A Refinement to Nonstandard Ultra-logic-systems Applied to the GGU-model.

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Abstract: This article develops and employs modern methods for mathematical modeling. In particular, the methods of nonstandard analysis are applied to general language logic-systems. This allows the operators for the only known mathematical cosmogony, the General Grand Unification Model (GGU-model), to more clearly exhibit their hyper-logical properties. All of the GGU-model entities and processes are predicted from observable entities and processes employed to construct physical entities. It is shown that, for each of the four cases for GGU-model interval construction and the four types of instruction paradigms $\underline{\mathcal{I}}_q$, there exists an ultra-word-like X_x^q such that when the hyper-algorithm ${}^*\underline{\mathcal{A}}'$ is applied to $({}^*\underline{S}^q, \{X_x^q\})$ an ultra-logic-system ${}^*\boldsymbol{\Lambda}^q(x)$ is hyper-deduced. Application of the hyper-algorithm ${}^*\underline{\mathcal{A}}$ to $({}^*\boldsymbol{\Lambda}^q(x), \{y\})$ yields a hyperfinite instruction paradigm $\mathcal{I}_x^q$ that contains $\underline{\mathcal{I}}_q$. The $\mathcal{I}_x^q$ is *-deduced in the required $\leq {}_{*\mathcal{I}_q}$ order. It is shown how a set of instruction paradigms $\{\mathcal{I}\}$ leads to the Patton and Wheeler required participator universe. The set of all ultra-propertons is defined and its properties examined. In this version, the set of all propertons that is sufficient for universe construction is properly established. Further, the GGU-model schemes are presented in diagram form. A process is introduced that leads, when applied, to the individual development of each universe-wide frozen-frame. An operator is shown to exist, which, via a substratum medium and processes, changes *instruction-information into a substratum info-field. From these, physical and physical-like systems are produced. In this refinement, the page 11 statement, in the original version of this paper on quantum theory, is completely modified and the previous statement is now essentially incorrect.

1. Logic-System Generation for Instructions

As is customary, the nonstandard model ${}^*\mathcal{M}$ as used in all of the articles for The General Grand Unification Model (GGU-model) is a saturated enlargement. In this paper, $q = 1, 2, 3, 4$. These numbers denote the four primitive-time intervals (Herrmann, 2006) employed for the GGU-model. The ultraword approach to generate a universe

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is replaced with an ultra-logic-system. As in Herrmann (2013b), this is a hyperfinite logic-system that, after application of the extended logic-system algorithm, generates each member of the hyperfinite developmental paradigm d_x^q in the proper $\leq_{d_x^q}$ order such that $\mathbf{d}_q \subset d_x^q \subset {}^*\mathbf{d}_q$, where $q = 1, 2, 3, 4$ and $x = \lambda, \nu\lambda, \mu\lambda, \nu\gamma\lambda$, respectively. Finally, the term "subparticle" was used previously. To prevent incorrect mental images as to models for subparticles, the term "properton" replaces the term "subparticle." Without visualizing, a properton is an entity characterized only by a list of properties.

The primitive entity, which yields physical reality for any GGU-model generated universe, is a collection of ultra-propertons. Their properties are neither based upon the particle physics of today nor any corresponding theory such as quantum theory. Although not originally presented in this manner, all of the GGU-model entities and processes are actually predicted from observable entities and processes employed to construct physical entities (Herrmann 2013a). They can be considered as existing in a **background universe** or **substratum world**. This world can be considered as a physical-like world, where the rules that govern universe formation are distinct from those processes and rules that govern the development of <u>any</u> physical universe. They are simple rules that refer mostly to counting. This substratum world is also interpreted philosophically in other ways.

If necessary for a specific physical theory, any continuity requirement is satisfied by the properton field (Herrmann, 1983, 1989). For our universe, a collection of propertons has been shown to be closely associated with relativistic effects (Herrmann, 2003). No other known primitive entities, such as strings, will have any affect upon the application of propertons as the primitive entities that generate a universe. The processes used to obtain particles and all other physical entities from ultra-propertons need not correspond to the rules of quantum field theory or any additional rules like how quarks combine to form particles.

Quantum field theory contains descriptions (rules or instructions) that produce particles from immaterial fields. Such fields are quantum mechanical systems and, when represented, have various degrees of freedom. These are but parameters which contribute to the overall state of the system. For various particles, parameters for physical measures, states or other descriptive modes are the characterizing features of propertons. As an example, the physical appearance and disappearance of particles are trivial applications of properton processes. For quantum field theory, one has the "creation" and "annihilation" operators that mathematically yield the same results obtained via properton processes.

For the GGU-model, quantum theory does not produce steps in a development since the method of production must be universe and physical law independent. For

our universe, the development "satisfies" the predictions of verified physical laws and theories.

The GGU-model can be based upon observable human behavior and the mathematics predicts, for our universe, behavior that satisfies the behavior predicted by verified physical laws and theories. The GGU-model satisfies all the requirements stated by Patton and Wheeler (1975). There is a vast amount of evidence for the predicted GGU-model processes. Whether such processes exist in some sort of reality is a philosophic choice. One can make this choice based upon various factors. One can choose to accept properton existence based upon the same philosophy expressed by those who accept that entities postulated in quantum field and particle theory exist.

The concept of instructions or rules is generalized to instructions that yield a physical reality from combinations of propertons. They are substratum laws. So as not to confuse these with physical laws, they are called instructions. Further, in what follows, the events that correspond to each $f^q(i,j)$, as completely defined in Section 2, are denoted by $E^q(i,j)$. This does <u>not</u> mean that the rules used in quantum theory (QT) actually yield each $E^q(i,j)$. The GGU-model rules are termed as "instruction-entities." As mentioned, what this signifies is that the QT rules are verified via the production of event sequences, which yield our universe.

Physical laws and theories are verified when each $f^q(i,j)$ is realized and they allow us to predict what behavior occurred in or will occur within other realized $f^q(p,k)$. Adjacent $E^q(i,j)$ satisfy the "best possible unification," $\bigvee_w \mathcal{H}$, for the collection $\mathcal{H}$ of all verified physical laws and theories (Herrmann, (2004, Corollary 2.10.1; 2006a, Theorem 2.2)). For the GGU-model, the "instructions" are rather simple ones that lead to all the characteristics that allow one to identify any material entity for any of the presently known cosmologies. A standard properton is modeled by a finite collection of numerical or coded descriptive physical characteristics. These characteristics are represented by coordinates within n-tuples. Other identifiers can also be included as specific coordinates.

What comes next is different than applying the GGU-model methods to the general notion of collections of descriptions taken from a "language" W′ and where the "general paradigm" approach investigates the composition of the words themselves. For propertons, the notion of the extended (general) language, in this case informal, is used with the appropriate cardinality (Mendelson, 1987). The GGU-model is just that a mathematical model, where the basic approach is to correspond the words or *words to entities within the substratum.

As previously done in Herrmann (1979-1993) section 9.3 on general paradigms, we often use symbols, denoted by a prime, in the extended alphabet to specify as

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constants the values for specific "variables" and other symbolic objects. Let W′ be the set of informal words as constructed by the Markov join operator, not specifically denoted, as in Herrmann (1979-1993). Throughout mathematical logic, at least, the completed set of natural numbers is employed. A completed set of this cardinality can be mentally imaged (Herrmann, 2013C). Potentially infinite portions of the mental scheme are used as represenions for the natural numbers.

Although within a finite environment only finitely many symbols are available, conceptually, for such representations, this can be extended, at last, to denumerably many natural numbers. Thus, this basic assumption, as applied within mathematical logic, holds conceptional. The (informal) set W′ is assumed to contain notation for denumerable many natural or rational numbers. For some aspects of mathematical logic, there are those who further extend this to the sets of greater cardinality.

Robinson (1963, p. 90) is the first to use a standard set of individuals U in the formal standard model in the following manner. He states, "[N]ow suppose that certain subsets of U are regarded as the constituents of a language of the first order predicate calculus." Then assuming the set U contains such language elements, he states that having such "individuals in a certain domain is entirely in keeping with the axiomatic approach to the syntax of a formal language." He differentiates between members of the formal language by simply stating that they are individuals and are different relative to all similar symbols used for other purposes. Further, Robinson assumes that there are subsets of U that comprise the "variables, connectives, well-formed-formula, sentences, etc." Robinson uses the same symbolic forms for the informal and formal structure.

As pointed out by Mendelson (1987, p. 28), mathematical logic modeling only requires members of a language to be "arbitrary objects rather than just linguistic objects." Thus, the Robinson approach can be viewed on a technical level as Mendelson states. On one hand, W′ are actual physical entities that satisfy various relations and, on the other hand, they are merely arbitrary "abstract" objects, such as sets or individuals, that satisfy corresponding abstract relations. In previous books and papers, these notions were differentiated by means of the bijection i that is still employed. The mathematical terminology employed referred to W′ and all necessary relations using what is usually termed as "informal" set theory as "informal" relations. Then objects in a more complex structure based only upon the natural numbers but directly corresponding to members of W′ and how they are formed, yields "formal" objects. However, in later research, the Robinson idea of employing W′ is added to the formal mix. This has created some terminology confusion.

As originally presented in Herrmann (1979-1993), the collection of all W′ is coded via a bijection i: W′ → $\mathbb{N}$ that codes each member of the "informal" or "intuitive"

words W′. (In Herrmann (1979-1993), the notation $\mathcal{W}'$ is used for the W′.) When formalized, this yields a mathematical model that employs only the natural numbers as its basic component. Comparative interpretations relate the nonstandard members of the formal model directly to members of W′ via the inverse of i. The Robinson approach considers only a propositional language L and states that it is a subset of its nonstandard extension L' (Robinson (1963, p. 91)). The same holds for defined "informal" relations.

Incorporating the Robinson approach is accomplished by considered two disjoint collections of atoms (individuals), W′, $\mathbb{R}$ ($\mathbb{R}$ represents the real numbers) and the construction of a superstructure $\mathcal{N} = \bigcup\{X_n \mid n \in \omega\}$ with ground set $X_0 = W' \cup \mathbb{R}$. This yields the standard structure $\langle \mathcal{N}, \in . = \rangle$. (In most cases, for the GGU-model, $\mathbb{R}$ can be replaced by the rational numbers $\mathbf{Q}$.) First, as usual, the nonstandard extension $^*X_0$ is obtained and a superstructure Y with ground set $^*X_0$, is constructed. This yields the structure $\mathcal{Y} = \langle Y, \in, = \rangle$. (The font change is defined below.) The ultrapower nonstandard structure $\mathcal{M}_1 = \langle \mathcal{N}^J, \in_{\mathcal{U}}, =_{\mathcal{U}} \rangle$ is obtained. The Mostowski Collapsing Lemma yields the nonstandard model $^*\mathcal{M} = \langle {}^*\mathcal{N}, \in . = \rangle$, where $\bigcup\{ {}^*X_n \mid n \in \omega \} = {}^*\mathcal{N}$. The standard structure $\mathcal{M}$ is a set-theoretic model for any bounded first-order statement that holds in $\mathcal{N}$. Further, each member of $^*\mathcal{N}$ is a member of Y.

Superstructure members of $\mathcal{N}$ are often termed as "informal" while members of Y are the "formal" objects. An additional strict formalizing can be added to these constructions by considering a superstructure generated by a corresponding "abstract" set of individuals Δ' in place of the W′. However, this very strict formalizing is based more upon the Philosophy of Mathematics and not upon an actual necessity. One would simply note that it all can be so formalized in this way if one desires. It now seems best to drop the additional Δ' notion. As previously, the set of equivalence classes $\mathcal{E}'$ of partial sequences can be employed, where each member of the class is a partial sequence that generates the same coded word as determined by the join operator.

In like manner, there are partial sequences that have as their images members of W′. These produce the significant equivalences classes defined in Herrmann (1979-1993) except that the images are not natural numbers but rather members of W′. In this revision, the symbol $\mathcal{W}'$ denotes the set of W′ generated equivalence classes. There is a trivial bijective correspondence between these two distinctly different sets of equivalence classes. (Note: It is usually assumed that the members of $\mathcal{W}'$ that correspond to the natural or other "numbers" employ distinctly different symbols than those used for such numbers as abstract entities. Also, there are sets in $\mathcal{E}'$ and $\mathcal{W}'$ that contain but one member and are here still termed as equivalence classes.)

Members of $\mathcal{E}'$ [resp. $\mathcal{W}'$] and all relations between them correspond bijectively to

members of W′, relations between them and other related entities. The corresponding members of $\mathcal{E}'$ [resp. $\mathcal{W}'$], relations between them and other related entities are denoted by **bold** font. Although, originally not done in any refined manner, members of W′ and all relations between them that do not carry the bold notation are usually denoted by Roman font. (Usually, if the "hyper" notation * is employed the original Roman or bold font is retained.) All other mathematical objects are usually denoted by mathematics italics. It is not necessary in this article, and almost all others, to differentiate between members $\mathcal{E}'$ or $\mathcal{W}'$ since the interpretations relate general nonstandard objects to members W′ and corresponding relations between these members.

(Often in Herrmann (1979-1993) the notation is highly simplified and much can only be understood from the context. For example, there is the expression $^*i[\mathcal{W}']$, where $\mathcal{W}'$ is the W′ in this article. Since the i is not a member of the formal structures, this can only mean, in this context, $^*(i[\mathcal{W}'])$. That is, the nonstandard extension of the formal set $i[\mathcal{W}'].$)

Although it is probably unnecessary to do so since all that follows is not dependent upon whether $\mathcal{E}'$ or $\mathcal{W}'$ is the underlying set used, when specification is not necessary, a standard set of equivalence classes is denoted, in this article, by a generic type symbol $\underline{\mathcal{W}}'$. As done in Herrmann (1979-1993), a member of $\underline{\mathcal{W}}'$ is bijectively related to a corresponding member of W′. Various bijective correspondences are usually indicated via font alterations.

Objects that correspond to members of W′ and relations between them have various standard "names." These names or appropriate modifications are used for the corresponding objects and relations, when viewed in $\underline{\mathcal{W}}'$, tend to carry the same names as the informal objects and the names for the informal relations that model the informal language characteristics. Symbolically, there are times when both the informal and formal symbols used are the same. Of course, in general, this is the customary approach throughout mathematical modeling. For W′, the equivalence class $\mathcal{W}'$ is produced by the projection map θ. In the coded case, such a map yields $\mathcal{E}'$. Obviously, there is a bijection from $\mathcal{W}'$ onto $\mathcal{E}'$ that preserves the identity on W′. Usually these bijections are not formally mentioned. It is the application of θ that is indicated by bold notation. This strict correspondence must be maintained.

