Cosmology without a cosmological constant.

John Hunter[1]

ABSTRACT

In order to deal with the coincidence problem a new scalefactor-redshift relation is considered. With the new relation a constant rate of expansion with no cosmological constant matches observations well. There is thus the possibility that the dark energy phenomenon is an illusion caused by the use of an incorrect relation.
The new relation arises naturally in a cosmological model which expands - yet appears static, incorporating the perfect cosmological principle. It is found that the new relation also leads to a natural resolution of the tension between Hubble constant values from the distance ladder and Cosmic Background Radiation.

INTRODUCTION

The LCDM model is currently favoured by the majority of cosmologists. The dark energy component, although lacking a theoretical understanding, has enabled the model to successfully match different sets of observations.

1) The distance moduli of supernovae (Betoule [1])

2) Measurements of the CMB, e.g. WMAP (Hinshaw [2]) and Planck (Ade [3]) appear to show a flat universe. The matter density parameter Ω_m is about 0.25 to 0.31, deduced from measurements of $\Omega_m h^2$, (where h is the Hubble parameter/100km/s/Mpc).

3) There are other observations (Weinberg [4]). Many of these also depend on the value of h or $\Omega_m h^2$

4) Baryon Acoustic Oscillations (Aubourg [5]), give a slightly lower value for h and a higher Ω_m, although there are inconsistences with the distance ladder (Riess [6]) and between the Lyα and galaxy redshift samples. These are reconciled in Appendix C.

Observations 1-4 seem to support each other, so it is understandable that many cosmologists support the dark energy conclusion. However a simple change to our notions of how scale factor relates to redshift can remove many of the arguments in favour of dark energy and a cosmological constant.

1. Independent Researcher. Huddersfield, U.K. john@gravity.uk.com

1. A NEW RELATIONSHIP BETWEEN REDSHIFT AND SCALE FACTOR

Traditionally the wavelength of light received is proportional to the ratio of the scale factor of the universe on emission and reception, giving the redshift of a photon

$$1 + z = \frac{\lambda_0}{\lambda_1} = \frac{a_0}{a_1} \qquad (1)$$

with the new proposal there is a different relation

$$1 + z = \frac{\lambda_0}{\lambda_1} = \left(\frac{a_0}{a_1}\right)^2 \qquad (2)$$

There is an example of a cosmological model which has relation (2) in Appendix B. This model expands, yet appears static, incorporating the perfect cosmological principle. However the consequences of relation (2) are the main subject of this paper, how any model which has relation (2) could lead to the apparent dark energy phenomenon if cosmologists are mistakenly using (1).

The rate of expansion of the universe

$$H(t) = \frac{da/dt}{a} \qquad (3)$$

If relation (2) is true

$$1 + z = (1 + Hdt)^2 = 1 + 2Hdt \qquad \text{for small } Hdt\text{, so}$$

$$v = 2Hd \qquad (4)$$

equation (4) relates the apparent velocity of a source to distance. So Hubbles constant H_0, is twice the rate of expansion $H(t)$, assumed constant, hereafter H.

$$H = \frac{H_0}{2} \qquad (5)$$

2. THE MATTER DENSITY

For a simple constant H solution, $a = a_0 \exp(Ht)$**,** Einsteins equations of General Relativity reduce to

$$8\pi G \frac{\rho}{c^2} = -\Lambda + \frac{3H^2}{c^2} + \frac{3k}{a^2} \qquad (6)$$

$$8\pi G \frac{p}{c^4} = \Lambda - \frac{3H^2}{c^2} - \frac{k}{a^2} \qquad (7)$$

so for a flat universe with $k = 0$, and $\Lambda = 0$

$$p = -c^2 \rho \qquad \text{(i.e. } \omega = -1\text{)} \tag{8}$$

and

$$\rho = \frac{3H^2}{8\pi G} \qquad \text{or} \qquad G = \frac{3H^2}{8\pi\rho} \tag{9}$$

for this, and other solutions, the inferred value for the matter density would be

$$\Omega_m = \frac{\rho}{\rho_{crit}} = \frac{3H^2 / 8\pi G}{3(H_0)^2 / 8\pi G} = 0.25 \tag{10}$$

whereas actually Ω_m is 1, as the denominator of (10) should contain H not H_0.

