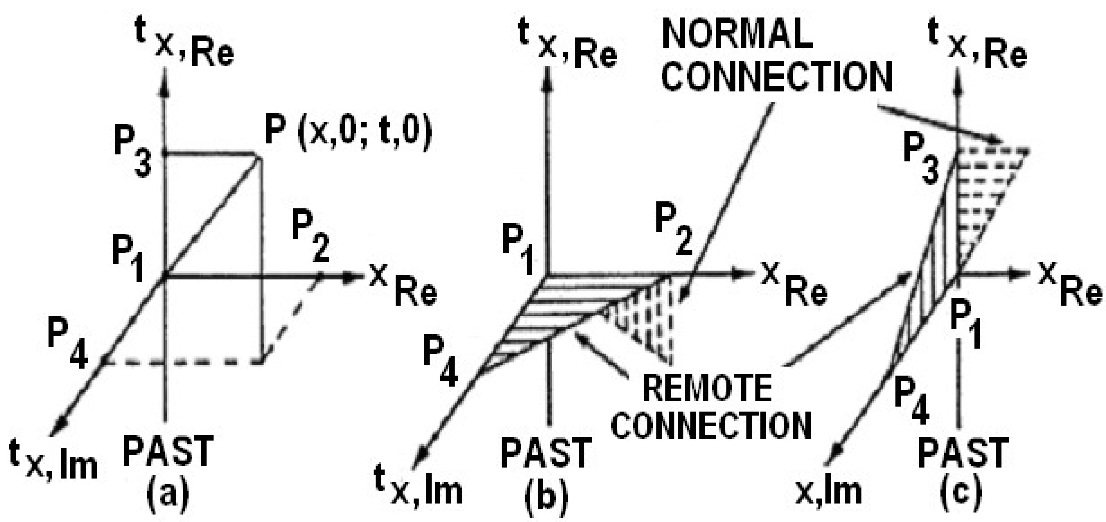

Figure 1. We represent the location of four points in the complex manifold. In figure 1a, point P_1 is the origin, and P is a generalized point which is spatially and temporally separated from P_1. In figure 1b, the Points P_1 and P_2 are separated in space but synchronous in time. This could be a representation of real-time nonlocal spatial separation. In figure 1c, points P_1 and P_3 are separated temporally and spatially contiguous. This represents an anticipatory temporal connection.

Figure 1c represents the case where anticipation occurs between P_1 and an apparent future anticipatory accessed event, P_3 on the t_{Re} axis. In this case, no physical spatial separation between observer and event is represented in the figure. Often such separation on the x_{Re} exists. In the case where

$x_{Re} = 0$, then access to anticipatory information, along t_{Re} can be achieved by access to the imaginary spatial component, x_{Im}. Hence, remote, nonlocal events in 4-space or the usual Minkowski space, appear contiguous in the complex 8-space and nonlocal temporal events in the 4-space appear as anticipatory in the complex 8-space metric. Both nonlocality and anticipatory systems occur in experimental tests of Bell's Theorem and perhaps in all quantum measurement processes.

5. Solitary Wave and Coherent Non-Dispersive Solutions in Complex Geometries

The properties and some of the implications of complex Minkowski spaces hold fundamental significance. We have presented the formalism for complex geometries in the previous section and also for superluminal x direction boosts in these geometries and the possible implications for remote connectedness, and anticipatory systems [11]. Also, the symmetry relations of the vector and scalar electromagnetic potential and other properties of Maxwell's equations, the x-directional superluminal boost, have been formulated [18]. The relationship of this approach to the Schrödinger equation in this work is of interest.

In this section we determine solutions to the Schrödinger equation formulated in a complex Minkowski space and demonstrate the relationship of the solutions to inter-connectedness and the nonlocality principle. The solutions are solitary or soliton waves which exhibit little or no dispersion over long distances. We present several implications of this formalism, for the test of Bell's Theorem, anticipatory processes and an explanation for some coherent, nonlinear, non-dispersive phenomena, such as nonlinear plasma phenomena [34,35].