The introduction of the "empty word" allows the set W′ with its join operator to be expressed as a monoid. There is one and only one function $f_\emptyset \in (W')^\emptyset$, the empty function. Consider $\mathcal{W}' \cup \{[f_\emptyset]\}$, where informal $[f_\emptyset] = \emptyset$ is the empty equivalence class. Then the informal join operator has a mimicking operator defined on $\mathcal{W}' \cup \{\emptyset\}$ [resp. $\mathcal{E}' \cup \{\emptyset\}$]. Each equivalence class [f] has a unique maximum f′, where $f' \in (W')^{[0,m]}$ and there is no $f \in [f']$ such that $f \notin (W')^{[0,k]}$, $k > m$. The partial sequence f′ intuitively

generates, in order right-to-left, each "alphabet" symbol that comprises a word.

Let $[f']$, $[g']$, $f' \in (W')^{[0,m]}$, $g' \in (W')^{[0,n]}$ be the unique members of $\mathcal{W}'$ [resp. $\mathcal{E}'$]. Define the binary operation $\circ$ as follows: $[f'] \circ [g'] = [h']$, where $h'(i) = g'(i)$, $i \in [0,n]$ and $h(n+j) = f'(j-1)$, $j \in [1, m+1]$. Then define $[f'] \circ \emptyset = [f'] = \emptyset \circ [f']$ and $\emptyset \circ \emptyset = \emptyset$. The collection $\mathcal{W}' \cup \{\emptyset\}$ [resp. $\mathcal{E}' \cup \{\emptyset\}$], with this binary operation defined, is a monoid structure $(\mathcal{W}' \cup \{\emptyset\}, \circ)$ due to the obvious associativity property it preserves.

As an example, consider the informal word "It|||is|||very|||good." (The ||| is a "spacing" symbol.) There is a member $f' \in [f]$, $n = 5$ such that f' yields $f'(4) \circ f'(3) \circ f'(2) \circ f'(1) \circ f'(0) =$ It|||is symbol string "It|||is|||" and $g' \in [g]$ such that g' yields $g'(9) \circ g'(8) \circ \cdots \circ g'(0) =$ very|||good., $m = 9$. Then $[h'] = [f'] \circ [g']$ and h yields h(15)h(14)h(13)h(12)h(11)h(10)h(9)h(8)h(7)h(6)h(5)h(4)h(3)h(2)h(1)h(0) = It|||is|||very|||good. $\in W'$. (Note that the actual correspondence to ordinary left-to-right word ordering corresponds to a reverse ordering of the numbers $\{0, \ldots, 15\}$.) This yields a model for the informal join operator via the monoid $(\mathcal{W}' \cup \{\emptyset\}, \circ)$.

Of course, relative to a physical or a non-physical world, all such mathematically modeled objects but "represent" entities or processes. Then their standing as members of a language follows from the characterizing relations such as the "ordering" notion and axiomatic join requirements represented on the equivalence classes by $\circ$. It is via an "interpretation" that the terminology is changed to linguistic notions. But, consistent with the usual practice, terms from the basic interpretation are used as names for $\underline{\mathcal{W}}'$, the members it contains and other representations for linguistic objects. As indicated above the previous results in books and papers dated before this one and that employ statements in terms of $\mathcal{E}'$ can be bijectively translated to $\mathcal{W}'$. The inverse relation θ^{-1} translates statements relative to the $\underline{\mathcal{W}}'$ into W' statements.

Consider the denumerable set all prime numbers P, a bijection $h \colon \mathbb{N}' \to P$, $\mathbb{N}' = \mathbb{N} - \{0\}$, and the sequence $g \colon \mathbb{N} \to \mathbf{Q}$, the set of rational numbers, where $g(n) = 1/10^n$.

Definition 1.1. For fixed even $K > 2$ and each $n \in \mathbb{N}'$, consider the sets of $K+2$-tuples $C_n = \{(h(j), 1, -1/10^n, 1/10^n, \ldots, -1/10^n, 1/10^n) \mid (j \in \mathbb{N})\}$.

From definition 1.1, via *-transform, we have for an $n \in {}^*\mathbb{N}$, the internal set

$$^*C_n = \{({}^*h(j), 1, -1/10^n, 1/10^n, \ldots, -1/10^n, 1/10^n) \mid (j \in {}^*\mathbb{N})\}.$$

Let $\omega \in {}^*\mathbb{N}' - \mathbb{N}' = \mathbb{N}_\infty$.

Definition 1.2. Ultra-propertons. The set of all ultra-propertons is (represented by) the internal $^*C_\omega = C = \{({}^*h(j), 1, -1/10^\omega, 1/10^\omega, \ldots, -1/10^\omega, 1/10^\omega) \mid (j \in {}^*\mathbb{N}')\}$. (Note: A coordinate can represent a numerically coded non-numerical

descriptive physical characteristic. When a properton process is applied to one collection of these special properton coordinates, then an intermediate properton with the descriptive characteristic is obtained. This special form is distinct from that used in Herrmann (1979-1993).)

For Definition 1.2, it is assumed that there is no more than K physical or physical-like numerical or coded descriptive characteristics for the any elementary entity.

Theorem 1.1. *Consider any nonempty internal $D \notin {}^*\mathbb{X}_0$ and $A \subset D$ such that $|A| < |\mathcal{M}_1|^+$. Then there exists a hyperfinite B_A such that $A \subset B_A \subset D$.*

Proof. Let $\mathcal{F}$ be the finite power set operator. That is for any set X, $\mathcal{F}(X)$ is the set of all finite subsets of X, where a set Y is finite if it is empty or there exists an $n \in \mathbb{N}' = \mathbb{N} - \{0\}$ and a bijection $f': [1, n] \to Y$.

Consider the internal binary relation $C = \{(x, y) \mid (x \in y) \wedge (x \in D) \wedge (y \in {}^*\mathcal{F}(D))\}$. Let $\{(x_1, y_1), \ldots, (x_m, y_m)\} \subset C$. Then $y' = y_1 \cup \cdots \cup y_m$ is an internal subset of ${}^*\mathcal{F}(D)$. Since the domain of C is D, $A \subset D$ and $|A| < |\mathcal{M}_1|^+$, then by saturation there exists a $B_A \in {}^*\mathcal{F}(D)$ such that $A \subset B_A \in {}^*\mathcal{F}(D)$ and $B_A \subset D$. This complete the proof. ∎

Corollary 1.1.1. *Consider any ${}^*E$ and $A \subset {}^*E$ such that $|A| < |\mathcal{M}_1|^+$. Then there exists a hyperfinite B_A such that $A \subset B_A \subset {}^*E$.*

Theorem 1.2. *Consider any nonempty hyperfinite $A \subset {}^*\mathbb{N}'$. Then there exists a $\gamma \in {}^*\mathbb{N}$ such that $A \subset [1, \gamma]$.*

Proof. Every nonempty finite subset F of $\mathbb{N}'$ has a greatest member $M_F \in \mathbb{N}'$. That is if $x \in F$, then $x \in [1, M_F]$. By *transfer, A has a *greatest member $\gamma \in {}^*\mathbb{N}'$ such that if $x \in A$, then $x \in [1, \gamma]$ ∎

In the Theory of Ultralogics (Herrmann (1979-1993)), the basic applications use $\mathbb{N}$ as a set of individuals. Due to the inclusion of propertons, it was necessary to extend the ground set by replacing $\mathbb{N}$ with $\mathbb{R}$. However, depending upon how physical characteristics are numerically measured, as mentioned, it is usually sufficient to replace the real numbers with the rational numbers $\mathbf{Q}$ as ground set individuals. Further, if $\mathcal{E}'$ is not employed, then it is necessary to numerically encode each of the non-numerically presented descriptive physical characteristics.

Let $r_1 \in \mathbb{R}$. By Theorem 11.1.1 in Herrmann (1979-93), there is a $\lambda_1 \in \mathbb{N}_\infty$ such that $\lambda_1/10^\omega \in \mu(|r|)$. Hence, $\mathtt{st}((\lambda_1/10^\omega)) = |r|$. Then there are K, λ_i, $i \in [1, K]$ that yield the K characteristics. For an elementary physical entity e_j, some characteristics can be 0, meaning that the measure has value 0. Throughout the combining processes, if a coordinate retains its infinitesimal value $\pm 1/10^\omega$, this indicates that the characteristic

has no meaning for e_j. In order to indicate these differences, any characteristic that has measure 0 is obtained from a combination of two ultra-propertons. The standard part (physical realization) operator St is only applied to coordinates of the intermediate properton representations with the form $\pm\lambda/10^\omega$, where $\lambda \geq 2$.

There are other characteristics such as spin, where the 0 takes on a different meaning. However, such coding is rather arbitrary and can be replaced with non-zero numbers or non-zero codings for the characteristics so as to not confuse them with a 0 measurement. For the needed intermediate properton, with a third coordinate characteristic under independent coordinate addition, the hyperfinite set of ultra-propertons $\{(\,^*h(j), 1, -1/10^\omega, \ldots, 1/10^\omega) \mid j \in [1, \lambda_1]\}$ is employed. Hence, the first intermediate properton is $(\Pi_{j=1}^{\lambda_1}\,^*h(j), \lambda_1, -\lambda_1/10^\omega, 1/10^\omega, \ldots, 1/10^\omega)$. For a forth coordinate intermediate properton for value r_2, consider $\{(\,^*h(j), \lambda_2, -1/10^\omega, \lambda_2/10^\omega, \ldots, 1/10^\omega) \mid j \in [\lambda_1 + 1, \lambda_1 + \lambda_2]\}$. This yields $(\Pi_{j=\lambda_1+1}^{\lambda_1+\lambda_2}\,^*h(j), \lambda_2, 1/10^\omega, \lambda_2/10^\omega, \ldots, 1/10^\omega)$. Continue these definition for each member of $[1, K]$. Thus, for the entire collection of ultra-propertons used to obtain e_1, one member of the set of elementary entities $\{e_j\}$, let $\lambda_1 + \cdots + \lambda_K = \delta_1 \in \mathbb{N}_\infty$. There is an injective correspondence, $p_1 \colon [1, \delta_1] \to C$, which, as indicated, is between $[1, \delta_1]$ and the set of all ultra-propertons C.

It is assumed that the collection $\{e_j\}$ of all elementary particles is nonempty and countable (i.e non-zero finite or denumerable). For the case that $|\{e_i\}| = n \geq 1$, by finite construction and presentation, there is the set of intervals $\{[\delta_{i-1} + 1, \delta_i] \mid 1 \leq i \leq n\}$, $n \in \mathbb{N}'$, $\delta_i \in \mathbb{N}_\infty$, $(\delta_0 = 0)$ such that if $j \neq k, 1 \leq j, k \leq n$, then $[\delta_{j-1} + 1, \delta_j] \cap [\delta_{k-1} + 1, \delta_k] = \emptyset$. Further, for each $j, k \in \mathbb{N}$, $1 \leq j, k \leq n$, $p_j[[\delta_{j-1} + 1, \delta_j]] \cap p_k[[\delta_{k-1} + 1, \delta_k]] = \emptyset$. Hence, there is the corresponding injection $\bigcup\{p_i \mid 1 \leq i \leq n\} = P_{\delta_n}$ such that $P_{\delta_n} \colon [1, \delta_n] \to C$.

If $\{e_i\}$ is denumerable, then, by induction, there is a denumerable set $\{\delta_i\}$, and the set of intervals $\{[\delta_{i-1}, \delta_i] \mid i \in \mathbb{N}'\}$ has the same property as in the finite case. Further, relative to common entities, the same additional properties hold. There is also an injection $P_\infty = \bigcup\{p_i \mid 1 \leq i \in \mathbb{N}'\}$ such that $P_\infty[\bigcup\{p_i \mid 1 \leq i \in \mathbb{N}'\}] \subset C$.

For the denumerable case, let $\Pi = \{(x, y) \mid (x < y) \wedge (x \in \,^*\mathbb{N}) \wedge (y \in \,^*\mathbb{N})\}$. This is an internal binary relation, which is well know to be concurrent. Consider the external denumerable set $\{\delta_i \mid 1 \leq i \in \mathbb{N}\}$. Then $\{\delta_i \mid 1 \leq i \in \mathbb{N}\}$ is a subset of the domain of Π. From $|\mathcal{M}_1|^+$-saturation, since $|\,^\sigma\mathbb{N}'| = |\mathbb{N}'|$, then there exists a $\Gamma \in \,^*\mathbb{N}$ such that $\delta_i < \Gamma$ for each $i \in \mathbb{N}$. Since there is a bijection from $[1, \gamma]$ into C, then there are "more than enough" ultra-propertons for universe generation. However, in general, elementary particles are not necessary to generate a physical universe.

For this application, it appears unnecessary to consider more than H, where $1 \leq H \in \mathbb{N}$, different types of elementary entities. The set of ultra-propertons

$\{(\,^*h(j), 1, -1/10^\omega, \ldots, 1/10^\omega) \mid j \in \,^*\mathbb{N}'\} = C$ is an internal set and as such the hyperfinite operator $^*\mathcal{F}$ is defined for it. For properton generation, a universe can be considered as a collection of physical-systems. Hence application of a hyperfinite iteration (> 0) of the hyperfinite powerset $^*\mathcal{F}^i$ to C yields $\mathcal{C} = \bigcup\{\,^*\mathcal{F}^i(C) \mid (1 \le i \le n)\}$, for an appropriate $n \in \,^*\mathbb{N}$, an internal collection that is sufficient to generate the physical-systems for any of the presently considered cosmologies. To accommodate the formation of the physical-like systems, infinite hyperfinite set X of internal sets that is disjoint from $\mathcal{C}$ is adjoined to $\mathcal{C}$. Then $\Pi^+ = \mathcal{C} \cup X$.

Recall that, for each (i, j) in primitive time (recently re-termed as a "primitive sequence"), $\mathrm{t}^q(i, j)$ is the rational number that identifies the actual moment when a description is realized and, hence, identifies a particular general description F^q for a developmental paradigm. The $\mathrm{f}^q = \mathrm{F}^q \circ \mathrm{t}^q$, where t^q is the primitive time double sequence and q varies from 1 to 4; the designations for the four types of "time" developments. Each $\mathrm{f}^q(i, j) \in \mathrm{W}'$ is a general description. Rather than the f^q being a general description, they are replaced by instructions or rules $\mathrm{f}^q(i, j) = \mathrm{I}^q \circ \mathrm{t}^q(i, j)$ - a nonempty finite subset of W', which is equivalent to a single word in W'.

These sets of instructions - instruction-entities - (also called instruction-information) are indexed in the same way as the general descriptions and determine the *instruction paradigm* $\mathcal{I}_q$. Thus, there is only a difference in the descriptive statements employed. In general, if not otherwise denoted as $\mathrm{I}^q(i, j)$, then for this paper the meaning of the $\mathrm{f}^q(i, j)$ is contextually controlled. There is one instruction paradigm for each universe and there can be a vast collection of such universes.