Measurements from WMAP9 alone, led to an inferred value for the matter density parameter of 0.279 ± 0.025. This value is deduced from measurements of $\Omega_m h^2$ and $\Omega_b h^2$ with, possibly, an incorrect value for h, and would be 1.12 ± 0.1 with h halved.

In this example the ‘coincidence problem’ is avoided. At all times the matter density is 1 and the cosmological constant is 0.

3. THE SUPERNOVAE DATA

With the new approach H is about 35km/s/Mpc and the comoving distance is

$$d_C = \frac{c}{H}\left(\sqrt{1+z} - 1\right) \tag{11}$$

(there is a derivation in Appendix A), so

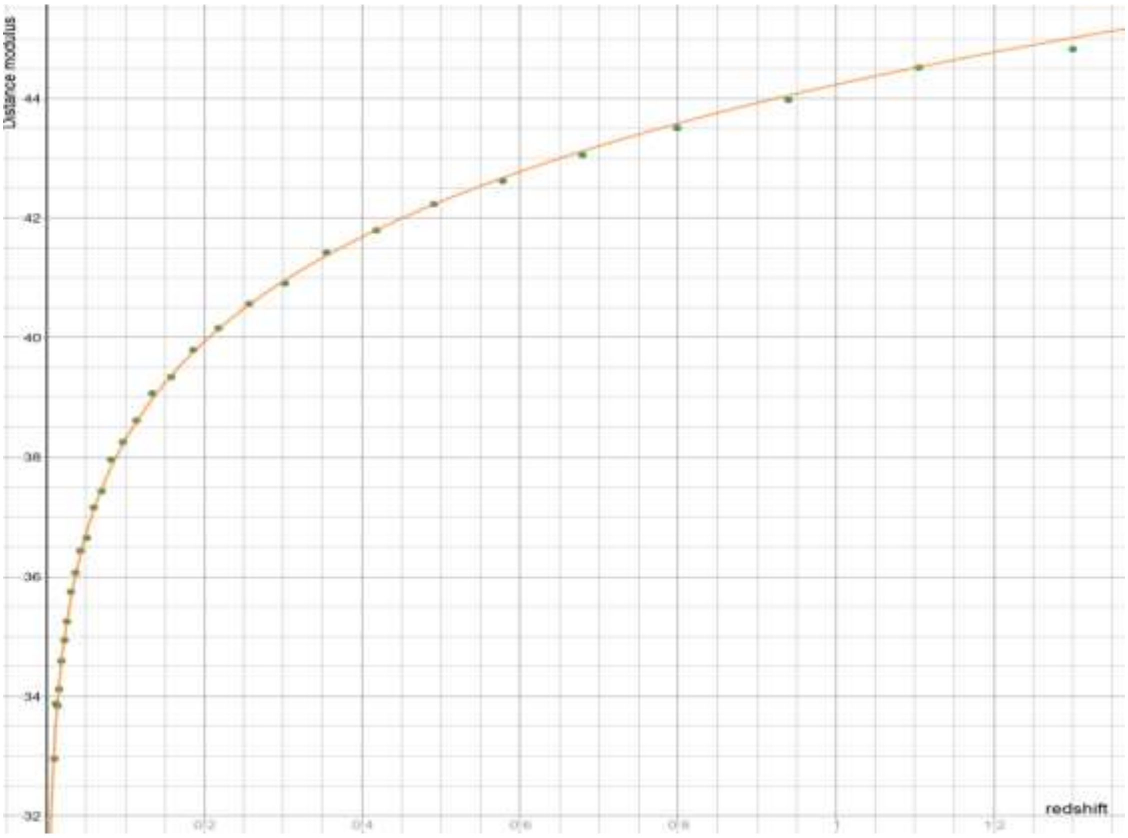
$$d_L = \frac{c}{H}(1+z)\left(\sqrt{1+z} - 1\right) \tag{12}$$

