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# $E = Mc^2$ DILEMMA

**Radwan M. Kassir** ©2018
radwan.elkassir@dar.com

**Abstract**

In its basic form, the Special Relativity's famous mass-energy equivalence equation doesn't seem to hold for massless particles. To get around this deficiency, another form (energy-momentum relation) of the same equation incorporating a momentum term has been introduced by the physics mainstream. The adopted mainstream derivation of the energy-momentum relation is based on certain assumptions using Minkowski spacetime four-momentum, forcing a solution towards the energy-momentum relation. This form of the mass-energy equivalence equation appears to hold for massless particles. However, in this paper, an analysis of the mainstream derivation of the energy-momentum relation is carried out, highlighting the made assumptions, and revealing a resulting contradiction. It is clearly shown, through a straight forward derivation, that the energy-momentum relation still doesn't hold for massless particles. Consequently, the energy-momentum relation requires that a photon must have an infinitesimal rest mass, for which its speed only approaches $c$. Thus, the speed of light will no longer be invariant with respect to all inertial reference frames, as postulated by Einstein in his formulation of the Special Relativity!

**Exposing Derivation of the Energy-Momentum Form of the Mass-Energy Equivalence Equation**

The mass-energy equivalence equation in its original form

$$E = Mc^2,$$

where $M$ is the relativistic mass (total mass), or

$$E = \gamma mc^2,$$

(where $m$ is the rest mass, and $\gamma = 1\big/\sqrt{1 - v^2/c^2}$) does not hold for massless particles $(m = 0)$, such as photons. In order to overcome this limitation, another form of the mass-energy equivalence equation has been derived, namely,

$$E^2 = \left(mc^2\right)^2 + \left(pc\right)^2$$

—where $p$ is the momentum of the particle—which gets around the aforementioned issue by simply yielding

$$E = pc$$

for massless particles (i.e., for $m = 0$).

The mainstream adopted derivation of the mass-energy equation in the energy-momentum relation form

$$E^2 = \left(mc^2\right)^2 + \left(pc\right)^2$$

is based on the four-momentum

$$p^\mu = \left(p^0, p^1, p^2, p^3\right) = \left(\frac{E}{c}, p_x, p_y, p_z\right),$$

with a norm assumed equal to $mc$, and equating Minkowski inner product $\left(p^\alpha \eta_{\alpha\beta} p^\beta\right)$ of the four-momentum to the square of the norm ($m^2c^2$)—$\eta_{\alpha\beta}$ being the Minkowski metric tensor.

The four-momentum therein is developed from the assumed four-velocity

$$u^\mu = \left(u^0, u^1, u^2, u^3\right) = \gamma\left(c, v_x, v_y, v_z\right)$$

and—by definition—

$$p^\mu = mu^\mu = \left(\gamma mc, \gamma mv_x, \gamma mv_y, \gamma mv_z\right),$$

which leads to the aforementioned four-momentum equation by replacing $E = \gamma mc^2$ in the first component (yielding $E/c$), and subsequently to $E = pc$ for $m = 0$.

On the other hand, had we replaced $E = \gamma mc^2$ in all of the four-momentum components, we would get

$$p^\mu = \frac{E}{c^2}\left(c, v_x, v_y, v_z\right),$$

which implies that even a body at rest $(v_x = v_y = v_z = 0)$ would have a momentum component of $E/c = \gamma mc$, which is the momentum of a massless particle (since, for a massless particle, $E = \gamma mc^2 = pc$), as well as $\gamma$ times the four-momentum norm —an obvious contradiction!

The above derivation is based purely on the aforementioned assumed four-velocity components and the assumed norm of the resulting four-momentum. In addition, evidently this derivation wouldn't be possible with $m = 0$.

***Simplified Derivation***

A straight forward derivation of the mass-energy equivalence in its energy-momentum relation form—still revealing it doesn't hold for massless particles—is introduced next.

Let's begin with the basic mass-energy equivalence equation in its original form

$$E = \gamma mc^2,$$

which is the same as

$$E = mc^2 + \delta mc^2$$

where $m$ is the rest mass, and $\delta m = m(\gamma - 1)$.
Squaring the above equation gives

$$E^2 = \left(mc^2\right)^2 + \left(\delta mc^2\right)^2 + 2mc^2 \delta mc^2$$

Or

$$E^2 = \left(mc^2\right)^2 + \left[m(\gamma-1)c^2\right]^2 + 2m^2c^2\left(\gamma-1\right)c^2;$$

$$E^2 = \left(mc^2\right)^2 + m^2c^2\left(\gamma-1\right)\left[(\gamma-1)c^2 + 2c^2\right];$$