As symbol strings, consider the set

$$\dagger\{\text{There}|||\text{are}|||\text{n}|||\text{ultra}|||\text{propertons}|||\text{combined}|||\text{to}|||$$

$$\text{produce}|||\text{an}|||\text{intermediate}|||\text{properton.} \mid \mathrm{n} \in \mathbb{N}\} = \mathrm{R.} \text{ Then}$$

using the single member equivalence class for an alphabet symbol with the monoid operator symbol suppressed this yields

$$\dagger\{\textbf{There}|||\textbf{are}|||\textbf{n}|||\textbf{ultra}|||\textbf{propertons}|||\textbf{combined}|||\textbf{to}|||$$

$$\textbf{produce}|||\textbf{an}|||\textbf{intermediate}|||\textbf{properton.} \mid n \in \mathbb{N}\} = \mathbf{R}$$

is a member of $\mathcal{N}$, where $\mathcal{M} = \langle \mathcal{N}, \in, = \rangle$. Indeed, there is some X_n such that standard $\mathbf{R} \in \mathrm{X}_n$ and $\mathbf{R} \subset \mathrm{X}_{n+1}$. But the terms "ultra-properton" and "properton" have no physical meanings within the physical world. Using the method of Theorem 9.3.1 in Herrmann (1979-93) and *-transform, this set corresponds to $^*\mathbf{R} \subset \,^*\mathrm{X}_{n+1}$.

Symbolically, let $\lambda \in \mathbb{N}_\infty$. This symbol λ represents a member of the hyper-language $^*\underline{\mathcal{W}}'$, since, symbolically, $\mathbb{N} \subset W'$. The $\lambda = [f] \in \,^*\underline{\mathcal{W}}'$. Then considering the inverses this yields the intuitive

‡There|||are|||λ|||ultra|||propertons|||combined|||to|||

produce|||an|||intermediate|||properton.

Such ‡ *instructions have "interpretations" in terms of the GGU-model language. There can be additional objects that behavior like symbol strings taken from $^*\underline{\mathcal{W}}'$ that, via the consistent interpretation, have no meanings until interpreted for the GGU-model.

Quantum Theory and Propertons

Any of the entities used in quantum physics can be produce by info-fields and appropriate physical-systems. Quantum physical entities that the positivists consider as imaginary, need not be imaginary.

This refinement is relative to the production of the physical-systems that yield the info-fields. The notion of what constitutes the intertwining of physical-systems can include the notion of related physical-theory interacting entities. How one describes such relations is not revehent to their properton formation that produce the info-fields. Such relations are exhibited when a universe develops.

Each physical-system can be considered as a physical-like system since their construction can include *instructions for, at present, unknowable processes or "things," as represented by the X, that seem to "force" these combinations to occur. These, at present, unknowable processes or "things" are modeled by the "gathering operator." Although instructions are modeled by a language when in standard form, these non-physical processes or "things" are distinct from the language itself. Notice that symbol-strings, diagrams, images and sensor information represented by an informal language W' are comprehensible only when they carry an additional component - meanings. "Meanings" are understood by the mind and cannot lead to mere circular thinking.

$\Diamond$ For the GGU-model, a specific form of symbolic expression was, in 1979, incorporated within each linguistic expression being modeled (Herrmann (1979-1993, Section 7.1).) **This additional aspect identifies the expression as being related to a specific moment during a development. Obviously, as the primitive time (the**

basic sequence) for such linguistic expressions is refined, the refined symbolic representation is considered attached as a more refined identifying symbolic representation for the development being represented by the language element. These additional sequence representations are also, obviously, directly related to the mathematically described notions of "order." In all cases, this special form of linguistic expression is maintained. When this feature needs to be recalled the symbol $\Diamond$ is appended.

For human endeavors, there is a five-step process. (1) An informal meaningful instruction-entity is given. (2) The instructions are mentally comprehended. (3) This mental comprehension is transmitted, via electro-chemical actions, to other human physical locations. (4) At these physical locations actions are performed. (5) These actions produce a physical entity that corresponds to the original instruction-entity. Errors can occur along this entire path. If these processes are performed in an errorless manner and the physical entity produced does not correspond to a desired physical object, then this alone cannot change the instruction-entity. The instruction-entity (1) would need to be altered in a meaningful way as required by (2). In this illustration, alteration is done by an intelligent physical entity. For the GGU-model, a standard meaningful instruction-entity contains operative statements.

From the modeling viewpoint, processes within the substratum are non-physical. Thus, step (1) is non-physical and steps (2) and (3) are considered as statements relative to a non-physical medium and a non-physical mode of transmission, respectively. For each universe-wide frozen-frame, step (4) corresponds to a subsets of $\mathcal{C}$ - the substratum info-fields. Step (5) is the application of the `St` operator and yields that physical products of the entire process. In order to be operative, the above displayed *instruction as well as similar ones have non-physical components.

The Eccles and Robinson (1984, p. 172) notion that there is empirical evidence for an immaterial medium and corresponding processes that influence physical brain activity can be employed. Additional comprehension is aided by this "medium" and it has been modeled via ultra-logic-systems (Herrmann, 2006b). In particular, there is a binary ultra-logic-system that rationally establishes the notion of creative mental activity as a product of a non-physical medium and processes. For specific information and the generation of a GGU-model universe, this corresponds to step (2). In general, the terms "non-physical" and immaterial, as used here, indicate that it is not part of a physical universe. But, interaction with a physical universe occurs.

2. Logic-System Generation for the Type-1 Interval.

[NOTE: For all GGU-model applications as originally presented in Herrmann (1979 - 1994), the developmental paradigm determining functions f and t, as discussed below, are defined on $Z \times \mathbb{N}$ and then the q notion, where $q = 1, 2, 3$ indicates a restriction of these functions to $Z_q \times \mathbb{N}$. For $q = 4$, the indicated functions are the original unrestricted ones. For the t function, the image is $R \subset \mathbf{Q}$. Then t^q is the appropriate restriction. Hence, the **I** domain is R and maps R into the language W'.]

In Herrmann (2006), there are two different t sequence notations. One t is in the informal world, while another t is in the formal standard superstructure. This is no longer necessary since the Robinson approach is now part of our standard structure. The term informal is often used when members of W' are being considered. The term standard, if used, usually refers to the corresponding equivalences. The informal composition $f^q = I^q \circ t^q$, when embedded relative to $\underline{\mathcal{W}}'$, is denoted by $\mathbf{f}^q = \mathbf{I}^q \circ t^q$. Each $t^q(i, j)$ is a rational number. Each $f^q(i, j)$ is an identified nonempty instruction-entity.

Each member of $\mathcal{I}_q$ is now considered as determined by a function defined on a set R_q of rational numbers, $\mathbf{Q}$. The members of R_q carry the rational number simple order and the order $\leq_{\mathcal{I}^q}$ for the members of $\mathcal{I}_q$ (the lexicographic order) is order isomorphic to R_q in the obvious way. Each interval partition is of the form (c_i, c_{i+1}) (with a closed interval in two cases), where $i \in \mathbf{Z}$ and $\mathbf{Z}$ is the set of integers, and $t^q(i, 0) = c_i$, $t^q(i + 1, 0) = c_{i+1}$. Then each member of (c_i, c_{i+1}) is a defined rational number $t^q(i, j)$ A fixed $K' \in \mathbb{N}'$ is selected. Then, for example, for $[0, c_1), [c_1, c_2), [c_2, c_3), \ldots$, let $0 \leq i \in \mathbf{Z}$ and $j \in \mathbb{N}$, then $t^q(i, j) = (1/K')(i + 1 - 1/2^j)$ and R_q is the appropriate q restriction of $\{x \mid (x = (1/K')(i + 1 - 1/2^j)) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbb{N})\}$. Hence, $[0, c_1), [c_1, c_2), [c_2, c_3), \ldots = [0, 1/K'), [1/K', 2/K'), [2/K', 3/K'), \ldots$. For a given q, the lexicographic order $\preceq$ on $\{(i, j)\}$ is also q restricted. For R_q this yields a bijection on the q restricted $\{(i, j)\}_q$ onto R_q such that, for each $(x, y), (z, w) \in \{(i, j)\}_q$, $(x, y) \preceq (z, w)$ if and only if $t^q(x, y) \leq t^q(z, w)$. The order $\leq_{\underline{\mathcal{I}}^q}$ on an instruction paradigm $\mathcal{I}_q$ is defined by $f^q(x, y) \leq_{\underline{\mathcal{I}}^q} f^q(z, w)$ if and only if $t^q(x, y) \leq t^q(z, w)$.

Let $\mathcal{I}_1$ be the standard instruction paradigm. An instruction paradigm is defined mathematically in the exact same manner as that of the developmental paradigm in Herrmann (2006) and is equivalent to the range of a sequence $g' \colon \mathbb{N} \to \mathcal{P}(W')$, where W' is our denumerable general language. The first case illustrated for the GGU-model is for a developing universe starting with a frozen segments (frame) instruction-entity $g'(0)$. For the other three GGU-model cases, this sequence is appropriately modified. In all cases, the $(f^q(i, j), f^q(p, k))$ is equivalent to "If $f^q(i, j)$, then $f^q(p, k))$. This notation will be simplified later.

For the type-1 case $[0, b]$, $b > 0$, as indicated above, a denumerable instruction paradigm displays a refined form. For $1 < m \in \mathbf{Z}$, $\mathcal{I}_1 = \{f^1(i, j) \mid (0 \leq i \leq m) \wedge (i \in$

$\mathbf{Z}) \wedge (j \in \mathbb{N})\}$.

Due to the simplicity and special nature of the logic-systems used, a simplified algorithm is employed. The basic logic-system algorithm is re-defined for sets of two distinct objects $\{A, B\}$. If a deduction yields C and C is a member of $\{A, B\}$, then the "other" member is a deduction. Hence, if A is deduced, then from $\{A, B\}$, B is deduced. This can be written as $\{A, B\} - \{A\}$ is deduced. In general, to avoid repetition, once such a two element set is employed for a deduction it is not used again. This approach is only valid for these special collections of two element sets. Binary relations can be used as logic-systems. This special algorithm for deduction mimics the propositional-logic modus ponens rule of inference for hypotheses A, and $A \to B$. The rule is expressed as $\{(X \to Y, X, Y) \mid X, Y \text{ are propositions}\}$. Using $\mathcal{I}_1$, consider the following logic-system.

Definition 2.1 Let $i \in \mathbf{Z}$ and $0 < m \in \mathbb{N}$. For each $n \in \mathbb{N}$, let $k_i^1(n) = \{\{f^1(i,j), f^1(i, j+1)\} \mid (0 \le j \le n-1) \wedge (j \in \mathbb{N})\}$, $K^1(m, n) = \bigcup\{k_i^1(n) \mid (0 \le i < m) \wedge (i \in \mathbf{Z})\}$. Finally, let finite $\Lambda^1(m, n) = \{f^1(0,0)\} \cup K^1(m, n) \cup \{\{f^1(p-1, n), f^1(p, 0)\} \mid (0 < p \le m) \wedge (p \in \mathbf{Z})\}$ and $\mathcal{L}^1 = \{\Lambda^1(m, x) \mid x \in \mathbb{N}\}$. The set $\{\{f^1(p-1, n), f^1(p, 0)\} \mid (0 < p \le m) \wedge (p \in \mathbf{Z})\}$ is called the "jump elements." Also, each $\Lambda^1(m, n)$ is a distinct finite set due to the specific identifying W$'$ language symbols employed.

In general, members in $\mathcal{L}^q$ can be characterized by a first-order sentence. When the deduction algorithm is applied to $\Lambda^1(m, n)$ the result is an ordered set of words from W$'$ - the ordered instruction paradigm. In accordance with the juxtaposition join operator that yields words in W$'$, this ordered instruction paradigm is a word in W$'$. It can be obtained using the spacing symbol where each member of this paradigm is considered a sentence. For a multi-universe cosmology, each such universe is a portion of each of the original members of the instruction paradigm.

In order to make the notation as simple as possible for the next construction, notice that $\mathcal{L}^1$ is denumerable. Let $\mathbb{N} - \{0\} = \mathbb{N}'$. Thus, there is a bijection $D^1 \colon \mathbb{N}' \to \mathcal{L}^1$. We use the subscript notation for this bijection. Thus, consider $\mathcal{L}^1 = \{D_i^1 \mid i \in \mathbb{N}'\}$. For each $n \in \mathbb{N}'$, define $M_n^1 = \{\{D_1^1, \ldots, D_n^1\}\}$. Let $\mathcal{M}^1 = \{M_n^1 \mid n \in \mathbb{N}'\}$. The set $M_n^1 = \{\{D_1^1, \ldots, D_n^1\}\}$, as before, can be considered as a single word-like object.

(There are a few typographic errors in Herrmann (2006). For example, in Herrmann (2006) Theorem 4.1, $m > 0$ should read $m > 1$, and $^*\mathbf{D}$, should read $^*\mathbf{D}_1$. These typographical error may be corrected in a future version.)

A finite consequence operator S is defined in Herrmann (1979 - 1993, p. 70; 65). However, new simplified logic-systems $\mathcal{S}_n^q$, $q = 1, 2, 3, 4$ are defined. When a logic-system is applied, it generates a specific finite consequence operator. It is the

logic-system algorithm that does this. In this article, this algorithm is explicitly noted since only logic-systems are used. In general, logic-systems are stated in terms of metamathematics n-tuples. If a set $\{A, B, C, \ldots, D\}$ is used as an hypothesis, then it is word-like since the objects the logical deduction models via the algorithm yields words or word-like objects.

What follows, through and including section 5, is for the Multi-Complexity GGU-model. Define $\mathcal{M}^q$, $q = 2, 3, 4$, in the same manner as $\mathcal{M}^1$, from members of $\mathcal{L}^q$. For each $G^q \in \mathcal{M}^q$, there exists a unique $n \in \mathbb{N}'$ such that $G^q \in M_n^q$. This $G^q = \{D_1^q, \ldots, D_n^q\}$, $D_i^q \in \mathcal{L}^q$, $1 \leq i \leq n$. (As I continue to mention, each D_i^q corresponds to a $\mathbf{D}_i^q$ a member of $\underline{\mathcal{W}}'$. Also, the usual standard deduction algorithm is applied to the member $D_1^q \wedge, \ldots, \wedge D_n^q$ (or some other language form with & or $|||$ taking the place of $\wedge$), which also corresponds to a member of $\underline{\mathcal{W}}'$.

For each $n \in \mathbb{N}'$, let $\mathcal{S}_n^q = \{\{x, y\} \mid (x \in M_n^q) \wedge (y \in \mathcal{L}^q) \wedge (y \in x)\}$. Then let $\mathcal{S}^q = \{\mathcal{S}_n^q \mid n \in \mathbb{N}'\}$. (This definition can be further described in order to characterize the doubleton set notion and can include all necessary bounds for the quantifiers.) Each member of M_n^q is directly related to a corresponding $\mathcal{S}_n^q$. Further, under the simplification used here, each member of an $\mathcal{S}_n^q$ is a propositional tautology. Notice that M^q is a function with values a singleton set containing an n-set (i.e. a set of "n" members).

Usually, such a logic-system would use ordered pairs to model the rules of inference. Within these rules, finite conjunctions are displayed as first coordinates via n-sets. Again the simplified doubleton-set approach is used here, where one of these sets is $\{\{D_1\}, D_1\}$.