the distance modulus is

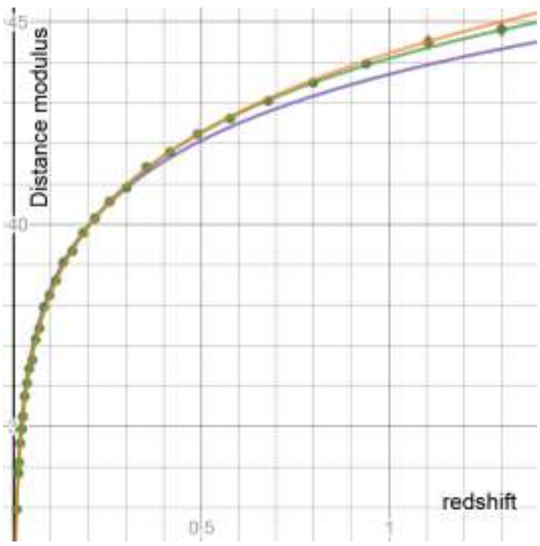
$$\mu = 25 + 5 \log d_L \tag{13}$$

Using (12) in (13), there is a good match to supernovae data. The Betoule binned data, is shown in Figure 1, with the new relation. The best fit is $2H$ constant, 70.9 km/s/Mpc, and no other variable parameters.

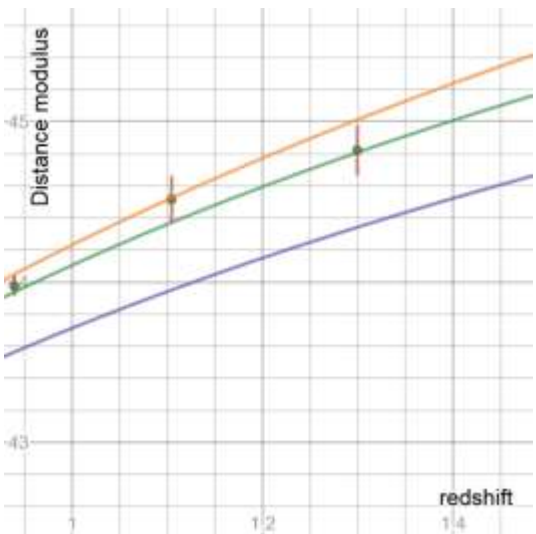
Figure 1a Distance modulus against redshift, (new relation, constant H)



1b New relation, LCDM Ω_m = 0.3 and 1.0



1c Enlargement of 1b



LCDM also gives a close match, middle curve 1b and c, but with two variable parameters, *h* and Ω_m.

The binomial expansions for the LCDM and the new relation are now compared. For LCDM, *E(z)=[m(1+z)^3+1-m]^0.5*, where *m* is short for Ω_m

$$d_C = \frac{c}{H_0}\int_0^z \frac{dz}{E(z)} \tag{14}$$

i.e.

$$d_C = \frac{c}{H_0}\left(z - \frac{3mz^2}{4} + \ldots\right) \tag{15}$$

Whereas for (11) the binomial expansion is

$$d_C = \frac{c}{H_0}\left(z - \frac{z^2}{4} + \ldots\right) \tag{16}$$

So for low *z* a match would occur if Ω_m =1/3 in LCDM. For *z*=1 the match occurs at 0.19, (using equal *H)*. Most of the data is at low redshift, so if LCDM is trying to match formula (12), by varying Ω_m, we would expect it to be slightly lower than 1/3.

5. CONCLUSIONS AND PREDICTIONS

There may have been a long-held misunderstanding of the relationship between the scale factor of the universe and redshift. The relation may be (2). If this is the case, we would expect the following.

1) The inferred value of Ω_m will be 0.25 from CMB data alone.

2) The distance moduli (13) of supernovae, will be according to (12), and the inferred value of Ω_m from supernovae will be slightly lower than 1/3.

3) Future measurements of Hubbles constant will be higher than currently expected, about 75-76 km/s/Mpc.

The cosmological constant may be zero.