We examine the relationship between our multi-dimensional remote connectedness geometry and possible coherent, non-dispersive solutions to the Schrödinger equation. These non-dissipative or non-dispersive solutions are termed soliton solutions, or solitary wave solutions, and are well known in macroscopic hydrodynamic phenomena. There has been some recent interest in the use of the soliton or instanton model to describe the gluon quark structure for "infinitely" bound quarks, in part, to explain the lack of experimentally observed free quarks.

The solution to linear wave equations are dispersive in space and time, that is, their amplitude diminishes and width at half maximum becomes larger as a function of time. The term soliton is commonly used to define a wave which retains its amplitude and "half width" over space and can interact and remain intact with other solitons. The term instanton, or evanescent wave, is used to describe a structure which experiences both spatial and temporal displacement. The term instanton seems to imply a short-lived structure but actually instantons can retain their spatial and temporal configuration indefinitely and interact with other instantons in a particle-like manner as do solitons. These unique solutions can explain the existence of long spatial and temporal phenomena such as Bell's remote connectedness phenomenon, Young's double slit experiment, plasma coherent collective states and other coherent phenomena.

Starting from the Schrödinger equation in complex spacetime, as seen previously [8,11], complex geometries have properties consistent with the above-mentioned phenomena. We proceed from the time-dependent Schrödinger equation in a vacuum with no potential term, $V\psi$. Which is considered later [21]. In real spacetime, we have

$$\frac{\hbar\nabla^2\psi}{2m} = \frac{1}{i}\frac{\partial\psi}{\partial t} \quad . \tag{11}$$

Monochromatic plane wave solutions for one dimension of space, or x-direction, such as

$$\psi = e\frac{i(kx-\omega t)}{\hbar} \tag{12a}$$

or

$$\psi^* = e\frac{-i(kx-\omega t)}{\hbar} \tag{12b}$$

which comprise the usual solutions. We can also write (12a) as

$$\psi = e^{i\alpha} \quad for \quad \alpha = \frac{kx-\omega t}{\hbar} \tag{13}$$

and we can write (13) as

$$\psi = e^{i\alpha} = \cos\alpha + i\sin\alpha \tag{14a}$$

and also

$$\psi = e^{i\alpha} = \sinh i\alpha + \cos i\alpha \,. \tag{14b}$$

Equation (11) is the usual linear form of the Schrödinger equation in which the superposition principle holds and the quantum measurement issue arises.

We proceed to formulate the Schrödinger equation in complex spacetime. The form of complex derivative utilized here is given in [8,11]. Only 1D forms of the derivative are considered in the del operator, ∇. We consider x-directional spatial dependence only for the real component of x as x_{Re}

$$\hbar\frac{\nabla^2}{2m}\psi \rightarrow \frac{\hbar}{2m}\frac{\partial^2}{\partial x_{\mathrm{Re}}^2}\psi \,. \tag{15}$$

Using the imaginary components of space and time x_{Im} and t_{Im}, we have

$$\hbar\frac{\nabla_{\mathrm{Im}}^2}{2m}\psi \rightarrow \frac{\hbar}{2m}\frac{\partial^2}{\partial x_{\mathrm{Im}}^2}\psi \,. \tag{16}$$

Note that the sign change occurs for the spatial second derivative for $ix \rightarrow x_{\mathrm{Im}}$. The imaginary time derivative yields

$$\frac{\partial}{\partial it^*} \rightarrow \frac{1}{i}\frac{\partial}{\partial t_{\mathrm{Im}}} \tag{17}$$

which is an imaginary term derivative.

The imaginary form of the Schrödinger equation becomes

$$\frac{\hbar}{2m}\nabla_{\mathrm{Im}}^2\psi = \frac{\partial\psi}{\partial t_{\mathrm{Im}}} \,. \tag{18}$$

Because the Schrödinger equation is second order in space and first order in time and no imaginary term occurs in Eq. (18), the harmonic solutions in Eqs. (13, 14a, 14b) are not solutions to the imaginary components of the Schrödinger equation. Since the Dirac equation is first order in space and time, and the Klein-Gordon equation and classical wave equation are second order in space and time, quite a different picture emerges.

