**EXISTENCE OF PRIME NUMBERS IN SUBSETS OF THE OPPERMANN'S INTERVALS**

ANIRILASY Méleste

***ABSTRACT**. We suggest that there exists, at least, one prime number in four intervals between $n^2$ and $(n+1)^2$ for any integer $n \geq 2$ such that :*

- *all intervals are half-open;*
- *the excluded endpoints are multiples of n;*
- *the number of elements in each interval is equal to the least even upper bound for the biggest prime number strictly less than n.*

*This conjecture is a strong form of Oppermann's one.*

**MSC : 11N05**
**Mathematical statement :** conjecture
**Keywords**: prime numbers, Legendre's conjecture, Oppermann's conjecture.



## 1. INTRODUCTION

For every positive integer n, Legendre's conjecture [1] states that there is, at least, one prime number between $n^2$ and $(n+1)^2$. We note that any integer strictly less than n has, at least, two multiples in that interval.
Oppermann's conjecture [2] is a strong form of Legendre's conjecture because there would be, at least, two prime numbers between $n^2$ and $(n+1)^2$. We also note that any integer strictly less than n has, at least, one multiple between $n^2$ and $n(n+1)$ and another one between $n(n+1)$ and $n(n+2)$.
In this paper, we consider as reference interval the Legendre's one by separating it into subsets in which any prime number strictly less than n has, at least, one multiple and we suggest the conjecture mentioned below . We then verified the proposed conjecture below 1,193,806,024.

## 2. RESULTS

**Conjecture 2.1.** Let m be the biggest prime number strictly less than n. Then

$\forall\ n > 2$ , $\exists\ p_1,p_2,p_3,p_4$ primes / $n^2 < p_1 \leq n^2 + (m+1)$ [first interval]
and $n(n+1) - (m+1) \leq p_2 < n(n+1)$ [second interval]
and $n(n+1) < p_3 \leq n(n+1) + (m+1)$ [third interval]
and $n(n+2) - (m+1) \leq p_4 < n(n+2)$ [fourth interval]

We tested the statement and the result shows that the statement holds to n = 1,700. Then, by using a table of maximal gaps, the conjecture is verified for all n up to 1,193,806,024.

**Examples 2.2.**
For n = 5 : m = 3 and the first interval is ]25;29] which contains the prime 29.
For n = 8 : m = 7 and the second interval is [64;72[ which contains the primes 67 and 71.

For n = 9 : m = 7 and the third interval is ]90;98] which contains the prime 97.
For n = 10 : m = 7 and the fourth interval is [112;120[ which contains the prime 113.

**Remark 2.3.** The value of n has to be strictly greater than 2 because no prime number is strictly less than 2.

**Remark 2.4.** $p_1$ and $p_2$ may not be necessarily distinct because the intersection of their sets is not empty.

**Proof.** According to the Bertrand-Chebyshev theorem [3] :

$$\frac{n}{2} < m < n$$

$$\Rightarrow \quad n^2 + \frac{n}{2} < n^2 + m < n^2 + n \qquad \text{with } n^2 + n = n(n+1)$$

$$\Leftrightarrow \quad n^2 + \frac{n}{2} < n^2 + m < n^2 + (m+1) < n(n+1)$$

By analogy :

$$\frac{n}{2} < m < n$$

$$\Rightarrow \quad -n < -m < -\frac{n}{2}$$

$$\Rightarrow \quad n(n+1) - n < n(n+1) - m < n(n+1) - \frac{n}{2}$$

$$\Leftrightarrow \quad n^2 < n(n+1) - m < n(n+1) - \frac{n}{2}$$

$$\Leftrightarrow \quad n^2 < n(n+1) - (m+1) < n(n+1) - m < n(n+1) - \frac{n}{2}$$

Since $n(n+1) - \frac{n}{2} = n^2 + n - \frac{n}{2} = n^2 + \frac{n}{2}$ ,

then $n^2 < n(n+1) - (m+1) < n^2 + \frac{n}{2} < n^2 + (m+1) < n(n+1)$

**Remark 2.5.** $p_3$ and $p_4$ may be the same prime number. This can be proved by the same way we prove that $p_1$ and $p_2$ may be the same prime numbers.

## 3. REFERENCES


[1] *J. R. Chen, ''On the distribution of almost primes in an interval", Sci. Sinica, vol. 18, no 5, 1975, p. 611-627*

[2] *L.Oppermann,"Om vor Kundskab om Primtallenes Mængde mellem givne Grændser", Oversigt over det Kongelige Danske Videnskabernes Selskabs Forhandlinger og dets Medlemmers Arbejder, 1882, p. 169–179, 1882.*

[3] *M.J.Bertrand, "Mémoire sur le nombre de valeurs que peut prendre une fonction quand on y permute les lettres qu'elle renferme", Journal de l'École royale polytechnique, vol. 18, cahier 30, 1845, p. 123-140.*

E-mail : meleste1@hotmail.com