ACKNOWLEDGEMENTS

With thanks to Professor Lidsey of Queen Mary College London, for lecture notes used in Appendix A and CosmoQuest members for useful discussions.

REFERENCES

Betoule M et al. Improved Cosmological constraints from a joint analysis of SDSS-II and SNLS supernova samples. 2014, arXiv:1401.4064
Hinshaw G et al. Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations. 2013, arXiv:1212.5226
Ade P et al. Planck 2015 results. XIII. Cosmological parameters. 2016 arXiv:1502.01589
Weinberg D et al. Observational Probes of Cosmic Acceleration. 2013 arXiv:1201.2434
Aubourg E et al. Cosmological Implications of Baryon Acoustic Oscillation Measurements. 2015 arXiv:1411.1074
Riess A et al. A 2.4% Determination of the Local Value of the Hubble Constant. 2016 arXiv:1604.01424
Ade P.et al. Planck 2015 results. XV. Gravitational lensing . 2015 arXiv:1502.01591
Matsuo K, Otsubo T. Temporal variations in Earth's gravity field. 2015 cddis.nasa.gov
Zwicky F. Theoretical Perspectives, PNAS, 90, 4827-4834. 1933

APPENDIX A: DERIVATION OF (12)

Starting from the Robertson-Walker metric

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \qquad \text{(A1)}$$

in terms of the co-moving co-ordinates, χ has the role of the radial co-ordinate

$r = \sin\chi$ if $k = +1$
$r = \chi$ if $k = 0$
$r = \sinh\chi$ if $k = -1$

For ray of light moving along a radial path with θ and φ constant , for a flat universe,

$$ds^2 = -c^2dt^2 + a(t)^2 d\chi^2 = 0 \qquad \text{(A2)}$$

so

$$ad\chi = -cdt = -c\frac{dt}{da}da = -c\frac{da}{\dot{a}} = -c\frac{da}{aH} \qquad \text{(A3)}$$

for the new relation (2)

$$1 + z = \left[\frac{a_0}{a_t}\right]^2 \qquad \text{(A4)}$$

$$dz = -2\frac{a_0^2}{a^3}da \qquad \text{(A5)}$$

so from (A3)

$$ad\chi = \frac{c}{2H}\frac{a^2}{a_0^2}dz \qquad \text{(A6)}$$

$$a_0 d\chi = \frac{c}{2H}\frac{a}{a_0}dz \qquad \text{(A7)}$$

from (2)

$$a_0 d\chi = \frac{c}{2H}\frac{1}{\sqrt{1+z}}dz \qquad \text{(A8)}$$

$$d_C = \int_0^z \frac{c}{2H}(1+z)^{-\frac{1}{2}}dz \qquad \text{(A9)}$$

$$d_C = \frac{c}{H}\left(\sqrt{1+z}-1\right) \qquad \text{(A10)}$$

which is (11), with H about 36km/s/Mpc and

$$d_L = \frac{c}{H}(1+z)\left(\sqrt{1+z}-1\right) \qquad \text{(A11)}$$

which is (12).

APPENDIX B: AN EXPANDING UNIVERSE WHICH APPEARS STATIC.

B1 Introduction.

A cosmological model is presented based on a symmetry principle. It provides an understanding of the origin of gravitation, and is an example of a type of model with the redshift-scalefactor relation equation (2).

Newtons Law of gravitation provides a description of the forces between masses. Einstein, with General Relativity, provided a more accurate description from how masses curve space-time. However neither of these theories deal with the question of why gravity exists.

Why does the attractive force depend on the size of a mass, or why does a larger mass curve spacetime more than a smaller mass? Why does gravitation exist at all? Why does G have its value? All these are unanswered.

A symmetry principle can help with these questions.

Einstein found that his equations did not allow a static universe. The universe must be expanding or contracting. Einsteins equations do not specify that only distances between galaxies are expanding. The expansion is not limited in principle to cosmological distances. If the expansion is for material objects and masses as well as distances between the masses, we find a simple and satisfying reason for the existence of gravitation, discussed in B4.