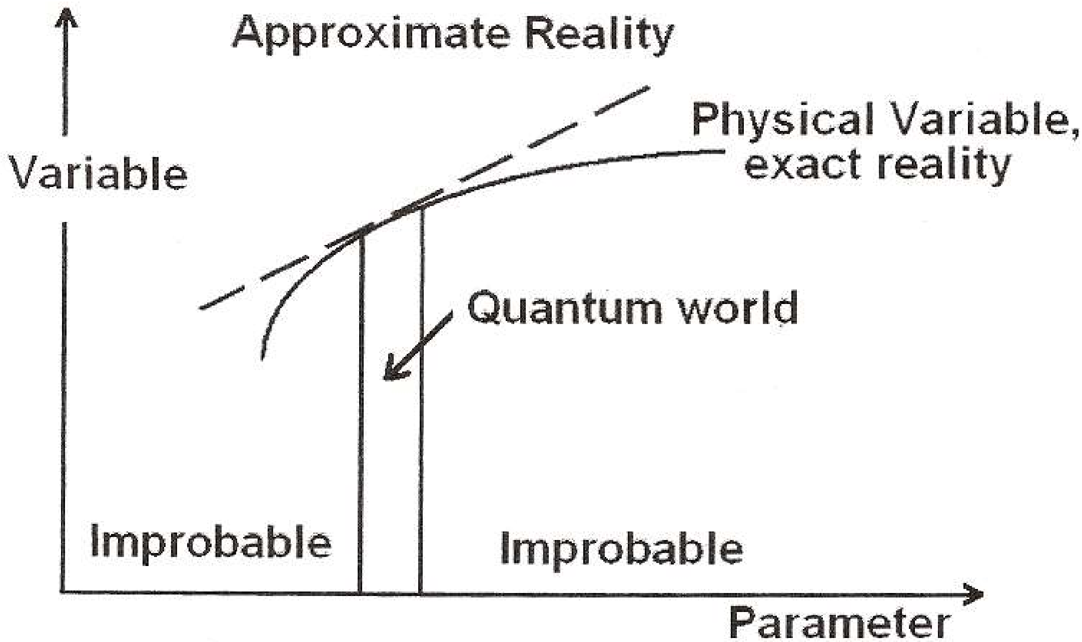

Figure 2. We approximate the quantum domain as a linear variable dependent on a parameter. The full "space" of exact reality is nonlinear.

Starting from a real solution, which is a plane exponential growth function

$$\psi = e^{\alpha} \quad for \quad \alpha \equiv \frac{kx + \omega t}{\hbar} \tag{19}$$

we then have from Eq. (18),

$$\frac{\partial \psi}{\partial x_{\text{Im}}} = \frac{k}{\hbar}\psi \quad or \quad \frac{\partial^2 \psi}{\partial x_{\text{Im}}^2} = \frac{k^2}{\hbar^2} \tag{20}$$

and

$$\frac{\partial \psi}{\partial t_{\text{Im}}} = \frac{\omega}{\hbar}. \tag{21}$$

and Eq. (19) satisfies (18). Note that $\alpha \equiv (kx - \omega t)/\hbar$ does not satisfy Eq. (18) because of the minus sign which then occurs in Eq. (21). All quantities $k^2, \hbar^2, \omega^2$ are real as is x_{Im} and t_{Im}.

The form of the solution in Eq. (19) for α positive definite, for all quantities greater than zero, yields an undamped growth function, that is we find that solutions in an imaginary spacetime geometry yield growth equations. Eq. (19) is of a linear form. We also have another solution in Eq. (25a), but Eq. (25b) is not a solution:

$$\psi = e^{-\alpha} \quad for \quad \alpha \equiv \left(\frac{kx - \omega t}{\hbar} \right) \tag{22a}$$

and

$$\psi = e^{-\alpha} \quad for \quad \alpha \equiv \left(\frac{kx + \omega t}{\hbar}\right) \tag{22b}$$

where in kx, x is x_{Im} and standing wave solutions cannot occur. Before we examined the solution of the Schrödinger equation in complex spacetime for $x' = x_{Re} + ix_{Im}$ and $t' = t_{Re} + it$. Let us briefly discuss the introduction of a nonlinear term with a small coupling constant.

