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# Invitation Letter

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To Whom It May Concern,

The International Scientific Committee has the pleasure of inviting you as oral presenter delegate to the ICGRG 2018: 20th International Conference on General Relativity and Gravitation to be held in Sydney, Australia on January, 29-30, 2018.

You are strongly urged to submit your proof of payment document to conference registration secretariat as soon as December, 26, 2017 at the very latest.

We look forward to your participation in the ICGRG 2018: 20th International Conference on General Relativity and Gravitation.

Sincerely,

International Scientific Committee
ICGRG 2018
Sydney, Australia

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# The Contribution of the Energy-Momentum Pseudotensors to the Total Mass Is Positive

R. I. Khrapko

***Abstract*****—**As is well known, the energy-momentum pseudotensors of the gravitational field allow you to calculate the total mass of a body and its gravitational field. We performed such a calculation and found a positive contribution of the pseudotensor to the mass.

***Keywords*****—**Conservation law, covariant integration, gravitation field, isolated system.

## I. Introduction. Energy of the Gravitation Field is Negative

When an electron is attracted to a proton from infinity by the force of the electric field, the energy of the electric field is converted into the kinetic energy, and a part of the electric field is eliminated. As a result, in harmony with the mass-energy conservation law, we have the excitation energy of the atom instead of the energy of the part of the electric field. This means that the total mass-energy of the system is conserved and the mass-energy of the matter is increased due to the disappearance of the field. Then 13.6 eV are radiated, and we get the neutral atom without an external electric field.

Now consider compression of a dilute dust cloud under the action of own gravitational attraction forces. If we adhere to the concept of gravitational field, the cloud is surrounded by its gravitational field. In the compression process, the kinetic energy appears. And this kinetic energy will be converted into the thermal energy when the compression is stopped by the pressure forces. As a result, the mass-energy of the matter is increased in the same way as it is increased at the electron-proton attraction. But in the case of gravitational attraction, the gravitational field of the dust is not eliminated. Instead, the gravitational field strengthened and extends to the volume that has become free from the cloud. So, if we trust that the mass-energy of the system "cloud + its gravitational field" is conserved in the process of compression, we have to ascribe a negative mass-energy to the gravitational field.

Apperdix A in [1] demonstrates the negativity of gravitational energy by the example of a thin sperical shell of mass.

The standard method to calculate the mass-energy of matter, $M$, implies the use of the energy-momentum tensor $T^\nu_\mu$. The standard method to calculate the mass-energy of gravitational field, $G$, implies the use of the Einstein pseudotensor [2]-[4] or Landau-Lifshitz pseudotensor [5] $t^\nu_\mu$. The standard method to calculate the total mass-energy of a system "matter + its gravitational field", $J = M + G$, implies the use of the sum $T^\nu_\mu + t^\nu_\mu$. The contribution of the pseudotensor to the mass of the system must be negative, $G < 0$, in order to compensate for the growth of the mass of the matter under compression if we trust that $J = M + G = \text{Const}$.

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## II. Calculation of the Mass-Energy of Matter

An infinitesimal space element $dV_\nu$ contans the 4-momentum element of the matter

$$dP_\mu = T^\nu_\mu \sqrt{-g}dV_\nu; \qquad (1)$$

here $T^\nu_\mu \sqrt{-g}$ is the energy-momentum *tensor density*, and $g$ is the determinant of the metric tensor (see e.g. § 96 in [5]). But a calculation of the total 4-momentum is not possible in general because we cannot sum up vectors (1), which components are defined in different points of a curved space-time. However, in the case of a static body, the infinitesimal 4-momentums (1) are parallel to each other and have only time components $dP_t$ in a static coordinate system. So, in this case it is possible to determine the total mass-energy of a body $M$ by integrating of modules of 4-momentums (1), $dM = dP_t / \sqrt{g_{tt}}$, over space in this coordinate system:

$$.M = \int T^t_t (\sqrt{-g} / \sqrt{g_{tt}}) dV_t . \qquad (2)$$

Note, that $T^t_t = \rho$ is the mass-energy *volume* density (see (95.10) in [3] or § 100 in [5]), and that $dV_t = dxdydz$ or $dV_t = drd\theta d\varphi$.