There now follows a fundamental reinterpretation of what Einstein's equations are telling us. Instead of the rate of expansion of the universe, H, being changed by gravitation – it is the expansion which causes gravitation. Gravitation is the result of the expansion hereafter called 'rescaling' and a symmetry principle, the rescaling symmetry principle.

The rescaling is of rate H, constant and unaffected by gravitation or any other force. It exists, postulated as a fundamental physical reality with H being a fundamental physical constant.

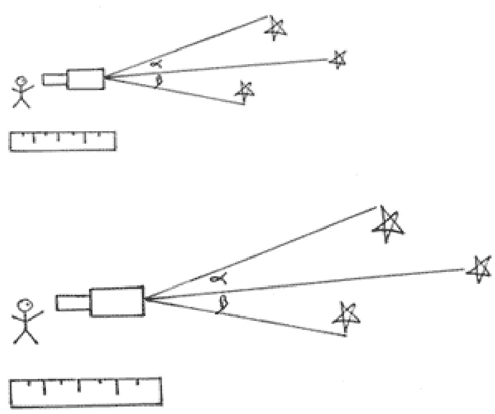
B2 The rescaling symmetry principle.

According to the rescaling symmetry principle, if every length in the universe were to increase or decrease in proportion there would be no noticeable effect to the inhabitants, (figure B1).

As can be seen in the cartoon, angles and measured distances are unaffected.

This continuous and ongoing change in length scale must happen to every length in the whole universe simultaneously, including the size of people, atoms and distances between all objects. Every physical constant must vary too, with the change depending on the number of length dimensions in the quantity.

Figure B1 Sketch to show a rescaling universe



A common cosmological time (t) is assumed.

Quantities rescale according to

$$\frac{dQ}{Q} = nHdt \qquad \text{(B1)}$$

where 'n' is the number of length dimensions in quantity Q. H is the rescaling constant, which is half of Hubbles constant H_0

$$Q = Q_0 \exp(nHt) \qquad \text{(B2)}$$

Table B1. The value of 'n' for various physical quantities.

Quantity	n
all lengths	1
speed of light	1
Plancks constant	2
particle masses	0
permittivity of free space	-3
scalefactor of universe 'a'	1
gravitational constant	3
Hubbles constant	0
Forces	1
quanity with n length dimensions	n

With this system the symmetry principle requires that any local experiment, to measure the change of any physical quantity, in a rescaling universe, would yield a null result. This is due to other relevant quantities rescaling too.

For example, imagine if an attempt were made to measure the change in the speed of light by timing the passage of a light beam over a given distance, 10 years apart. Since both the distance and the speed of light rescale in proportion the time of passage would remain the same.
It could be argued that it's meaningless to consider such a changing of scale – that because it's immeasurable it cannot be happening. As the rescaling symmetry principle applies to the whole universe simultaneously it could also be argued that the universe could be regarded as static, with no change of any physical quantity. However there is an important difference. In a rescaling universe (assumed expanding), the universe and various physical constants are larger now than they used to be.

This leads to some observational differences between the static and rescaling universe cases. These arise from the conservation of energy, as described below.

B2 The redshift of light.

In a rescaling universe, a photon of light arriving from a distant star, would be emitted at a time when Plancks constant was lower.

Figure B2 The redshift of light

By the time it has arrived at earth Plancks constant would be

$$h_0 = h_1 \exp(2Ht) \quad \text{(B3)}$$

where t is the travel time of the photon.

If the energy of the photon is conserved

$$f_0 = f_1 \exp(-2Ht) \quad \text{(B4)}$$

So light received from a distant source becomes redshifted. In this model the redshift of light is due to the rescaling universe, instead of an expanding universe.

The redshift of light is from

$$1 + z = \frac{\lambda_0}{\lambda_1} = \exp(2Ht) = \left(\frac{a_0}{a_1}\right)^2 \quad \text{(B5)} \qquad \text{also (2)}$$

every quantity Q with n length dimensions, at redshift z , has the relation

$$Q(z) = Q_0/(1+z)^{n/2} \quad \text{(B6)}$$

As we can only observe angles, timings and photons of light arriving from distant sources it is not easy to verify the rescaling this way.