5.1 NONLINEAR SCHRÖDINGER EQUATION WITH COMPLEX TEMPORAL PERTURBATION

We introduce a 'potential' like term which is coupled by a small coupling constant, g^2, and is associated with an attractive force. If the coupling term is small, then solutions can be determined in terms of a perturbation expansion. A $g^2 > 0$ implies an attractive force when it is regarded as a second quantized Fermi field. This field satisfies the Dirac equation and introduces an additional term in the Lagrangian. In reference [11] we detail this formalism, in which causality conditions in terms of analytic continuation in the energy plane gives motivation for identifying the nonlinear coupling term with the imaginary temporal coordinate, as $t^* = it_{Im}$.

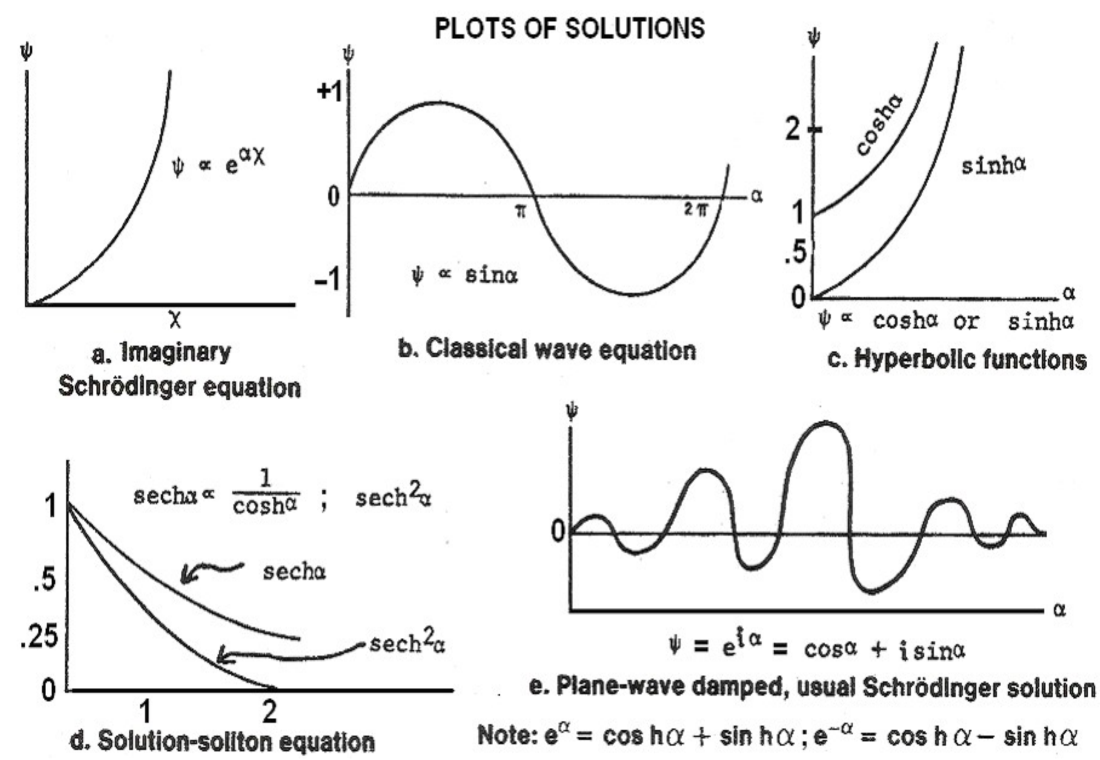

Figure 3. Five plots of various solutions related to the Schrödinger equation.