Let us use (2) to calculate the mass-energy of a sphere of perfect fluid. The space-time of such a body is discribed by the Schwarzschild's internal and external solutions of the Einstein equation. The internal solution [6] (see also § 96 in [3]) depends on two parameters, $R$ and $r_1$:

$$ds^2 = \left(\frac{3}{2}\sqrt{1-\frac{r_1^2}{R^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{R^2}}\right)^2 dt^2$$

$$-\left(1-\frac{r^2}{R^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\varphi^2, \qquad (3)$$

$$\sqrt{-g}_{\wedge} = \sqrt{-g_{tt}g_{rr}}\, r^2 \sin\theta . \qquad (4)$$

Here $0 \le r \le r_1 \le R$, $r_1$ is the coordinate of the surface where the external Schwarzschild spaceitime is attached, and $R$ is the curvature radius of the inner space determined by the fluid volume density

$$.T_t^t = \rho = 3/(8\pi R^2). \quad (5)$$

The external solution,

$$ds^2 = (1-2m/r)dt^2 - (1-2m/r)^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\varphi^2 \quad (6)$$

depends on the single (Schwarzschild) parameter $m = r_g/2$, which describes external gravitational field. It is seen that at smooth matching

$$m = r_1^3 / 2R^2. \quad (7)$$

In view of the Birkhoff's theorem, the parameter $m$ remains constant under the compression of the sphere.

Now the mass-energy of the sphere can be calculated by using (2):

$$M = \int T_t^t (\sqrt{-g}/\sqrt{g_{tt}}) dr d\theta d\varphi = \int_0^{r_1} T_t^t \sqrt{-g_{rr}} r^2 dr 4\pi \quad (8)$$

Using (5) and $-g_{rr} = (1-r^2/R^2)^{-1}$ yields

$$M = \int_0^{r_1} \frac{3}{2R}\frac{r^2 dr}{\sqrt{R^2 - r^2}} = \frac{3R}{4}(\sin^{-1}\xi - \xi\sqrt{1-\xi^2}), \quad \xi = r_1/R. \quad (9)$$

Using the series expansions

$\sin^{-1}\xi = \xi + \xi^3/6 + 3\xi^5/40 + \ldots$,

$\sqrt{1-\xi^2} = 1 - \xi^2/2 - \xi^4/8 + \ldots$ yields

$$M = R\xi^3/2 + 3R\xi^5/20 + .. = m + 3m^2/(5r_1) + \ldots \quad (10)$$

So, $M > m$. This excess of the matter mass $M$ over the Schwarzschild parameter $m$ was named the (positive) gravitational mass defect (§ 100 in [5]). The point is the parameter $m$ is the *total* mass, i.e. mass of matter *and* of its gravitational field, and this total mass $m$ does not change when compressing of the sphere, according to the Birkhoff's theorem. This mean that the energy of the gravitation field must be equal $G = -3m^2/(5r_1) + \ldots < 0$ if we wish the total mass, $J = M + G$, to be conserved.

## III. Calculation of the Mass-Energy of the Gravitation Field

Now we must evaluate the integral

$$.G = \int t_t^t (\sqrt{-g}/\sqrt{g_{tt}}) dV_t, \quad (11)$$

which is analogical to (2). Here $t_t^t\sqrt{-g}$ is the component of the Einstein or Landau-Lifshitz *pseudotensor density*. The Einstein pseudotensor density is [2]-[4]

$$t_\mu^\nu\sqrt{-g} = \frac{1}{16\pi}\left(-\partial_\mu(g^{\alpha\beta}\sqrt{-g})\frac{\partial\mathfrak{L}}{\partial(\partial_\nu(g^{\alpha\beta}\sqrt{-g}))} + \delta_\mu^\nu\mathfrak{L}\right),$$

$$\mathfrak{L} = g^{\alpha\beta}\sqrt{-g}(\Gamma^\nu_{\alpha\mu}\Gamma^\mu_{\beta\nu} - \Gamma^\mu_{\alpha\beta}\Gamma^\nu_{\mu\nu}), \quad (12)$$