Hypotheses are considered as members of a subset of W' (a unary relation), when part of a logic-system. They are, usually, considered as a list of the members of this set. In general, a logic-system, when considered as an operator, is defined on subsets of the language employed.

From the definitions employed for the logic-systems used here, the properties of the logic-system algorithm $\mathcal{A}$ can be explicitly described in set-theoretic notation. For these applications, $\mathcal{A}$ is a function defined on various defined logic-systems and a set of hypotheses. For example, the entire set of deductions or the order in which the deductions are made, among a few other characteristics. In our application to a logic-system, the notation used signifies all of the ordered deductions the algorithm produces when the logic-system is applied to a set of hypotheses. This yields the same results as a corresponding finite consequence operator. What the notation indicates is that the finite consequence operator is being displayed in a more refined and explicit manner. Hence, the algorithm and its relation to the logic-system can be embedded into the

formal structure via formalizable characteristics.

When the application characteristics are *-transferred, then the notation $^*\underline{\mathcal{A}}$ is employed. The process of applying the algorithm to each $\mathcal{S}_n^q$, that is applying it to a set of hypotheses Y, is denoted by $\mathcal{A}((\mathcal{S}_n^q, \mathrm{Y}))$. (Note: Although the algorithm is actually a sequence of algorithms, in each case, the process itself when described in a first-order statement is independent from n. Notationally, the algorithm is not denoted as a sequence.) Hence, $\mathcal{A}$ is defined upon a set of ordered pairs. The result of $\mathcal{A}((\mathcal{S}_n^q, \mathrm{Y}))$ is a set. An additional step can be included for this specific algorithm, where Y is removed as a deductive conclusion. When this is done the algorithm is denoted by $\mathcal{A}'$. The necessary informally and, hence, formally described properties are specifically displayed. In general, the q notion is not included as part of the $\mathcal{A}$ notation unless confusion would result.

For the denumerable set $\mathcal{L}^1$, notice that for any $\Lambda^1(m, k)$, $k \in \mathbb{N}$ there exists an $k' \in \mathbb{N}$ and $\mathrm{X}_{k'}^1 \in \mathrm{M}_{k'}^1$, such that $\Lambda^1(m, k) \in \mathcal{A}'((\mathcal{S}_{k'}^1, \{\mathrm{X}_{k'}^1\}))$ and, in this case, finite choice yields the $\Lambda^1(m, k)$ logic-system. Notice that the logic-system $\Lambda^1(m, k)$ is considered as a set-theoretic object. Then the logic-system algorithm $\mathcal{A}$ is applied to $(\Lambda^1(m, k), \{\mathrm{f}^1(0, 0)\})$, where $\mathrm{f}^1(0, 0)$ is the only hypothesis contained in the logic-system. This yields $\mathrm{f}^1(i, j) \in \mathcal{I}_1$ as a deduction from $\mathrm{f}^1(0, 0)$. Conversely, if $\mathrm{f}^1(i, j) \in \mathcal{I}_1$, then there is an $\mathrm{X}_{k'}^1 \in \mathrm{M}_{k'}^1$ and a logic-system $\Lambda^1(m, k) \in \mathcal{A}'(\mathcal{S}_{k'}^1, \{\mathrm{X}_{k'}^1\})$ such that application of the logic-system algorithm $\mathcal{A}$ to $(\Lambda^1(m, k), \{\mathrm{f}^1(0, 0)\})$ yields $\mathrm{f}^1(i, j)$ as a deduction from $\mathrm{f}^1(0, 0)$.

The informal algorithm $\mathcal{A}$ is defined on any logic-system that contains an hypothesis and, in this paper, such a logic-system is $\Lambda^q(x)$ and application is on $(\Lambda^q(x), \mathrm{Y})$ where Y contains the hypotheses in the logic-system and, in this case, it contains but one member. Due to the construction of the $\Lambda^q(x)$, this yields a partial sequence of members of $\mathcal{I}_q$. This sequence and any other sequence in this article that represents steps in such a deduction is denoted by $[\![\mathcal{A}(\Lambda^q(x), \mathrm{Y})]\!]$. These sequences satisfy orders of the type $\leq_{\mathcal{I}_x^q}$. **It is the rational process being displayed upon application of $\mathcal{A}$ that models the process of ordering descriptive entities.** Also, for this case, $\mathcal{A}((\Lambda^q(x), \mathrm{Y})) = \mathcal{I}_x^q \subset \mathcal{I}_q$. Significantly, for $n, k \in \mathbb{N}$, $n \leq k$, $\mathcal{A}((\Lambda^1(m, n), \mathrm{Y})) \subset \mathcal{A}((\Lambda^1(m, k), \mathrm{Y}))$ and $[\![\mathcal{A}[(\Lambda^1(m, k), \mathrm{Y})]\!]|[1, n] = [\![\mathcal{A}(\Lambda^1(m, n), \mathrm{Y})]\!]$. [Note: Herrmann (2013b) such sequences are denoted by $\mathcal{A}[(\Lambda^q(x), \mathrm{Y})]$.

Further, the (intuitive) dictionary definition the word "sequence" is "(1) The coming of one thing after another, succession. (2) The order in which this occurs." However, in much of mathematics, the definition is a restriction of this intuitive notion. In order to retain the intuitive notion, throughout my writings for the GD, GGU and GID models, I may use this term in a generalization form. A sequence for me need not be

just a function defined on the natural numbers. In many cases, it "produces" things. An "ordered" set D is given and this set need not be the set of counting (or natural) numbers. That is, D may have a simple order defined on it, like the one for the integers. This ordered set is always stated and understood if this generalization is employ. *The successive order of what is produced is reflected by the order on the domain, not by any order that may be perceived for the set of produced results.* The term "net" is not applied to this generalized notion.]

In the usual way, all of the above informally defined objects are embedded relative to $\underline{\mathcal{W}}'$. When the informal set-theoretic expressions are considered as embedded into the standard superstructure, all of the bold font conventions defined in Herrmann (1979-1993) are observed. All other embedded symbols retain their math-italics form. Where script notation is used, an underline is used in place of the bold face font. All the following results are relative to our nonstandard model $^*\mathcal{M}$ or $^*\mathcal{M}_1$ (Herrmann, (1979 - 1993)).

Theorem 2.1 *Consider primitive time interval* $1 = [0,b], b > 0$. *It can always be assumed that interval 1 is partitioned into two or more intervals* $[c_0, c_1), \ldots$ $[c_{m-1}, c_m]$, $c_m = b$, $m > 1$, $m \in \mathbf{Z}$. *Let* $\underline{\mathcal{I}}_1$ *be an instruction paradigm order isomorphic to the rational numbers* $R_1 \subset [0,b]$. *For any* $\lambda \in \mathbb{N}_\infty$, *there exists a unique hyperfinite* $^*\mathbf{\Lambda}^1(m, \lambda) \in {}^*\underline{\mathcal{L}}^1$ *and a* $\lambda' \in {}^*\mathbb{N}'$ *such that the ultra-word-like* $X^1_{\lambda'} \in {}^*\mathbf{M}^1_{\lambda'}$, *and ultra-logic-system* $^*\mathbf{\Lambda}^1(m, \lambda) \in {}^*\underline{\mathcal{A}}'(({}^*\underline{\mathcal{S}}^1_{\lambda'}, \{X^1_{\lambda'}\}))$ *and* $^\sigma\underline{\mathcal{I}}_1 \subset$ $^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(m, \lambda), \{{}^*\mathbf{f}^1(0,0)\})) = \mathcal{I}^1_{m,\lambda} \subset {}^*\underline{\mathcal{I}}_1$. *Also the* $[\![{}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(m, \lambda), \{{}^*\mathbf{f}^1(0,0)\}))]\!]$ *steps satisfy the* $\leq_{\mathcal{I}^1_{m,\lambda}}$ *order and* $({}^*\underline{\mathcal{I}}_1 - {}^\sigma\underline{\mathcal{I}}_1) \cap {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(m, \lambda), \{{}^*\mathbf{f}^1(0,0)\})) = an$ *infinite set.*

Proof. This follows in the same manner as Theorem 4.1 in Herrmann (2006) by *-transfer of the appropriate first-order statements that precede this theorem statement. Also note that since for every $n \in \mathbb{N}'$, the $\Lambda^1(m, n)$ is finite, then, via the identification process, $^\sigma\mathbf{\Lambda}^1(m, n) = \mathbf{\Lambda}^1(m, n)$. It also follows that $^*\mathbf{\Lambda}^1(m, n) = \mathbf{\Lambda}^1(m, n)$ under the customary conventions. Since for any n, $k \in \mathbb{N}'$, $n \leq k$, $\underline{\mathcal{A}}((\mathbf{\Lambda}^1(m, n), \{\mathbf{f}^1(0,0)\})) \subset \underline{\mathcal{A}}((\mathbf{\Lambda}^1(m, k), \{\mathbf{f}^1(0,0)\}))$, from the above and, via *-transfer, it follows that $^\sigma\underline{\mathcal{I}}_1 \subset {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(m, \lambda), \{{}^*\mathbf{f}^1(0,0)\})) = \mathcal{I}^1_{m,\lambda} \subset {}^*\underline{\mathcal{I}}_1$. From the definition of $\Lambda^1(m, n)$, these steps numbers are order isomorphic the set of rational numbers R_1. Hence, $^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(m, \lambda), \{{}^*\mathbf{f}^1(0,0)\}))$ is *order isomorphic to a hyperfinite subset of $^*\mathbf{Q}$. Since there are infinitely many $i < \lambda$ and $i \in \mathbb{N}_\infty$, there are infinitely many $^*\mathbf{f}(i, j) \in {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^1(m, \lambda), \{{}^*\mathbf{f}^1(0,0)\})) \subset {}^*\underline{\mathcal{I}}_1$, where $^*\mathbf{f}(i, j) \in {}^*\underline{\mathcal{I}}_1 - {}^\sigma\underline{\mathcal{I}}_1$. These are interpreted as ultranatural events but in some cases may differ from physical events only in their primitive time identifications. This completes the proof. ∎

By considering the definition of $\mathcal{L}^1$, it follows that the given $1 < m \in \mathbb{N}$, $^*\mathbf{\Lambda}^1(m, \lambda)$

is precisely $\{\,{}^*\mathbf{f}^1(0,0)\} \cup \{\bigcup\{\,{}^*\mathbf{k}_i^1(\lambda) \mid (0 \leq i < m) \wedge (i \in {}^*\mathbf{Z})\}\} \cup \{\{\,{}^*\mathbf{f}^1(p-1,\lambda), {}^*\mathbf{f}^1(p,0)\} \mid (0 < p \leq m) \wedge (p \in {}^*\mathbf{Z})\}$. Of significance is the fact that the steps in the *-deduction ${}^*\underline{\mathcal{A}}((\,{}^*\mathbf{\Lambda}^1(m,\lambda), \{\,{}^*\mathbf{f}^1(0,0)\}))$ preserve the order $\leq_{\,{}^*\underline{\mathcal{I}}_1}$. Notice that ${}^*\mathbf{\Lambda}^1(m,\lambda)$ is obtained by hyperfinite choice. Further, any ${}^*\mathbf{f}^1(i,j) \in \{\,{}^*\mathbf{f}^1(x,y) \mid (0 \leq x < m) \wedge (0 \leq y \leq \lambda) \wedge (x \in {}^*\mathbf{Z}) \wedge (y \in {}^*\mathbf{N})\} \cup \{\,{}^*\mathbf{f}^1(m,0)\}$ is a hyperfinite *-deduction from ${}^*\mathbf{f}^1(0,0)$. And, it also follows that the set of all such *deductions yields a hyperfinite set $\mathcal{I}_\lambda^1$ such that ${}^\sigma\underline{\mathcal{I}}_1 \subset \mathcal{I}_\lambda^1 \subset {}^*\underline{\mathcal{I}}_1$.

3. Logic-System Generation for the Type-2 Interval

For the type-2 case $[0, +\infty)$, a denumerable instruction paradigm displays a refined form. For this case, $\mathcal{I}_2 = \{\mathbf{f}^2(i,j) \mid (0 \leq i) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbf{N})$. Using $\mathcal{I}_2$, consider the following logic-system.

Definition 3.1 Let $0 \leq i \in \mathbf{Z}$. For each $n \in \mathbf{N}$, let $\mathrm{k}_i^2(n) = \{\{\mathrm{f}^2(i,j), \mathrm{f}^2(i,j+1)\} \mid (0 \leq j \leq n-1) \wedge (j \in \mathbf{N})\}$. For $0 < m \in \mathbf{Z}$, let $\mathrm{K}^2(m,n) = \bigcup\{\mathrm{k}_i^2(n) \mid (0 \leq i < m) \wedge (i \in \mathbf{Z})\}$. Finally, let $\Lambda^2(m,n) = \{\mathrm{f}^2(0,0)\} \cup \mathrm{K}^2(m,n) \cup \{\{\mathrm{f}^2(p-1,n), \mathrm{f}^2(p,0)\} \mid (0 < p \leq m) \wedge (p \in \mathbf{Z})\} \cup \{\{\mathrm{f}^2(m,j), \mathrm{f}^2(m,j+1)\} \mid (0 \leq j < n) \wedge (j \in \mathbf{N})\}$, and $\mathcal{L}^2 = \{\Lambda^2(x,y) \mid (0 \leq x \in \mathbf{Z}) \wedge (y \in \mathbf{N})\}$. Notice that if $0 \leq i < k$, i, $k \in \mathbf{Z}$, then $\mathcal{A}((\Lambda^2(i,j), \{\mathrm{f}^2(0,0)\})) \subset \mathcal{A}((\Lambda^2(k,n), \{\mathrm{f}^2(0,0)\}))$ for any j, $n \in \mathbf{N}$. Also, each $\Lambda^2(m,n)$ is a distinct finite set due to the specific identifying $\mathcal{W}'$ language symbols employed. (Notice that members in $\mathcal{L}^2$ can be characterized by a first-order sentence.)

Consider any $\Lambda^2(q,k)$. Then there exists an $q', k' \in \mathbf{N}'$ and the q', k'-set $\mathrm{X}_{q',k'}^2 \in \mathrm{M}_{q',k'}^2$, such that $\Lambda^2(q,k) \in \mathcal{S}_{q',k'}^2(\{\mathrm{X}_{q',k'}^2\})$ and, in this case, finite choice yields the $\Lambda^2(q,k)$ logic-system. Then the logic-system algorithm $\mathcal{A}$ applied to $(\Lambda^2(q,k), \{\mathrm{f}^2(0,0)\})$ yields $\mathrm{f}^2(q,k)$ as a deduction from $\mathrm{f}^2(0,0)$. Further, $\mathrm{f}^2(q,k) \in \mathcal{I}_2$. Conversely, if $\mathrm{f}^2(q,k) \in \mathcal{I}_2$, then there exists an $q', k' \in \mathbf{N}'$ and an $\mathrm{X}_{q',k'}^2 \in \mathrm{M}_{q',k'}^2$ and a logic-system $\Lambda^2(q,k) \in \mathcal{A}'((\mathcal{S}_{q',k'}^2, \{\mathrm{X}_{q',k'}^2\}))$ such that application of the logic-system algorithm $\mathcal{A}$ to $(\Lambda^2(q,k), \{\mathrm{f}^2(0,0)\})$ yields a deduction of $\mathrm{f}^2(q,k)$ from $\mathrm{f}^2(0,0)$.