By analogy to this form of the Dirac equation, we can write

$$\frac{\hbar\nabla_{\mathrm{Im}}^2\psi}{2m}+g^2(\psi^+\psi)\psi=0 \tag{23}$$

for the time-dependent equation where ψ^+ is the Hermitian conjugate of ψ. For the real time-dependent equation, we have

$$\frac{\hbar\nabla_{\mathrm{Im}}^2\psi}{2m}+g^2(\psi^+\psi)\psi=\frac{1}{i}\frac{\partial\psi}{\partial t_{\mathrm{Im}}}. \tag{24}$$

For the Schrödinger and Dirac equation, we can find solutions which we can identify in a field theory, in which each point is identifiable with a kinetic, potential and amplitude function. Linearity can be approximated for $g^2\sim 0,$ for g^2 expressed in terms of it_{Im}. In the following subsection we examine the complexification of the Schrödinger equation.

5.2 THE SCHRÖDINGER EQUATION IN COMPLEX SPACE AND TIME

Returning to our definition of complex space and time,

$$x'=x_{\mathrm{Re}}+ix_{\mathrm{Im}},\quad t'=t_{\mathrm{Re}}+it_{\mathrm{Im}} \tag{25}$$

where x_{Re} and t_{Re} are the real parts of space and time and x_{Im} and t_{Im} are the imaginary parts of space and time and are themselves real quantities. In the most general case we have functional dependencies x_{Im} (x,t) and t_{Im} (x,t) where x and t are x_{Re} and t_{Re}. With the quantum superposition principle, we can combine real and imaginary parts. For the x-directional form of Eq. (11), we have

$$\frac{\hbar}{2m}\frac{\partial^2\psi_1}{\partial x_{\mathrm{Re}}^2}=\frac{1}{i}\frac{\partial\psi_1}{\partial t_{\mathrm{Re}}}. \tag{26}$$

For the imaginary part, we have from Eq. (18)

$$\frac{\hbar}{2m}\frac{\partial^2\psi_1}{\partial x_{\mathrm{Im}}^2}=\frac{\partial\psi_2}{\partial t_{\mathrm{Im}}}. \tag{27}$$

By linear superposition, we can combine the above equation, as

$$\frac{\hbar}{2m}\left(\frac{\partial^2}{\partial x_{\mathrm{Re}}^2}+\frac{\partial^2}{\partial x_{\mathrm{Im}}^2}\right)\psi=\left(\frac{1}{i}\frac{\partial}{\partial t_{\mathrm{Re}}}+=\frac{\partial}{\partial t_{\mathrm{Im}}}\right)\psi\,. \tag{28}$$

Note that we make an assumption that the mass in Eq. (26) is the same as in Eq. (27). We discuss this assumption and tachyonic implications in [11]. We now form solutions $\psi(x_{\mathrm{Re}},x_{\mathrm{Im}},t_{\mathrm{Re}},t_{\mathrm{Im}})$ in terms of linear combinations of $\psi_1(x_{\mathrm{Re}},t_{\mathrm{Re}})$ and $\psi_2(x_{\mathrm{Im}},t_{\mathrm{Im}})$.

Equation (27) is defined on a 4D space $(x_{\mathrm{Re}},x_{\mathrm{Im}},t_{\mathrm{Re}},t_{\mathrm{Im}})$. In the first approximation, we will choose $\partial^2\psi/\partial x_{\mathrm{Im}}^2=0$ so that we have

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_{\mathrm{Re}}^2}\psi = \left(\frac{1}{i}\frac{\partial}{\partial t_{\mathrm{Re}}} + \frac{\partial}{\partial t_{\mathrm{Im}}}\right)\psi . \tag{29}$$

Motivation for this approximation can be found in our discussion of remote connectedness properties, diagrammed in Figs. 1c and 1b of the previous section.

Let us rewrite Eq. (26) as

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_{\mathrm{Re}}^2}\psi - \frac{\partial}{\partial t_{\mathrm{Im}}}\psi = \frac{1}{i}\frac{\partial}{\partial t_{\mathrm{Re}}}\psi \tag{30}$$

where ψ is a function of $(x_{\mathrm{Re}}, t_{\mathrm{Re}}, t_{\mathrm{Im}})$. From examination of the forms of Eq. (24) and (29), we can identify the g^2 term with the imaginary time derivative $\partial/\partial t_{\mathrm{Im}}$. This result is similar to the more comprehensive field theoretic argument for the Dirac equation. The associated metric space for $(x_{\mathrm{Re}}, t_{\mathrm{Re}}, t_{\mathrm{Im}})$ defines a remote connectedness geometry. We then have