The error made in the derivation of (1) found in textbooks is that the expansion has been applied only to cosmological distances, not universally to all distances. With the traditional redshift-scalefactor relation, the photon loses energy as it travels, whereas in the above the energy of the photon is conserved.

The energy of each mass m is also conserved during the rescaling, due to gravitation as discussed below.

B3 The origin of gravitation

Newtonian arguments are used. Consider a mass m in a rescaling universe, by its presence the mass generates gravitational potential energy – but why?

Figure B3 A small mass m and the surrounding universe

The total energy due to each mass m is

$$mc^2 - \frac{GMm}{R} \qquad \text{(B7)}$$

at a later time the total energy would be

$$(mc^2 - \frac{GMm}{R})\exp(2Ht) \qquad \text{(B8)}$$

the second term in (B7) represents the combined contributions to the potential energy, due to m, of the rest of the universe, of mass M, up to the Hubble radius R. If energy is conserved in a rescaling universe the term (B7) must be zero

$$G = \frac{Rc^2}{M} \qquad \text{(B9)}$$

Small numerical constants are omitted for simplicity.

The significance of equation (B9) is that gravity is caused by rescaling – i.e. the phenomenon of gravitation and the value of G, is a result of the conservation of energy in a rescaling universe.

This naturally leads to a universe near critical density, and a natural solution of the ‘flatness problem’. An exact treatment would need General Relativity and others are invited to attempt it. The equivalent of (B9) is (9).

There is a reduction in the value of G for masses of high mass to radius ratio.

For a large stationary mass, (B7) is amended to

$$mc^2 - \frac{GMm}{R} - \frac{Gm^2}{r} = 0 \qquad \text{(B10)}$$

giving

$$G_{effective} = \frac{c^2}{\left(M/R + m/r\right)} \tag{B11}$$

so, from (B7)

$$G_{effective} = \frac{c^2}{\left(\frac{c^2}{G} + \frac{m}{r}\right)} \tag{B12}$$

Equation (B12) indicates that a version of General Relativity which incorporates the rescaling symmetry principle will have an effective value of G (or active gravitational mass of a body), will varies from object to object. For masses of extremely high m/r ratio, G will decrease.

Whilst the value of G is constant within General Relativity, a future understanding of the theory or an amended version of it, may incorporate this feature. Some scalar tensor theories (Brans [7]) have a variable gravitational constant. They have apparently been ruled out by Lunar Laser Ranging (LLR). If the rescaling interpretation is valid, such theories may merit further consideration as the change in R and the variation in G may not be a measurable one.

Equation (B12) predicts an annual variation in Earth's gravity, as the earth-sun distance changes, of $3.3 \times 10^{-10}G$. This variation may already have been detected. Measurements from Satellite Laser Ranging (SLR) data (Matsuo [8]), show a variation in geoid height of about that magnitude, although the cause is uncertain. The annual variation of the earth-moon distance is much larger due to other factors.

Such a mechanism allows large collapsing masses to 'bounce' giving rise to explosive, or ejection phenomenon on various scales. A future theory may be able to account for the foam-like large scale structure, ejections from AGNs and incorporate the successes of Big Bang cosmology - an expanding universe model, which appears static on the largest scales, yet is in dynamic equilibrium and includes the perfect cosmological principle.

If and when the nature of dark matter is understood, there may remain the outstanding question of why it is distributed in such a way as to give the flat rotation curves (Zwicky [9]). The distribution of dark matter may be determined by (B12). Matter approaching a galactic centre could only spiral in at such a rate, so as to give a constant m/r ratio for every value of r. If matter approached faster, the value of $G(m,r)$ would be reduced, allowing matter to drift away from the centre, reducing the m/r ratio. A constant m/r ratio is thus maintained, this leads to the constant velocity of rotation. Incoming matter being periodically ejected perpendicular to the disk.

APPENDIX C: THE TENSION IN HUBBLE CONSTANT MEASUREMENTS.

There is currently tension between the Hubbles constant values from the distance ladder approach (Riess[6]), hereafter R16 and the Planck consortium (Ade[3]), hereafter Planck.