Theorem 3.1 *Consider primitive time interval* $2 = [0, +\infty)$. *It can always be assumed that interval 2 is partitioned into intervals* $[c_0, c_1), \ldots [c_{m-1}, c_m)$, $m > 1$, $m \in \mathbf{Z}$. *Let* $\underline{\mathcal{I}}_2$ *be an instruction paradigm order isomorphic to the rational numbers* $R_2 \subset [0, +\infty)$. *For any* $\lambda \in \mathbf{N}_\infty$ *and* $\nu \in {}^*\mathbf{Z} - \mathbf{Z}$, $\nu > 0$, *there exists a unique hyperfinite* ${}^*\mathbf{\Lambda}^2(\nu, \lambda) \in {}^*\underline{\mathcal{L}}^2$ *and* $\nu', \lambda' \in {}^*\mathbf{N}'$ *such that the ultra-word-like* $X_{\nu',\lambda'}^2 \in {}^*\mathbf{M}_{\nu',\lambda'}^2$ *and ultra-logic-system* ${}^*\mathbf{\Lambda}^2(\nu, \lambda) \in {}^*\underline{\mathcal{A}}'((\,{}^*\mathcal{S}_{\nu',\lambda'}^2, \{X_{\nu',\lambda'}^2\}))$ *and* ${}^\sigma\underline{\mathcal{I}}_2 \subset {}^*\underline{\mathcal{A}}((\,{}^*\mathbf{\Lambda}^2(\nu, \lambda), \{\,{}^*\mathbf{f}^2(0,0)\})) = \mathcal{I}_{\nu,\lambda}^2 \subset {}^*\underline{\mathcal{I}}_2$. *Also the* $[\![{}^*\underline{\mathcal{A}}((\,{}^*\mathbf{\Lambda}^2(\nu, \lambda), \{\,{}^*\mathbf{f}^2(0,0)\}))]\!]$ *steps satisfy the* $\leq_{\mathcal{I}_{\nu,\lambda}^2}$ *order and* $(\,{}^*\underline{\mathcal{I}}_2 - {}^\sigma\underline{\mathcal{I}}_2) \cap {}^*\underline{\mathcal{A}}(\,{}^*\mathbf{\Lambda}^2(\nu, \lambda), \{\,{}^*\mathbf{f}^2(0,0)\})) =$ *an infinite set.*

Proof. As in Theorem 2.1, the proof follows by *-transfer of the appropriate formally presented material that appears above in this section 3.

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By considering the definition of $\mathcal{L}^2$, it follows that the $^*\mathbf{\Lambda}^2(\nu, \lambda)$ is precisely $\{^*\mathbf{f}^2(0,0)\} \cup \{\bigcup\{^*\mathbf{k}_i^2(\lambda) \mid (0 \leq i < \nu) \wedge (i \in {}^*\mathbf{Z})\}\} \cup \{(\{^*\mathbf{f}^2(p-1, \lambda), {}^*\mathbf{f}^2(p, 0)\} \mid (0 < p \leq \nu) \wedge (p \in {}^*\mathbf{Z})\} \cup \{\{^*\mathbf{f}^2(\nu, j), {}^*\mathbf{f}^2(\nu, j+1)\} \mid (0 \leq j < \lambda) \wedge (j \in {}^*\mathbb{N})\}$. Of significance is the fact that the steps in the *-deduction $^*\underline{\mathcal{A}}[^*\mathbf{\Lambda}^2(\nu, \lambda), \{^*\mathbf{f}^2(0,0)\})]$ preserve the order $\leq {}_*\underline{\mathcal{I}}_2$. Notice that $^*\mathbf{\Lambda}^2(\nu, \lambda)$ is obtained by hyperfinite choice. Further, any $^*\mathbf{f}^2(i, j) \in \{^*\mathbf{f}^2(x, y) \mid (0 \leq x \leq \nu) \wedge (0 \leq y \leq \lambda) \wedge (x \in {}^*\mathbf{Z}) \wedge (y \in {}^*\mathbb{N})\}$ is a hyperfinite *-deduction from $^*\mathbf{f}^2(0,0)$. And, it also follows that the set of all such *deductions yield a hyperfinite set $\mathcal{I}_{\nu,\lambda}^2$ such that $^\sigma\underline{\mathcal{I}}_2 \subset \mathcal{I}_{\nu,\lambda}^2 \subset {}^*\underline{\mathcal{I}}_2$.

4. Logic-System Generation for the Type-3 Interval

For the type-3 case $(-\infty, 0]$, a denumerable instruction paradigm displays a refined form. For this case, $\mathcal{I}_3 = \{\mathbf{f}^3(i, j) \mid (i \leq 0) \wedge (i \in \mathbf{Z}) \wedge (j \in \mathbb{N})$. Using $\mathcal{I}_3$, consider the following logic-system.

Definition 4.1 Let $i \in \mathbf{Z}$, $i \leq 0$. For each $n \in \mathbb{N}$, let $\mathbf{k}_i^3(n) = \{\{\mathbf{f}^2(i, j), \mathbf{f}^1(i, j + 1)\} \mid (0 \leq j \leq n-1) \wedge (j \in \mathbb{N})\}$. For $m \in \mathbf{Z}$, $m < 0$, let $\mathbf{K}^3(m, n) = \bigcup\{\mathbf{k}_i^3(n) \mid (m \leq i < 0) \wedge (i \in \mathbf{Z})\}$. Finally, let $\Lambda^3(m, n) = \{\mathbf{f}^3(m, 0)\} \cup \mathbf{K}^3(m, n) \cup \{\{\mathbf{f}^3(p-1, n), \mathbf{f}^3(p, 0)\} \mid (m < p \leq 0) \wedge (p \in \mathbf{Z})\}$, and $\mathcal{L}^3 = \{\Lambda^2(x, y) \mid (0 \leq x \in \mathbf{Z}) \wedge (y \in \mathbb{N})\}$. Notice that if $i < k \leq 0$, $i, k \in \mathbf{Z}$, then $\mathcal{A}((\Lambda^3(i, j)), \{\mathbf{f}^3(m, 0)\})) \subset \mathcal{A}((\Lambda^3(k, n), \{\mathbf{F}^3(m, 0)\}))$ for any $j, n \in \mathbb{N}$. Also, each $\Lambda^3(m, n)$ is a distinct finite set due to the specific identifying $\mathcal{W}'$ language symbols employed. (Notice that members in $\mathcal{L}^3$ can be characterized by a first-order sentence.)

Consider any $\Lambda^3(q, k)$. Then there exists an $q', k' \in \mathbb{N}$ and $\mathrm{X}_{q',k'}^3 \in \mathrm{M}_{q',k'}^3$, such that $\Lambda^3(q, k) \in \mathcal{A}'((\mathcal{S}_{q',k'}^3\{\mathrm{X}_{q',k'}^3\}))$ and, in this case, finite choice yields the $\Lambda^3(q, k)$ logic-system. Then the logic-system algorithm $\mathcal{A}$ applied to $(\Lambda^3(q, k), \{\mathbf{f}^3(q, 0)\}))$ yields $\mathbf{f}^3(q, k)$ as a deduction from $\mathbf{f}^3(q, 0)$. Further, $\mathbf{f}^3(q, k) \in \mathcal{I}_3$. Conversely, if $\mathbf{f}^3(q, k) \in \mathcal{I}_3$, then there is an $\mathrm{X}_{q',k'}^3 \in \mathcal{M}_{q',k'}^3$ and a logic-system $\Lambda^3(q, k) \in \mathrm{S}^3(\{\mathrm{X}_{q',k'}^3\})$ such that application of the logic-system algorithm $\mathcal{A}$ to $(\Lambda^3(q, k), \{^*\mathbf{f}^3(q, 0)\}))$ yields $\mathbf{f}^3(q, k)$ as a deduction from $\mathbf{f}^3(q, 0)$.

Theorem 4.1 *Consider primitive time interval* $3 = (-\infty, 0]$. *It can always be assumed that interval 3 is partitioned into intervals* $\ldots, [c_{-2}, c_{-1}), [c_{-1}, c_0]$. *Let* $\underline{\mathcal{I}}_3$ *be an instruction paradigm order isomorphic to the rational numbers* $R_3 \subset (-\infty, 0]$. *For any* $\lambda \in \mathbb{N}_\infty$, $\mu \in {}^*\mathbf{Z} - \mathbf{Z}$, $\mu < 0$, *there exists a unique hyperfinite* $^*\mathbf{\Lambda}^3(\mu, \lambda) \in {}^*\underline{\mathcal{L}}^3$ *and* $\mu', \lambda' \in {}^*\mathbb{N}'$ *such that the ultra-word-like* $\mathrm{X}_{\mu',\lambda'}^3 \in {}^*\mathbf{M}_{\mu',\lambda'}^3$ *and ultra-logic-system* $^*\mathbf{\Lambda}^3(\mu, \lambda) \in {}^*\underline{\mathcal{A}}'(({}^*\mathcal{S}_{\mu',\lambda'}^3, \{\mathrm{X}_{\mu',\lambda'}^3\}))$ *and* $^\sigma\underline{\mathcal{I}}_3 \subset {}^*\underline{\mathcal{A}}({}^*\mathbf{\Lambda}^3(\mu, \lambda), \{^*\mathbf{f}^3(\mu, 0)\})) = \mathcal{I}_{\mu,\lambda}^3 \subset {}^*\underline{\mathcal{I}}_3$. *Also the* $[\![{}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^3(\mu, \lambda), \{^*\mathbf{f}^3(\mu, 0)\}))]\!]$ *steps satisfy the* $\leq_{\mathcal{I}_{\mu,\lambda}^3}$ *order and* $({}^*\underline{\mathcal{I}}_3 - {}^\sigma\underline{\mathcal{I}}_3) \cap {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^3(\mu, \lambda)), \{^*\mathbf{f}^3(\mu, 0)\})) =$ *an infinite set.*

Proof. As in Theorem 3.1, the proof follows by *-transfer of the appropriate formally presented material that appears above in this section 3.

By considering the definition of $\mathcal{L}^3$, it follows that the $^*\boldsymbol{\Lambda}^3(\mu, \lambda)$ is precisely $\{\,^*\mathbf{f}^3(\mu, 0)\} \cup \{\bigcup\{\,^*\mathbf{k}_i^3(\lambda) \mid (\mu \leq i < 0) \wedge (i \in \,^*\mathbf{Z})\}\} \cup \{\{\,^*\mathbf{f}^3(p-1, \lambda), \,^*\mathbf{f}^3(p, 0)\} \mid (\mu < p \leq 0) \wedge (p \in \,^*\mathbf{Z})\}$. Of significance is the fact that the steps in the *-deduction $^*\underline{\mathcal{A}}[(\,^*\boldsymbol{\Lambda}^3(\mu, \lambda), \{\,^*\mathbf{f}^3(\mu, 0)\}]$ preserve the order $\leq_{\,^*\underline{\mathcal{I}}_3}$. Notice that $^*\boldsymbol{\Lambda}^3(\mu, \lambda)$ is obtained by hyperfinite choice. Further, any $^*\mathbf{f}^3(i, j) \in \{\,^*\mathbf{f}^3(x, y) \mid (\mu \leq x < 0) \wedge (0 \leq y \leq \lambda)\} \cup \{\,^*\mathbf{f}^3(0, 0)\}$ is a hyperfinite *-deduction from $^*\mathbf{f}^3(\mu, 0)$. And, it also follows that the set of all such *deductions is a hyperfinite set $\mathcal{I}_{\nu,\lambda}^3$ such that $^\sigma\underline{\mathcal{I}}_3 \subset \mathcal{I}_{\nu,\lambda}^3 \subset \,^*\underline{\mathcal{I}}_3$.

5. Logic-System Generation for the Type-4 Interval

Theorem 5.1 *Consider primitive time interval* $4 = (-\infty, +\infty)$. *It can always be assumed that interval 4 is partitioned into intervals* $\ldots, [c_{-2}, c_{-1}), [c_{-1}, c_0), \ldots$. *Let* $\underline{\mathcal{I}}_4$ *be a instruction paradigm order isomorphic to the rational numbers* $R_4 \subset (-\infty, +\infty)$. *For any* $\lambda \in \mathbb{N}_\infty$, $\nu, \gamma \in \,^*\mathbf{Z} - \mathbf{Z}$, *such that* $\nu \leq 0$, $\gamma \geq 0$, *there exists a unique hyperfinite* $^*\boldsymbol{\Lambda}^4(\nu, \gamma, \lambda) \in \,^*\underline{\mathcal{L}}^4$ *and* $\nu', \gamma', \lambda' \in \,^*\mathbb{N}'$ *such that the ultra-word-like* $X_{\nu',\gamma',\lambda'}^4 \in \,^*\mathbf{M}_{\nu',\gamma',\lambda'}^4$ *and ultra-logic-system* $^*\boldsymbol{\Lambda}^4(\nu, \gamma, \lambda) \in \,^*\underline{\mathcal{A}}'((\,^*\underline{\mathcal{S}}_{\nu',\gamma',\lambda'}^4, \{X_{\nu',\gamma',\lambda'}^4\}))$ *and* $^\sigma\underline{\mathcal{I}}_4 \subset \,^*\underline{\mathcal{A}}((\,^*\boldsymbol{\Lambda}^4(\nu, \gamma, \lambda), \{\,^*\mathbf{f}^4(\nu, 0)\})) = \mathcal{I}_{\nu,\gamma,\lambda}^4 \subset \,^*\underline{\mathcal{I}}_4$. *Also the* $\llbracket\,^*\underline{\mathcal{A}}((\,^*\boldsymbol{\Lambda}^4(\nu, \gamma, \lambda), \{\,^*\mathbf{f}^4(\nu, 0)\}))\rrbracket$ **steps satisfy the* $\leq_{\mathcal{I}_{\nu,\gamma,\lambda}^4}$ *order and* $(\,^*\underline{\mathcal{I}}_4 - \,^\sigma\underline{\mathcal{I}}_4) \cap \,^*\underline{\mathcal{A}}((\,^*\boldsymbol{\Lambda}^4(\nu, \gamma, \lambda), \{\,^*\mathbf{f}^4(\nu, 0)\})) =$ *an infinite set.*

By considering the definition of $\mathcal{L}^4$, it follows that the $^*\boldsymbol{\Lambda}^4(\nu, \gamma, \lambda)$ is precisely $\{\,^*\mathbf{f}^4(\nu, 0)\} \cup \{\bigcup\{\,^*\mathbf{k}_i^4(\lambda) \mid (\nu \leq i < \gamma) \wedge (i \in \,^*\mathbf{Z})\}\} \cup \{\{\,^*\mathbf{f}^4(p-1, \lambda), \,^*\mathbf{f}^4(p, 0)\} \mid (\nu < p \leq \gamma) \wedge (p \in \,^*\mathbf{Z})\} \cup \{\{\,^*\mathbf{f}^4(\gamma, j), \,^*\mathbf{f}^4(\gamma, j+1)\} \mid (0 \leq j < \lambda) \wedge (j \in \,^*\mathbb{N})\}$. Of significance is the fact that the steps in the *-deduction $^*\underline{\mathcal{A}}[(\,^*\boldsymbol{\Lambda}^4(\nu, \gamma, \lambda), \{\,^*\mathbf{f}^4(\nu, 0)\})]$ preserve the order $\leq_{\,^*\underline{\mathcal{I}}_4}$. Notice that $^*\boldsymbol{\Lambda}^4(\nu, \gamma, \lambda)$ is obtained by hyperfinite choice. Further, any $^*\mathbf{f}^4(i, j) \in \{\,^*\mathbf{f}^4(x, y) \mid (\nu \leq x \leq \gamma) \wedge (0 \leq y \leq \lambda)\}$ is a hyperfinite *-deduction from $^*\mathbf{f}^4(\nu, 0)$. And, it also follows that the set of all such *deductions is a hyperfinite set $\mathcal{I}_{\nu,\gamma,\lambda}^4$ such that $^\sigma\underline{\mathcal{I}}_4 \subset \mathcal{I}_{\nu,\gamma,\lambda}^4 \subset \,^*\underline{\mathcal{I}}_4$.