$$\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x_{\mathrm{Re}}^2} + G^2\psi = \frac{1}{i}\frac{\partial\psi}{\partial t_{\mathrm{Im}}} \tag{31}$$

where $G^2 = g^2(\psi^+\psi)$ is identified with the $\partial/\partial t_{\mathrm{Im}}$ term. We proceed from the assumption that $(x_{\mathrm{Re}}, t_{\mathrm{Re}}, t_{\mathrm{Im}})$ are independent variables of each other.

We can define three cases for the right side of Eq. (31), that is, the real time-dependent case, (a) zero, time dependent cases, (b) $\frac{1}{i}E_n\psi$, and (c) $\frac{1}{i}\frac{\partial\phi}{\partial t}$. In determining the coupling constant G^2, we define solutions $\phi(x_{\mathrm{Re}}, t_{\mathrm{Re}}, t_{\mathrm{Im}})$ for the third case. We have, in general,

$$\frac{\partial^2\phi}{\partial x^2} + G^2(\tau)\frac{\partial\phi}{\partial\tau} = \frac{1}{i}\frac{\partial\phi}{\partial t}. \tag{32}$$

We define the quantity $\xi = kx_{\mathrm{Re}} - \omega t_{\mathrm{Re}} - \alpha t_{\mathrm{Im}}$. For case (a) above we have solutions

$$\phi = \phi_0 + A\sec h^2 a\xi \tag{33}$$

where

$$G^2(t_{\mathrm{Im}}) = \frac{a\hbar^2k^2}{2\alpha m}\tanh a\xi \tag{34}$$

where k is the wave number or k_{Im}. The constant, a can be expressed in terms of $\hbar$ and m where $m' = im = m^* = m_{Im}$ which is the tachyonic mass, which we formulate in complex 8-space. For case (c), we find a similar solution for ϕ for

$$G^2(t_{\mathrm{Im}}) = \frac{\frac{a\hbar^2k^2}{2m}}{(\alpha + \frac{1}{2}\omega)}\tanh a\xi . \tag{35}$$

Solutions and the form of G^2 (t_{Im}) is more complicated for case (b). Note the analogy to the solutions for the Korteweg-deVries equation [21] for

$$u(x,t) = A\sec h^2 k \quad for \quad K = x - ct/L \tag{36}$$

where L is a characteristic length dimension of a soliton wave which is expressed in terms of the amplitude A and the hydrodynamic media depth h or $L = \sqrt{h^3/3A}$ [35].