Using ideas from above, consistency can be found from R16 and Planck as follows.

4.1 Planck and BAO data.

Planck uses BAO data to determine H_0. Section 5.2 formula (25) of Planck XIII involves the expression $\frac{cz}{H}$ for distance, which should be $\frac{2c(\sqrt{1+z})-1)}{H}$ from (11).

BAO data is used to break the degeneracy between matter density and Hubbles constant. The BAO likelihood is dominated by two BOSS measurements, in particularly from BOSS CMASS with a redshift z of 0.57.
Hence BAO might overestimate distances by a factor 1.1265 (0.57 compared to 0.505993) and hence underestimate H_0 by the same factor. Plancks value of 67.3km/s/Mpc, their formula (27), multiplied by 1.1265 becomes 75.8 km/s/Mpc.

In 'Cosmological implications of BAO measurements' Aubourg [5], at the bottom of page 9, finds that the quantity $cln(1+z)/D_M(z)$ is 151 km/s/Mpc at z = 1090. Cosmological BAO measurements determine the product $H_0 r_d$ where r_d is the radius of the sound horizon. If the value H_0 used is twice the expansion rate, r_d should be halved.

Since $D_M(z)$ is proportional to r_d (their equation (19)), the 151 km/s/Mpc, is twice Hubbles constant giving a value for of H_0 of 75.5 km/s/Mpc.

The same result is from the model in Appendix B.

For the universe of Appendix B, which obeys the perfect cosmological principle, an observer must see a static universe. If the observer and the distant sources expand together, that is what would be observed. However when time dilation of factor $(1+z)$ is included, a distant source would appear to expand more slowly than the observer, so the universe would no longer appear static. To maintain the perfect cosmological principle we must have

$$H(z) = (1+z)H \qquad \text{(C1)}$$

Since

$$H(z) = H_0 E(z) \qquad \text{(C2)}$$

$$d_M = \frac{c}{H}\int_0^{\sqrt{1+z}-1} \frac{1}{1+z} dz \qquad \text{(C3)}$$

$$d_M = \frac{c}{2H}\ln(1+z) = \frac{c}{H_0}\ln(1+z) \qquad \text{(C4)}$$

So we would expect $cln(1+z)/D_M(z)$ to be measured as H_0 if the correct value of r_d is used and it is concluded that the true value of H_0 is close to 75.5 km/s/Mpc.

4.2 Planck and Lensing data.

The results from Planck lensing data (Ade[7]), Planck XV, are consistent with the higher Hubble constant value. They find $\sigma_8\Omega_m^{0.25}$ to be 0.591.
With Ω_m of 0.25, σ_8 is 0.836. Figure 18, page 29 of Planck XIII shows that these values in the $\sigma_8 - \Omega_m$ plane lie on the crossing point of the CHFTLenS data and the Planck lensing data (grey bands), corresponding to a Hubble constant value of approximately 76 km/s/Mpc. This can also be seen on Figure 7, page 9 of Planck XV.

4.3 Plancks $\Omega_m h^2$ Value.

With Ω_m as 0.25 and using the Planck XIII value for $\Omega_m h^2$ of 0.1428 gives H_0 as 75.6 km/s/Mpc.

4.4 Distance Ladder measurements.

R16 has an H_0 of 73.24 $\pm$ 1.74 km/s/Mpc, but in the analysis uses a deceleration parameter q_0 of -0.55 , derived from their 2007 paper, which uses the assumption of the LCDM model. The q_0 should be -1. When this value is applied to their equation (5) it changes a_x and so from their equation (4) changes H_0, by a factor 10^{a_x}.
The data has redshifts between 0.023 and 0.15, with more data near the 0.15 side.
The required change is: for z = 0.05 it's by a factor 1.011, for z = 0.1 it's 1.024 and for z = 0.15 it's 1.036 - if a factor of 1.025 is applied we find a corrected value of 73.24 × 1.025 = 75.1 km/s/Mpc

4.1 - 4.4 are consistent with each other given the errors involved. They have values of about 75–76 km/s/Mpc.