6. Necessary refinements.

For the GGU-model, a universe is a nonempty collection of empty-systems, physical-systems, physical-like systems or other-systems. In general, an infinite hyperfinite set X of internal sets, disjoint from $\bigcup\{\,^*\mathcal{F}^i(\mathcal{C}) \mid (1 \leq i \leq n) \wedge (i \in \,^*\mathbb{N})\}$, for an appropriate n, is adjoined when info-fields are employed.

It is now necessary that a more refined definition for each $\mathbf{f}^q(i, j)$, which yields each $^*\mathbf{f}^q(i, j)$, be given. The notion of the "non-operative" instruction is used. Using the alphabet symbol yX $\in$ W$'$, consider the meaningless "word" yX. If this word

is considered an instruction, then it has no operative content and **yX** yields neither properton combinations of any form nor any entities that require the adjoined set X. Its application yields an empty-system. This word is introduced in order to simplify the following refinement.

Although thus far $^*\mathbf{f}^q(i,j)$ has been considered as an *instruction-entity and any further refinements as to how it is constructed were unnecessary, this is no longer the case. None of the previous results are altered by this refinement. All members of $^*\underline{\mathcal{W}}'$ being considered in this section are *instructions. Let $\mathbf{Z}_q$, $q = 1, 2, 3, 4$, be as employed to define the $\mathcal{I}_q$. Consider, as with q and defined in the same manner, $\mathbf{Z}_r \subset \mathbf{Z}$, where $r = 1, 2, 3, 4$. Notationally, let $\mathbf{T} = \mathbf{Z}_r \times \mathbb{N}$. Both $\mathbf{Z}_q \times \mathbb{N}$ and $\mathbf{T}$ carry the simple lexicographic order $\preceq$ (Herrmann, 2006).

[As with the previous NOTE, the following g and v functions are compositions, where the respective t functions are defined on $\mathbf{Z} \times \mathbb{N}$ with the same values as members of R. In the case of g, the values are the v sequences, where the values of v are members of W'. The (q, r) notation indicates the appropriate restrictions.]

In what follows, given a particular q, then r is fixed for all universe-wide frozen-frames. **For this approach, the Theorems 2.1, 2, 3, 4 are highly refined.** Consider functions $\mathrm{g}^{(q,r)} \colon \mathbf{Z}_q \times \mathbb{N} \to (\mathrm{W}')^{\mathbf{T}}$ with the following properties. For $(i, j) \in \mathbf{Z}_r \times \mathbb{N}$, $\mathrm{g}^{(q,r)}((i, j)) = \mathrm{v}$. Then for each $(k, s) \in \mathbf{Z}_r \times \mathbb{N}$, $\mathrm{v}(k, s)$ is an appropriate instruction-entity. Notationally, the instruction-entity is $\mathrm{g}^{(q,r)}((i,j))(k, s) = \mathrm{v}(k, s) = \mathrm{g}^{(q,r)}(i, j; k, s)$. For applications and a particular (i, j), the k identifies the system and varying s identify system constituents.

Definition 6.1. Depending upon the application, each $(i.j)$-universe-wide frozen-frame (UWFF) is one of three general types. **(1) It can be empty. (2) It can contain but repetitions. (3) It can be composed of, at the least, one different object.** The same three general types of $; r, s)$ physical, physical-like or other-systems can occur.

Definition 6.2. UWFF Associations. All UWFF and the various types of systems can be related relative to their (i, j) and $; k, s)$ identifiers. However, there can be three special types of relations termed as **associations**. For a given $(i, 0)$-UWFF, each of the (i, j)-UWFF is **associated** with the $(i, 0)$. In this i-case and for standard j and j', (i.e. $\{j, j'\} \subset \mathbb{N}$), each (i, j) and (i, j'), is **closely associated** with each other. Standard j and $j' \in \mathbb{N}$ are closely associated. Then for $\{j, j'\} \subset \mathbb{N}_\infty$, each (i, j) and (i, j'), is **exquisitely association**. Further, due to required convergence, each (i, j), $j \in \mathbb{N}_\infty$ is also exquisitely associated with the $(i + 1, 0)$-UWFF.

Definition 6.3. System Associations. For a nonempty (i,j)-UWFF, the three types of systems identified by the quasi-ordered pairs $;k,s)$ can also carry the same "association" terminology. Further, the "other-systems" often have an additional feature. They can be conceived of a k **internal structures**. On the other hand, via specific interpretations, they need only be considered as associated physical or physical-like systems.

> Although defined in this article, the use of the "association" term usually appears when the GGU-model is interpreted, where an intuitive meaning is applied.

Using this notation, for a fixed (i,j), the $g^{(q,r)}(i,j;k,s)$ represents the same type of primitive time ordering (lexicographic) as does the universe-wide frozen-frames when generated by the deduction algorithm process. For purposes of analysis, physical-systems and universes-wide frozen-frames are also viewed as subsets of W$'$. For each $(i,j,k) \in \mathbf{Z}_q \times \mathbb{N} \times \mathbf{Z}_r$, consider the set $\{g^{(q,r)}(i,j;k,s) \mid (s \in \mathbb{N})\}$. Define $h^{(q,r)}(i,j,k) = \{g^{(q,r)}(i,j;k,s) \mid s \in \mathbb{N}\}\diamondsuit$. For each $(i,j) \in \mathbf{Z}_q \times \mathbb{N}$, let $f^{(q,r)}(i,j) = \bigcup\{h^{(q,r)}(i,j,k) \mid k \in \mathbf{Z}_r\}\diamondsuit$. By embedding and *-transfer, for $(i,j) \in {}^*\mathbf{Z}_q \times {}^*\mathbb{N}$ and $(k,s) \in {}^*\mathbf{Z}_r \times {}^*\mathbb{N}$, consider the composed function ${}^*\mathbf{g}^{(q,r)}(i,j;k,s)$, with its *instruction-entity property. Thus, there is an internal ${}^*\mathbf{h}^{(q,r)}(i,j,k) = \{{}^*\mathbf{g}^{(q,r)}(i,j;k,s) \mid s \in {}^*\mathbb{N}\}$. Then, for each $(i,j,k) \in {}^*\mathbf{Z}_q \times {}^*\mathbb{N} \times {}^*\mathbf{Z}_r$, the *instruction-entity ${}^*\mathbf{f}^{(q,r)}(i,j) = \bigcup\{{}^*\mathbf{h}^{(q,r)}(i,j,k) \mid k \in {}^*\mathbf{Z}_r\}$. On appropriate $\{{}^*\mathbf{g}^{(q,r)}(i,j;k,s)\}$, the relation $\leq_{(q,r)}$ is also satisfied. (It is costmary for such relations to be expressed this way, where one actually means ${}^*\leq_{(q,r)}$.)

The notion of the "extended" language is necessary when certain single word representations are employed. This means that denumerably long words are allowed as members of W$'$. Further, in this case, the equivalence class representation for such a word is composed of total sequences. (Note: When such an equivalence class $[f]$ is embedded into the nonstandard model, then, general;y, it does not hold that ${}^*[f] = [f]$.) With respect to the original method used to obtained words in W$'$, note that from the definitions and when considered as restrictions, for fixed (i,j,k), each $g^{(q,r)}(i,j;k,s)$ is an instruction-entity, each $h^{(q,r)}(i,j,k)$ is an instruction-entity (of instruction-entities) as is each $f^{(q,r)}(i,j)$. For fixed (i,j), (k,s), there is a single word $W^{(q,r)}_{(i,j;k,s)}$ in W$'$ that corresponds to $g^{(q,r)}(i,j;k,s)$. However, due to how the intervals and interval partition methods are employed, the various single word forms are considered as but a logical form constructed by the addition of a "conjunction" (and, &, $\wedge$) notation. Mathematically from $\diamondsuit$ any two conjuncts can be compared relative to the simple order as symbolically expressed.

Then there is a word $W^{(q,r)}_{(i,j;k)} \in$ W$'$ corresponding to each $h^{(q,r)}(i,j,k)$ the in-

dividual symbol-strings, the spacing symbol, diagrams, images, or sensory information determined by each word $g^{(q,r)}(i,j;k,s)$ as s varies. This word yields a written instruction-entity for each of the finite k systems.

In like manner for fixed (i,j), there is a member of W$'$ that yields, in written word-ordered form, via the lexicographic order, a single word $W^{(q,r)}_{(i,j)} \in W'$. Then there is a single word $W^{(q,r)}$ that yields in written word-ordered form, again via $\diamondsuit$, a word that corresponds to a complete development for a generated standard universe. These results are immediately extended to the hyperfinite cases. Then, for the $W^{(q,r)}(n)$, this immediately yields the existence of an ultraword ${}^{*}W^{(q,r)}(\lambda) \in {}^{*}\underline{\mathcal{W}}'$ with the same first-order properties as $W^{(q,r)}(n)$.

For each $(i,j) \in {}^{*}\mathbf{Z}_q \times {}^{*}\mathbb{N}$ and $k \in {}^{*}\mathbf{Z}_r$, ${}^{*}\mathbf{h}^{(q,r)}(i,j,k)$ yields an empty-system, physical-system, physical-like system or an other-system. (Note that physical-like properties include physical-like behavior relative to non-physical entities, where the entities have no other known properties.) Of course, each hyperfinite ${}^{*}\mathbf{f}^{(q,r)}(i,j)$ is an *instruction-entity as is each ${}^{*}\mathbf{h}^{(q,r)}(i,j,k)$. By application of the word $\mathbf{yX}$ and the GGU-model construction of each universe-wide frozen frame, there are only finitely or hyperfinitely many physical or physical-like systems. Further more, various physical-like systems can be physical-systems in that only physical events exist. Which physical-like systems are but physical depends upon choice since the GGU-model is not dependent upon what science classifies as "physical." Such a choice is dependent upon a chosen interpretation. The same holds for what are considered as empty-systems or other-systems.

As previously done by considering the finite case, for each $(i,j) \in {}^{*}\mathbf{Z}_q \times {}^{*}\mathbb{N}$ that yields a universe-wise frozen-frame, ${}^{*}\mathbf{g}^{(q,r)}(i,j;k,s)$ <u>produces</u> an info-field in the same manner as an entire universe is considered as corresponding to a *developmental paradigm. These info-fields can also be considered in system form via the ${}^{*}\mathbf{h}^{(q,r)}(i,j,k)$ *instruction-entities. If empty-systems are used, only a type-r $= 4$ ultra-logic-system need be considered. Then as based upon the finite case, this yields for each such (i,j), $\lambda \in {}^{*}\mathbb{N}$, and $\{\nu, \gamma\} \subset {}^{*}\mathbf{Z}$ a generating ultra-logic-system

$${}^{*}\mathbf{F}^{(q,r)}(i,j,\nu,\gamma,\lambda) = \{\, {}^{*}\mathbf{g}^{(q,r)}(i,j;\nu,0)\} \cup \{\bigcup {}^{*}\mathbf{k}^{(q,r)}_x(\lambda) \mid (\nu \leq x < \gamma) \wedge (x \in {}^{*}\mathbf{Z})\}\}$$

$$\cup\{\{\, {}^{*}\mathbf{g}^{(q,r)}(i,j;k-1,\lambda), {}^{*}\mathbf{g}^{(q,r)}(i,j;k,0)\} \mid (\nu < k \leq \gamma) \wedge (k \in {}^{*}\mathbf{Z})\}\cup$$

$$\{\{\, {}^{*}\mathbf{g}^{(q,r)}(i,j;\gamma,s), {}^{*}\mathbf{g}^{(q,r)}(i,j;\gamma,s+1)\} \mid (0 \leq s < \lambda) \wedge (s \in {}^{*}\mathbb{N})\}, \ \lambda \in \mathbb{N}_\infty.$$

The set ${}^{*}\mathbf{F}^{(q,r)}_{\nu,\gamma,\lambda}(i,j) = {}^{*}\mathbf{F}^{(q,r)}(i,j,\nu,\gamma,\lambda)$ is a hyperfinite set of internal entities and, as such, it is internal. For the previous ultra-logic-system ${}^{*}\mathbf{\Lambda}^q(x,\lambda)$, where x

depends on the q, the $^*\mathbf{F}^{(q,r)}_{\nu,\gamma,\lambda}$ replaces the $^*\mathbf{f}^q$. Thus, this gives an ultra-logic-system $^*\mathbf{\Lambda}^{(q,r)}(x,\lambda)$ that is composed of ultra-logic-systems. As such, it is an hyperfinite set of internal entities and, hence, internal. Note again that the basic entities correspond to ultrawords in $^*\underline{\mathcal{W}}'$.