The expression (12) is awkward. Luckily, we can use a result of a calculation that was performed by Tolman. For a quasi-static isolated system with respect to quasi-Galilean coordinates, Tolman integrated the sum of *the time components* of the mass-energy matter element $T_t^t\sqrt{-g}dV_t$ and of the mass-energy field element $t_t^t\sqrt{-g}dV_t$

$$dJ_t = (T_t^t + t_t^t)\sqrt{-g}dV_t. \quad (13)$$

The integration gives (see (91.1), (92.4), (97.5) in [3] or [7])

$$U = \int dJ_t = \int (T_t^t + t_t^t)\sqrt{-g}dV_t = \int (T_t^t - T_1^1 - T_2^2 - T_3^3)\sqrt{-g}dV_t = \int (\rho + 3p)\sqrt{-g}dV_t = m \quad (14)$$

where $p$ is pressure.

This integral quantity is not the mass of the system and even is not a component of a vector because it is obtained by adding up the components $dJ_t$ belonging to different points where the coordinate bases differ from each other. And there is no coordinate basis to which the quantity (14) could belong as a component. We cannot write $J_t$ because it is senseless. Nevertheless, this result shows that the integral

$$\int t_t^t\sqrt{-g}dV_t = \int 3p\sqrt{-g}dV_t > 0 \quad (15)$$

is *positive* because pressure $p$ is positive. And this means a very important fact: the contribution of the Einstein pseudotensor to the mass of the system, (11), is also positive because $\sqrt{g_{tt}} > 0$:

$$.G = \int t_t^t(\sqrt{-g}/\sqrt{g_{tt}})dV_t > 0. \quad (16)$$

The Landau-Lifshitz pseudotensor gives the same result; it is stated by formula (105.23) in [5].

So, the pseudotensors give positive value for the mass-energy of the gravitational field. The conservation law turns out to be broken.

$$M > m,\ G > 0, \text{ so, } J = M + G > m. \quad (17)$$

This means that the pseudotensors do not represent gravitational mass-energy, which is negative. (If we adhere to the concept of gravitational field and trust that the mass-energy of the system is conserved.)

## IV. Why the Invalidity of the Mass-Energy was not Noticed?

Equation (14), $U = \int (T_t^t + t_t^t)\sqrt{-g}dV_t = m = \text{Const}$, creates an illusion that the pseudotensor $t_t^t$ ensures conservation of the sum of the matter mass $\int T_t^t\sqrt{-g}dV_t$ and the field mass $\int t_t^t\sqrt{-g}dV_t$. But, in reality, the integral (14) does not represent masses at all. Equation (14) is an inadmissible integral of covariant components of the infinitesimal vectors. The correct total mass $J$ (17) is larger than the quantity (14) $U$ because $g_{tt} < 1$ [8]:

$$J = M + G = \int dJ_t/\sqrt{g_{tt}}$$

$$= \int (T_t^t + t_t^t)(\sqrt{-g} / \sqrt{g_{tt}}) dV_t > m \,. \quad (18)$$

## V. Conclusion

We have to admit that we do not know how to describe the energy of a stationary gravitational field in the frame of the General Relativity. The pseudotensors are not suitable for this.

The noncovariant equation (35) in [9]

$$\partial_\nu [(T_\mu^\nu + t_\mu^\nu)\sqrt{-g}] = 0 \qquad (19)$$

means the conservation of the quantity (14)

$$U = \int (T_t^t + t_t^t)\sqrt{-g} dV_t = m = \text{Const} \,,$$

which has no geometrical and physical sense.

And here we do not touch the problem of the energy of gravitational waves.

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