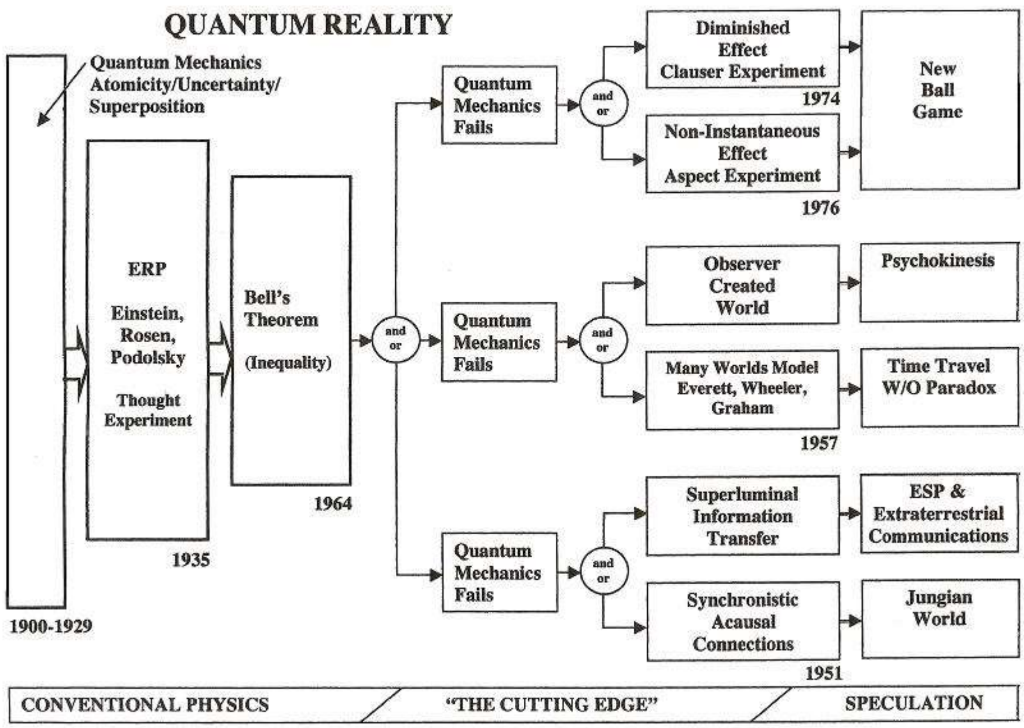

Figure 4. Historical development of quantum theory.

The form of $G^2(t_{\mathrm{Im}})$ is nonlinear and is compatible with the soliton solutions. The non-dispersive nature of the solutions may be associated with a complex space "signal" which defines the connection of remote parts of the multi-dimensional geometric space [11]. Several types of solutions are displayed in Fig. 3. See Fig. 4 for the implications of the Quantum Theory and Bell's theorem.

5.3 DISCUSSION AND APPLICATION OF COHERENT STATE SOLUTIONS

The soliton solution is a unique solution in that it is non-dispersive. All other solutions to the Schrödinger equation are dispersive to various degrees. Each state solution has a particular amplitude at a specific point in space and instant in time. One can calculate the probability of this existence of a specific amplitude as a function of x and t. A unique feature of the soliton is that it retains its amplitude in space and time and therefore we have a reasonable certainty in our measure of it for each space and time.

In practice, there are no completely non-dispersive waves but soliton solutions are defined in terms of coherent, non-dispersive states that retain their identity and amplitude over many iterations. Hence the soliton acts like a particle, in that soliton solution collisions do not disrupt the wave form or amplitude in elastic processes [36].

In hydrodynamics, the interpretation of the soliton or solitary wave is not completely clear [37]. One possible interpretation of this particular type of solution to the wave equation in this particular complex geometry, including the small coupling nonlinear term, is that the geometry selects the particular wave function. Note that this possible interpretation may have deep implications for the quantum measurement issue or the "collapse of the wave function". In the usual nuclear energy levels, a particular state may be composed of a sum of various states of angular momentum and spin which sum to the total I and l values. The amplitude of these states vary, with one predominant term [38]. In the current case, the soliton non-dispersive wave could represent the predominant, fixed amplitude solution with other smaller dispersive terms.

We have examined coherent collective states in plasmas with high temperature fusion media and electron gases in metal conductors. It is felt that these and other types of collective, coherent, dynamical phenomena can be explained by the soliton formalism. Other such phenomena which may also involve an intermediate temperature plasma is the illustrative so-called "ball lightning" [32].

6. Conclusion

We have formulated a complex multi-dimensional Minkowski space and associated twistor algebra which has nonlocal and anticipatory properties. One unique property of this geometry is its remote connectedness. We have formulated the Schrödinger equation in this multi-dimensional geometry. We identify the imaginary temporal component term as a small nonlinear term and determine soliton or solitary wave solutions. These non-dispersive, coherent waves are appropriate to define signals, in the space, which exhibit remote connectedness properties. Phenomena which involve remote correlation of events, such as Bell's Theorem, Young's double slit experiment, and super-coherence phenomena, demand nonlocality. The twistor algebra can be constructed to be mapable 1:1 with the spinor calculus and allows us to develop a unique formalism of Bell's inequality.

We also speculate that the nonlinear quantum model with coherent non-dispersive solutions to the Schrödinger equations, which is an expression of the remote nonlocality property of the space, may lend insight into the quantum measurement problem. A mechanism may be formulated which defines a connection between the observer and the observed. The properties of certain systems appear to demand a nonlinear, nonlocal anticipatory description.

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