For this refined approach, the algorithm $\mathcal{A}$ has an extended definition. It is applied to these special logic-systems where the members are themselves logic-systems. This is how modified $\mathcal{A}$ is applied. As each logic-system is obtained in the indicated ordered manner, $\mathcal{A}$ is applied to it. This is what would occur in the standard definition for application of $\mathcal{A}$ to a collection of logic-systems in n-tuple form except that the logic-systems are obtained deductively in a specific order and then as each is deduced the deduction is completed for the deduced logic-system. This is a rather natural way one would precede. Under this extended definition for $\mathcal{A}$ all of previous Theorems 2.1, 2, 3, 4 in sections 2, 3, 4, 5 are modified as follows:

*Consider an appropriate (q,r), x, a, b and any $\lambda \in \mathbb{N}_\infty$. Then there exists a unique hyperfinite $^*\mathbf{\Lambda}^{(q,r)}(x,\lambda) \in {}^*\underline{\mathcal{L}}^{(q,r)}$ and an $x',\lambda' \in {}^*\mathbb{N}'$ such that the ultra-word-like $X^{(q,r)}_{x',\lambda'} \in {}^*\mathbf{M}^{(q,r)}_{x,\lambda'}$ and ultra-logic-system $^*\mathbf{\Lambda}^{(q,r)}(x,\lambda) \in {}^*\underline{\mathcal{A}}'(({}^*\underline{\mathcal{S}}^{(q,r)}_{x',\lambda'}, \{X^{(q,r)}_{x',\lambda'}\}))$ and $^\sigma\underline{\mathcal{I}}_{(q,r)} \subset {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), \{{}^*\mathbf{F}^{(q,r)}_{\nu,\gamma,\lambda}(a,b)\})) = \mathcal{I}^{(q,r)}_{\nu,\gamma,\lambda} \subset {}^*\underline{\mathcal{I}}_{(q,r)}$. Also the $[\![{}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), \{{}^*\mathbf{F}^{(q,r)}_{\nu,\gamma,\lambda}(a,b)\}))]\!]$ *steps satisfy the $\leq_{\mathcal{I}^{(q,r)}_{\nu,\gamma,\lambda}}$ order and $({}^*\underline{\mathcal{I}}_{(q,r)} - {}^\sigma\underline{\mathcal{I}}_{(q,r)}) \cap {}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), \{{}^*\mathbf{F}^{(q,r)}_{\nu,\gamma,\lambda}(a,b)\})) =$ an infinite set.*

Let $D_i \subset \mathbb{R}$ be a countable set of non-zero numerical or coded values for a particular ultra-properton coordinate $i \in K$ or, in some cases, a coded descriptive member of W'. (Note. It is possible that such coded descriptions can also be modeled as members of $^*\underline{\mathcal{W}}'$.) Let $D = \bigcup\{D_i \mid i \in K\}$. For each $p \in D$, let $\Lambda_p = \{x \mid (x \in {}^*\mathbb{N}) \wedge (\mathtt{st}(x/10^\omega) = p)\}$. If $p \neq 0$, then $x \in \mathbb{N}_\infty = {}^*\mathbb{N} - \mathbb{N}$. The sets Λ_p are disjoint and, by choice, consider distinct $\lambda_p \in \Lambda_p$ for each $p \in D$. For any $p \in D$, consider the λ_p-finite set P_p of ultra-propertons. There exists a bijection from $[1, \lambda_p]$ onto P_p. From the definition of the set of all ultra-propertons, for each $i', j' \in [1, K]$ that yield characteristics and each $p, y \in D$, there exist λ_p-finite $x(p, i') \in \mathcal{C}$ and a λ_y-finite $x(y, j') \in \mathcal{C}$ sets of ultra-propertons such that if $i \neq j$, then $x(p, i') \cap x(p, j') = \emptyset$. The $x(p, i')$ are intermediate propertons. The same holds if coordinates are but coded descriptive members of W'.

All of the previous appropriate results hold for the case of a corresponding developmental paradigm by substituting corresponding developmental paradigm $f^{(q,r)}$, $g^{(q,r)}(i, j; k, s)$, and the like for instruction paradigm notation.

For a particular $\lambda \in \mathbb{N}_\infty$, i such that $i \in {}^*\mathbf{Z}_q$, and j such that $j \in {}^*\mathbb{N}$, $j \leq \lambda$, consider $^*\mathbf{f}^{(q,r)}_\lambda(i, j) = \bigcup\{{}^*\mathbf{h}^{(q,r)}(i, j, k) \mid k \in {}^*\mathbf{Z}_r\}$. Let hyperfinite $f^{(q,r)}_\lambda = \{{}^*\mathbf{f}^{(q,r)}_\lambda(i, j) \mid$

$(i \in {}^{*}\mathbf{Z}_q) \wedge (j \in {}^{*}\mathbb{N}) \wedge (j \leq \lambda)\}$. Then let

$$G_\lambda^{(q,r)} \colon f_\lambda^{(q,r)} \to \Pi^+.$$

The image $G_\lambda^{(q,r)}({}^{*}\mathbf{f}_\lambda^{(q,r)}(i,j)) = IF_\lambda^{(q,r)}(i,j)$ is an info-field. Depending upon how a result of $IF_\lambda^{(q,r)}(i,j)$ is analyzed, it can be viewed as collections of ultra-propertons, collections of intermediate propertons, collection of these collections . . . . However, it can also contain non-properton generated physical-like systems and the empty set.

For physical-systems only, the function $G_\lambda^{(q,r)}$ is the "gathering" or "binding" operator. Its properties for the generation of physical-like systems are unknown. From a set-theoretic viewpoint for physical-systems, this operator simply gathers ultra-propertons into bound subsets, the intermediate propertons. For our universe, these subsets are further gathered into bound sets that represent the elementary particles, this continues, as necessary to bound sets that, when realized, yield, for "meaningful" *instructions-sets, various disjoint physical-systems. As previously noted, empty-systems are produced when the "*instructions" are meaningless in that no members of a info-field are produced by them.

For a physical universe, the intermediate propertons $x(p, i')$ determine physical objects and this can be viewed two ways. First, a collection of such sets is viewed as a collection of ultra-propertons and the independent coordinate addition is coupled with the standard part operator. Or, independent coordinate addition is applied in a separate step to sets of ultra-propertons and the sets are replaced with a single entity - an intermediate properton. Since for physical objects within our universe, what are considered as "particles" are, from the GGU-model, but bound collections, of collections of ultra-propertons, then introducing the intermediate subparticle as but an appropriate collection of ultra-propertons seems, for comprehension, to be the best approach. **But, the standard part operator, St, is applied only to the bound collections of ultra-propertons that yield the intermediate propertons. This will automatically yield each physical-system.** This approach can eliminate the virtual particle or process concept within reality models. The models that need such concepts to predict behavior can be considered as just that "models" for predicted behavior and that the virtual "stuff" does not exist in physical reality.

Application of the standard part operator, the physical realization operator, to a specific $IF_\lambda^{(q,r)}(i,j)$ causes the immediate info-field sequential predecessor, if any, to cease in that the propertons are no longer bound. For the intermediate propertons, this can be modeled by vector space subtraction and this would be followed by independent *-finite subtraction.

The function $G_\lambda^{(q,r)}$ represents a substratum medium and the processes that yield the gathering action. This correspondence produces a correlation between the ultra-logic-system, ordered *deduction, via $^*\underline{\mathcal{A}}$, that mimics these processes with respect to its range. This further corresponds to the appearance and the behavior of the physical-systems that will display the same properties as the mimicked entities. Further, this substratum medium has other properties. In all that follows, this refinement is assumed.

7. GGU-model Schemes.

he following schemes are not expressed in complete composition form. In what follows, for $q = 1, 2, 3, 4$, the a, b, c take the appropriate value for a specific q. The relations (operators) such as $^*\underline{\mathcal{A}}$ and the others presented in the following left-to-right sequential form, represent processes. There is also the process, here denoted by Ch, that, depending upon an interpretation, represents a characterizable choice process.

For the multi-complexity cosmogony, the scheme is

$$(\text{MC}) \quad {}^*\mathbf{M}_{\lambda'}^{(q,r)} \Rightarrow \text{Ch}({}^*\mathbf{M}_{x',\lambda'}^{(q,r)})) = ({}^*\underline{\mathcal{S}}_{x',\lambda'}^{(q,r)}\{X_{x',\lambda'}^{(q,r)}\}) \Rightarrow$$

$$\text{Ch}({}^*\underline{\mathcal{A}}'({}^*\underline{\mathcal{S}}^{(q,r)}, \{X_{x',\lambda'}^{(q,r)}\})) = ({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), \{{}^*\mathbf{F}_{\nu,\gamma,\lambda}^{(q,r)}(b,c)\})$$

$$\Rightarrow (\text{St}G_\lambda^{(q,r)})([\![({}^*\underline{\mathcal{A}}({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), \{{}^*\mathbf{F}_{\nu,\gamma,\lambda}^{(q,r)}(b,c)\}))]\!]) \Rightarrow \mathcal{U}.$$

In (MC), when $G_\lambda^{(q,r)}$ is sequentially applied to an ${}^*\mathbf{F}_{\nu,\gamma,\lambda}^{(q,r)}(i,j)$, the St is applied. This scheme can be further refined in that the info-fields are set-theoretically gathered into an ultra-logic-system $\Gamma^{(q,r)}(x,\lambda)$.

Then $(\text{St}G_\lambda^{(q,r)})([\![({}^*\underline{\mathcal{A}}({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), \{{}^*\mathbf{F}_{\nu,\gamma,\lambda}^{(q,r)}(b,c)\}))]\!]) \Rightarrow \mathcal{U}$ is replaced with

$$G_\lambda^{(q,r)}([\![({}^*\underline{\mathcal{A}}({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), \{{}^*\mathbf{F}_{\nu,\gamma,\lambda}^{(q,r)}(b,c)\}))]\!]) = (\Gamma^{(q,r)}(x,\lambda), IF_\lambda^{(q,r)}(b,c)) \Rightarrow$$

$$\text{St}([\![({}^*\underline{\mathcal{A}}((\Gamma^{(q,r)}(x,\lambda), IF_\lambda^{(q,r)}(b,c)))]\!]) \Rightarrow \mathcal{U}.$$

For a particular $\lambda \in \mathbb{N}_\infty$ and appropriate x and a, b one **single-complexity** scheme is

$$(\text{S}) \quad (\text{St}G_\lambda^{(q,r)})([\![({}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), {}^*\mathbf{F}_{\nu,\gamma,\lambda}^{(q,r)}(a,b)))]\!]) \Rightarrow \mathcal{U} \text{ or}$$

$$(\text{S}') \quad G_\lambda^{(q,r)}([\![({}^*\underline{\mathcal{A}}(({}^*\mathbf{\Lambda}^{(q,r)}(x,\lambda), \{{}^*\mathbf{F}_{\nu,\gamma,\lambda}^{(q,r)}(a,b)\})))]\!]) = (\Gamma^{(q,r)}(x,\lambda), IF_\lambda^{(q,r)}(a,b)) \Rightarrow$$

$$\text{St}([\![({}^*\underline{\mathcal{A}}((\Gamma^{(q,r)}(x,\lambda), IF_\lambda^{(q,r)}(a,b))))]\!]) \Rightarrow \mathcal{U}.$$

This scheme indicates that only one complexity level, λ, is considered for a particular universe. This is can be considered a fixed complexity for the entire GGU-model.

This is a significant simplification. For (S) and a standard finite complexity k case, there is a concrete model for corresponding standard $\underline{\mathcal{A}}((\Lambda^{(q,r)}(m,n), \{\mathrm{F}_{p,t,k}^{(q,r)}(a,b)\}))$ in the form of a book with chapters identified.

The minimal scheme, where only info-fields exist, is

$$\text{(M)} \ \mathtt{St}(\llbracket ({}^*\underline{\mathcal{A}}((\Gamma^{(q,r)}(x,\lambda), IF_{\lambda}^{(q,r)}(a,b)))) \rrbracket) \Rightarrow \mathcal{U}.$$

The mathematics yields a mathematical model for behavior that is most likely expressed differently. In the usual way, abbreviate "If P, then Q" as P → Q. Let A, B, C, D, . . . be members of a standard logic-system as represented by two element sets and ||| is a spacing, where each A, B, C, D, . . . represent an instruction-entity for the physical-systems that comprise a specific (i,j) universe-wide frozen-frame. For this standard case, there is a word, $\mathrm{W}_{(i,j)}^{(q,r)} = \mathrm{A}|||\mathrm{A} \to \mathrm{B}|||\mathrm{B} \to \mathrm{C}|||\mathrm{C} \to \mathrm{D}||| \ldots \in \mathrm{W}'$, and each expression, other than |||, in A|||A → B|||B → C|||C → D||| . . . is considered, in the usual way, an hypothesis. The propositional rule for deduction yields, in order, $\{A, B, C, D, \ldots\}$. Let the members of a finite set $\{\mathrm{W}_{(i,j)}^{(q,r)}\}$ of these words be denoted by P, Q, R, S, T, ... This is a finite collection of instruction-entities for finitely many universe-wide frozen-frames. Hence, using the words P, Q, R, S, . . . there exists in informal W' a word $\mathrm{W}^{(q,r)}(n) = \mathrm{P}.|||\mathrm{P} \to \mathrm{Q}.|||\mathrm{Q} \to \mathrm{R}.|||\mathrm{R} \to \mathrm{S}.|||...$ Applying modus ponens throughout the members of this word yield, in order, the instruction-entities for each universe-wide frozen-frame and each physical-system contained therein.

Of course, although technically ${}^*\underline{\mathcal{W}}'$ is a set of *equivalences classes within the model, these results yield ultrawords $W_{(i,j)}^{(q,r)}$ and $W^{(q,r)}(x,\lambda)$, when these results are extended using the notion of a "hyper" language. Thus, rationally, one can state that ${}^*\Lambda^{(q,r)}(x,\lambda)$ corresponds to an ultraword $W^{(q,r)}(x,\lambda)$ in a hyper-language to which a type of hyper-deduction is applied. This type of hyper-deduction is the exact same rule as modus ponens except it is applied to an ultraword and has hyperfinitely many steps.

8. The Algorithm $\mathcal{A}$ (Also Denoted Elsewhere as A).

[Note: When I formally express set-theoretic statements in a first-order language, I usually use "clarifying" parentheses that need not be part of the formal language. For example, $\forall x((x \in \mathrm{A}) \wedge (x \in \mathrm{B}))$, as slightly abbreviated, need only be written as $\forall x(x \in \mathrm{A} \wedge x \in \mathrm{B}$ since $x \in \mathrm{A}$ and $x \in \mathrm{B}$ are often formally expressed by predicate notation $\mathrm{E}(x,y)$ and this statement is expressed as $\forall x(\mathrm{E}(x, \mathrm{A}) \wedge \mathrm{E}(x, \mathrm{B}))$. There is a major axiom used in informal mathematics that is not expressible in this first-order formal manner. It is the Peano-Dedekind induction axiom for the complete collection of natural numbers.]

Given nonempty B, the doubleton operator $\mathcal{D}$ on B, $\mathcal{D}(B)$, is defined as

$\forall x((x \in \mathcal{D}(B)) \leftrightarrow ((x \in \mathcal{P}(B)) \wedge (\exists y \exists z((y \in B) \wedge (z \in B) \wedge (y \in x) \wedge (z \in x) \wedge (\forall w((w \in B) \wedge (w \in x) \rightarrow ((w = y) \vee (w = z))))))))$.