$$E^2 = \left(mc^2\right)^2 + m^2c^2c^2\left(\gamma^2 - 1\right).$$

Now, let

$$p = \gamma mv$$

Then (with $\gamma = 1\big/\sqrt{1 - v^2/c^2}$)

$$p^2 = \frac{m^2v^2}{1 - \frac{v^2}{c^2}};$$

$$p^2 = \frac{m^2c^2}{\frac{c^2}{v^2} - 1};$$

$$p^2 = m^2c^2\left(\gamma^2 - 1\right).$$

Hence, the above energy-mass equivalence equation becomes

$$E^2 = \left(mc^2\right)^2 + \left(pc\right)^2,$$

where

$$p = \gamma mv.$$

It follows that, the above derivation reveals the mass-energy equivalence equation invalid for massless particles, since $p$ in the same equation is nothing but $\gamma mv$!

**Failure of the Mass-Energy Equivalence Equation to Predict Massless Particles Energy**

According to Special Relativity, massless particles travel at the speed of light. A typical demonstration is provided by considering the Special Relativity mass-energy equivalence equation of a particle in the form derived above —yet $p$ is not explicitly expressed as $\gamma mv$ —

$$E^2 = \left(mc^2\right)^2 + \left(pc\right)^2,$$

where $m$ is the rest mass of the particle, and arguing that if the rest mass was zero, the particle will have no energy stored as mass, and its energy $pc$ will be purely due to its momentum, with the speed of light $c$ being its velocity. If this massless particle internally acquired a certain mass, it will lose an amount from its "momentum" energy, reducing its speed to $v < c$, so as to maintain the above energy-mass equation.

However, there's a hidden trick in the above interpretation. In fact, considering the mass-energy equivalence equation in the following two forms

$$E^2 = \left(mc^2\right)^2 + \left(pc\right)^2,$$

where $m$ is the rest mass of the particle, and

$$E = Mc^2,$$

where $M$ is the relativistic mass, $\gamma m$, and plugging the latter equation

$$E = \gamma mc^2$$

in the former one, we get

$$\gamma^2 \left(mc^2\right)^2 = \left(mc^2\right)^2 + \left(pc\right)^2;$$

$$p^2c^2 = \left(mc^2\right)^2 \left(\gamma^2 - 1\right);$$

$$p^2 = m^2c^2 \left(\gamma^2 - 1\right).$$

But, since

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2},$$

Then

$$\left(\gamma^2 - 1\right) = \frac{v^2}{c^2}\gamma^2,$$

where $v$ is the velocity of the particle. Then

$$p^2 = m^2c^2\frac{v^2}{c^2}\gamma^2;$$

$$p = \gamma mv.$$

The mass-energy equivalence equation can therefore be written as

$$E^2 = \left(mc^2\right)^2 + \left(\gamma mvc\right)^2,$$

and for massless particle (for which $\gamma$ takes the undefined form $1/0$), it takes the undefined form $E = 0/0$!

For instance, photon, a massless particle, does have momentum, but not according to the mass-energy equivalence equation, in which the momentum is given by $\gamma mv$, and which predicts undefined energy for massless particles! Imposing $p$ to be independent of the mass in the mass-energy equivalence equation is a hoax used to get around Special Relativity deficiency in predicting the energy of a massless particle!

**Photon must be Massive – within the SR Framework!**

This dilemma of the Special Relativity's mass-energy equivalence equation deficiency might be resolved if the photon rest mass was infinitesimal (its mass approaches zero instead of being massless), in which case its speed can only approach the speed of light $c$ (otherwise it would acquire an undetermined relativistic mass if it traveled at $c$), and $\gamma$ becomes infinitely large. In such a condition only, we can set, for a photon, the limit of $\gamma mv$ in the energy-mass equivalence equation, to the momentum $p$ of the photon — when its rest mass $m$ is infinitesimally small, its speed $v$ approaches the speed of light $c$, and hence $\gamma$ becomes infinitely large!

Therefore, the photon energy should be expressed as

$$E^2 = \lim_{m \to 0}\left[\left(mc^2\right)^2 + \left(\gamma mvc\right)^2\right] = \left(pc\right)^2$$

It follows that, for the mass-energy equivalence equation to seem to hold for a photon, and yield the adopted photon energy formula $(E = pc)$, the photon must have an infinitesimal rest mass, for which its speed only approaches the speed of light!

Hence, the speed of light is no longer the upper speed limit. What will then be the value of the upper speed limit $c$? It remains unknown, and could be any value greater than the light (photon) speed!
It follows that all calculations of time dilation, length contraction, relativistic mass, and Doppler shift, assuming the upper speed limit $c$ to be the speed of light (photon), will be wrong!

The implication of the speed of light being not the upper speed limit, is that the speed of light will no longer be invariant with respect to all inertial reference frames, as postulated by Einstein in his formulation of the Special Relativity!