Extension of Proposition 23 from Euclid's Elements

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Proposition 23 states that two parallel lines in a plane never intersect. We use this definition with first and second postulate of Euclid to prove that two distinct lines through a single point cannot be parallel.

Proof:

Consider a point P. Let A and B be two points such that lines PA and PB are distinct from one another. So line PA does not contain B and line PB does not contain A. Therefore these two distinct lines have a common point P on them or they intersect at P. Therefore PA and PB are not parallel to one another.

Hence any two distinct lines that pass through the same point cannot be parallel to one another. A non-zero angle APB is always present between two such lines.