Previously, I merely stated that algorithm $\mathcal{A}$ can be formally characterized and its basic properties captured. I now show how this can be done. Let $\mathbf{\Lambda}^4(p,r,n)$ be the logic-system defined on $\mathcal{I}_4$. Then the algorithm $\mathcal{A}$ has various properties. (Recall that underlined symbols indicates "bold" font.) One statement relating these properties is

$\forall n \forall p \forall t(((n \in \mathbb{N}) \wedge (p \in \mathbf{Z}) \wedge (p \leq 0) \wedge (t \in \mathbf{Z}) \wedge (t \geq 0)) \rightarrow (\forall z((z \in \mathcal{A}(\mathbf{\Lambda}^4(p,r,n), \{\mathbf{f}^4(p,0)\})) \leftrightarrow ((z \in \{\mathbf{f}^4(p,0)\}) \vee (\exists x \exists y((\neg(x = y) \wedge (x \in \mathbf{\Lambda}^4(p,t,n)) \wedge (y \in \mathbf{\Lambda}^4(p,t,n)) \wedge (x \in \mathcal{D}(\underline{\mathcal{I}}_4)) \wedge (y \in \mathcal{D}(\underline{\mathcal{I}}_4)) \wedge (z \in x) \wedge (z \in y)))) \vee (\exists w((w \in \mathcal{D}(\underline{\mathcal{I}}_4)) \wedge (z \in w) \wedge (\forall v((v \in \mathcal{D}(\underline{\mathcal{I}}_4) \wedge \neg(v = w) \rightarrow \neg(z \in v)))))))))))$.

Then there is the less informative statement

$\forall n \forall p \forall t(((n \in \mathbb{N}) \wedge (p \in \mathbf{Z}) \wedge (p \leq 0) \wedge (t \in \mathbf{Z}) \wedge (t \geq 0)) \rightarrow (\forall z((z \in \mathcal{A}(\mathbf{\Lambda}^4(p,t,n), \{\mathbf{f}^4(p,0)\})) \leftrightarrow ((z \in \{\mathbf{f}^4(p,0)\}) \vee (\exists x((x \in \mathbf{\Lambda}^4(p,r,n)) \wedge (x \in \mathcal{D}(\underline{\mathcal{I}}_4)) \wedge (z \in x)))))))$.

Then upon embedding and *-transfer these two formal statements become

$\forall n \forall p \forall r(((n \in {}^*\mathbb{N}) \wedge (p \in {}^*\mathbf{Z}) \wedge (p \leq 0) \wedge (r \in {}^*\mathbf{Z}) \wedge (r \geq 0)) \rightarrow (\forall z((z \in {}^*\underline{\mathcal{A}}({}^*\mathbf{\Lambda}^4(p,r,n), \{{}^*\mathbf{f}^4(p,0)\})) \leftrightarrow ((z \in \{{}^*\mathbf{f}^4(p,0)\}) \vee (\exists x \exists y((\neg(x = y) \wedge (x \in {}^*\mathbf{\Lambda}^4(p,r,n)) \wedge (y \in {}^*\mathbf{\Lambda}^4) \wedge (x \in {}^*\mathcal{D}({}^*\underline{\mathcal{I}}_4)) \wedge (y \in {}^*\mathcal{D}({}^*\underline{\mathcal{I}}_4)) \wedge (z \in x) \wedge (z \in y)))) \vee (\exists w((w \in {}^*\mathcal{D}({}^*\underline{\mathcal{I}}_4)) \wedge (z \in w) \wedge (\forall v((v \in {}^*\mathcal{D}({}^*\underline{\mathcal{I}}_4) \wedge \neg(v = w) \rightarrow \neg(z \in v)))))))))))$.

$\forall n \forall p \forall r(((n \in {}^*\mathbb{N}) \wedge (p \in {}^*\mathbf{Z}) \wedge (p \leq 0) \wedge (r \in {}^*\mathbf{Z}) \wedge (r \geq 0)) \rightarrow (\forall z((z \in {}^*\underline{\mathcal{A}}({}^*\mathbf{\Lambda}^4(p,r,n), \{{}^*\mathbf{f}^4(p,0)\})) \leftrightarrow ((z \in \{{}^*\mathbf{f}^4(p,0)\}) \vee (\exists x((x \in {}^*\mathbf{\Lambda}^4(p,r,n)) \wedge (x \in {}^*\mathcal{D}({}^*\underline{\mathcal{I}}_4)) \wedge (z \in x)))))))$.

Since the GGU-model is based upon the finite, then the methods used are probably the "simplest" that exist within nonstandard analysis. Of course, the *-transferred statements and the original ones are stating that the same properties hold but when examined from the meta-world more can be stated. From that world, it is observed that the p, t, n can be the hyper-numbers ν, γ, λ, respectively.

Of course, one can also introduce the notion of the "step-order" for the algorithm. For each nonempty finite instruction paradigm $\mathcal{I}^4_{p,t,n}$ (not in refined notation), there is an $m \in \mathbb{N}'$ and a bijection S:$[1, m] \rightarrow \mathcal{I}^4_{p,t,n}$ that preserves the order $\leq$ on the natural numbers and the lexicographic order $\leq_{\mathcal{I}^4_{p,t,n}}$ on $\mathcal{I}^4_{p,t,n}$.

$\forall n \forall p \forall t \forall k \forall m(((n \in \mathbb{N}) \wedge (p \in \mathbf{Z}) \wedge (p \leq 0) \wedge (t \in \mathbf{Z}) \wedge (t \geq 0) \wedge (k \in \mathbb{N}') \wedge (m \in \mathbb{N}') \wedge (k \leq m)) \rightarrow (\mathbf{S}(k) \leq_{\underline{\mathcal{I}}^4_{p,t,n}} \mathbf{S}(m)))$.

Hence,

$$\forall n \forall p \forall t \forall k \forall m(((n \in {}^*\mathbb{N}) \wedge (p \in {}^*\mathbf{Z}) \wedge (p \leq 0) \wedge (t \in {}^*\mathbf{Z}) \wedge (t \geq 0) \wedge (k \in {}^*\mathbb{N}') \wedge (m \in {}^*\mathbb{N}') \wedge (k \leq m)) \rightarrow ({}^*\mathbf{S}(k) \leq {}_{*\underline{\mathcal{I}}^4_{p,t,n}} {}^*\mathbf{S}(m))).$$

Thus, for the particular ν, γ, λ in Theorem 5.1, algorithm $\underline{{}^*\mathcal{A}}$ "deduction" takes place in $\mu \in \mathbb{N}_\infty \leq$ steps.

9. The Participator Universe.

For the GGU-model, one of the most difficult requirements is to include the concept of the "participator" universe. As stated at the May 1974 Oxford Symposium in Quantum Gravity, Patton and Wheeler describe how the existence of human beings alters the universe to various degrees. "To that degree the future of the universe is changed. We change it. We have to cross out that old term 'observer' and replace it with the new term 'participator.' In some strange sense the quantum principle tells us that we are dealing with a participator universe." (Patton and Wheeler (1975, p. 562).) This aspect of the GGU-model is only descriptively displayed in section 4.8 in Herrmann (2002). It is now possible to obtain formally the collection of universes that satisfies this participator requirement. For simplicity and for our universe, it is assumed in this section that a single-complexity type universe is used throughout. The universe in which we dwell is one that needs to satisfy the notion of participator alterations.

For our universe as presently accepted, this is now extended, for each $n \in \mathbb{N}$, to a sequence $\Lambda^{(q,r)}$ defined on $[1, m]$, $m \in \mathbb{N}'$, with range $\{\Lambda_p^{(q,r)}(x, n) \mid 1 \leq p \leq m\}$. By *-transfer, for $\lambda \in \mathbb{N}_\infty$, appropriate x, the finite (hence, hyperfinite) sequence has range $\{{}^*\Lambda_p^{(q,r)}(x, \lambda) \mid 1 \leq p \leq m\}$. (Of course, the x and λ are fixed relative to the (q, r).)

A logic-system need not be confined to language elements. Using a similar procedure as appears in section 2, there is a $Y_m^{(q,r)} \in {}^*\mathbf{M}_m^{y(q,r)}$ such that for each p, $1 \leq p \leq m$, ${}^*\Lambda_p^{(q,r)}(x, \lambda) \in \underline{{}^*\mathcal{A}}'({}^*\underline{\mathcal{S}}^{y(q,r)}, \{Y_m^{(q,r)}\})$. Hence, the members of the $\{\Lambda_p^{(q,r)}(x, n) \mid 1 \leq p \leq m\}$ can be considered as *deduced.

An original alteration can be miniscule and made in one or more of the necessary parameters that are satisfied by a specific cosmology. This can be done in such a way that only miniscule alterations in physical-systems satisfy the alterations. On the other hand, a highly altered cosmology can also occur. Alterations are considered as those initiated by a collection of human mental activities via the Eccles and Robinson notion of "mental intentions" and is local prior to it being propagated during a universe's development. Each *instruction paradigm is consider as the result of the corresponding *developmental paradigm.

For various interpretations, only the set $\{\,^*\mathbf{\Lambda}_p^{(q,r)}(x,\lambda) \mid 1 \leq p \leq m\}$ needs to be considered. Further, there are different approaches as to the activation of the members of $\{\,^*\mathbf{\Lambda}_p^{(q,r)}(x,\lambda) \mid 1 \leq p \leq m\}$ (Herrmann, (2013a)). Notice that, for each type of cosmology, there is a "last" step in the development of each member of $\{\,^*\mathbf{\Lambda}_p^{(q,r)}(x,\lambda) \mid 1 \leq p \leq m\}$.

Let fixed $K' \in \mathbb{N}'$. Then for each appropriate $i \in {}^*\mathbf{Z}$ and $j \in {}^*\mathbb{N}$, $j \leq \lambda$, each moment in primitive-time ${}^*t^{(q,r)}(i,j) = 1/K'(i + 1 - 1/2^j)$, there is a defined ${}^*\mathbf{\Lambda}_{p'}^{(q,r)}(x,\lambda) \in \{\,^*\mathbf{\Lambda}_p^{(q,r)}(x,\lambda) \mid 1 \leq p \leq m\}$ that generates, if any, all previous universe-wide frozen-frames, whether they stem from alterations or not. This particular ${}^*\mathbf{\Lambda}_{p'}^{(q,r)}(x,\lambda)$ also contains at ${}^*t^{(q,r)}(i,j+1)$, or if $j = \lambda$, then at ${}^*t^{(q,r)}(i+1,0)$, any altered universe-wide frozen-frame. For certain theological interpretations, this or the corresponding *developmental paradigm also can correspond to a "history" file. Thus, as a participator universe progresses, preceding, if any, universe-wide frozen-frame can either come about relative to the unique ${}^*\mathbf{\Lambda}_p^{(q,r)}(x,\lambda)$ or other similar unique members of $\{\,^*\mathbf{\Lambda}_p^{(q,r)}(x,\lambda) \mid 1 \leq p \leq m\}$ that satisfy participator alterations.

Depending upon the q, each universe has, with respect to $\preceq$, a "first" universe-wide frozen-frame at $(x,0)$. Hence, for a hyperfinite restricted *rational number interval $[\,^*t^{(q,r)}(x,0), {}^*t^{(q,r)}(i,j)]$ and relative to realized alterations, there is a defined member ${}^*\mathbf{\Lambda}_{p''}^{(q,r)}(x,\lambda)$ of $\{\,^*\mathbf{\Lambda}_p^{(q,r)}(x,\lambda) \mid 1 \leq p \leq m\}$ and a complete hyperfinite developmental paradigm that descriptively details the specific members of the realized universe that includes all realized alterations. The application of a scheme such as (S′) can be sequentially preceded by ${}^*\underline{\mathcal{A}}'(\,^*\underline{\mathcal{S}}^{y(q,r)}, \{Y_m^{(q,r)}\}) = \{\,^*\mathbf{\Lambda}_p^{(q,r)}(x,\lambda) \mid 1 \leq p \leq m\}$.

10. The Physical Book Model

For the book and chapter model that appears in Herrmann (2013a), each member of $\mathcal{U}$ (and any corresponding *developmental paradigm), corresponds to a book, where a chapter determines the step-by-step construction of a specific universe-wide frozen-frame. Sequentially prior to an alteration of a universe-wide frozen-frame, there is a specific p-book chapter with (i,j) realized. The altered chapter is a member of a k-book and realization begins either with chapter $(i+1,0)$ or chapter $(i,j+1), 0 < j+1 \leq \lambda$.

11. More GGU-model Schemes.

This is a slight modification of the *instruction-information (GGU) model (Herrmann, 2013). Recall that $G_\lambda^{q,r}[\{h^{(q,r)}(i,j;k) \mid k \in \mathbf{Z}_r\}] = IF_\lambda^{(q,r)}(i,j)$, where it is assumed that this info-field has λ-complexity. In the standard schemes (S), (S′) above, the $\mathtt{St}$ operator is applied to each intermediate properton or the coded *-descriptions for physical characteristics. It applies to nothing else. For (S), (S′), one considers a

complete (i,j) "universe-wide frozen-frame" as denoted by $\mathtt{St}(IF_{\lambda}^{(q,r)}(i,j))$. The info-fields are for a participator universe, if the universe is such. As with the notion of quantum fields, for what follows, the info-fields are considered as present in the substratum. Thus, notationally, for the entire primitive time sequence, a complete universe is denoted by

$$\text{(PWM)}\ \{\mathtt{St}(IF_{\lambda}^{(q,r)}(i,j)) \mid (\alpha \le i \le \beta) \wedge (0 \le j \le \lambda)\} \Rightarrow \{E^{(q,r)}(i,j)\},$$

for the appropriate α, β and λ as previous defined. Notice that for each (i,j), (n,m),

$$(i,j) \preceq (n,m) \leftrightarrow E^{(q,r)}(i,j) \le E^{(q,r)}(n,m),$$

where $\le$ is order isomorphic to $\preceq$. That is, the physical events are ordered with respect to primitive-time, and, when restricted, to observer-time.

For the participator universe, there are sets of info-fields $\{\{IF_{p}^{(q,r)}(i,j) \mid (\alpha \le i \le \beta) \wedge (0 \le j \le \lambda)\} \mid 1 \le p \le m\}$. One considers $\mathtt{St}$ as p dependent. The hyper-fast properton selection process preserves the order $\preceq$. The $\mathtt{St}_p$ is applied to the appropriate (i,j) member of the p member of $\{\{IF_{p}^{(q,r)}(i,j) \mid (\alpha \le i \le \beta) \wedge (1 \le j \le \lambda)\} \mid 0 \le p \le m\}$.

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Herrmann, R. A., (1989), Fractals and ultrasmooth microeffects, J. Math. Physics, 30(4), :805-808. (Note that there are typographical errors in this paper. The statement of Theorem 4.1 (iii) should read "*-differentiable of order n." In the proof of Theorem 4.1, in equations $h(x, c, d)$, $G_j(x)$, the $) +)$ should be $)) +)$. In $G_j(x)$, the second c should be replaced with a_j. On page 808, the second column, second paragraph, line six, $\mathtt{st}(D)$ should read $\mathtt{st}(^*D)$ and, trivially, $x \in \mu(p)$, should read $x \in \mu(p) \cap {}^*D$. In the proof of Theorem 3.1, first line $^*\mathbf{R}^m$ and should read $\mathbf{R}^m$.

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