A new model for quantum mechanics and the invalidity of no-go theorems

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Abstract

In this paper we define and study the new model for quantum mechanics (QM) – the hybrid epistemic model. We describe in detail its axiomatic definition and its properties. The new feature of this model consists in the fact that it does not contain the formal definition of the measurement process (as it is standard in other models) but the measurement process is one of possible processes inside of QM.
The hybrid-epistemic model of QM is based on two concepts: the quantum state of an ensemble and the properties of individual systems. It is assumed that the quantum state (i.e. the wave function) can be attributed only to ensembles (with some exceptions) and not to individual systems. On the other hand, the properties of individual systems can be described by properties which are collected into classifications. Properties are assumed to be exclusive, i.e. a given individual system having certain property cannot have another property.
We shall describe the internal measurement process in the hybrid-epistemic model of QM in all details. This description substitutes the formal definition of the measurement process in the standard QM.
We show the local nature of EPR correlations in the hybrid-epistemic model of QM in all details. We show that the anti-correlations between measurements at the Alice's part and the Bob's part is completely analogical to the standard classical local anti-correlations originated in the correlation in the past.
We define precisely the epistemic and the ontic models of QM for the goal to prove that these three models give the same empirical predictions, i.e. that they are empirically equivalent. This theorem on the empirical equivalence is proved in all details.
We show that the no-go theorems (Bell's theorem, the Leggett-Garg's theorem and others theorems) cannot be proved in the hybrid-epistemic model of QM. This is one of the main results of this paper. We interpret this as the invalidity of no-go theorems in QM. This interpretation is sound since the true consequences of QM must be provable in all models of QM.
We shall consider the possible inconsistences of the ontic model of QM. We show that there are many consequences of the ontic model of QM which are dubious or controversial. There are many such controversial consequences. In the next part we consider the internal inconsistency of the ontic model which is more serious and we consider this argument against the ontic model as the most serious.
We introduce the property-epistemic model of QM which is the special case of the hybrid-epistemic model. We describe this model in all details and we show that this model of QM is the most suitable and most elegant model of QM. In this model many proofs are extremely simplified and almost trivial.
Then we discussed possible arguments in this field and our answers to these arguments.
We summarize our conclusions.
At the end there are three appendices. In the first appendix we give proofs of all theorems. In the second appendix we give our conjectures, opinions and suggestions. In the third appendix we describe the ontic model for the Brownian motion. We think that this model shows clearly (by analogy) the absurdity of the ontic model of QM.

Content

1. Introduction

In this paper we shall use some simplifications which we would like to declare at the beginning:

- We shall consider only finite-dimensional systems, i.e. systems whose Hilbert space of states is finite-dimensional
- We shall consider only the von Neumann's measurement schema based on the projection-valued measures (we shall not use the POV measures)
- We shall exclude any considerations connected with the idea of hidden parameters
- We shall assume that quantum mechanics (QM) is a valid theory.

It is well-known that there are many foundational problems in quantum mechanics (QM). We shall consider some of them. The main model for QM is the standard Dirac – von Neumann's model [5] which will be called here the ontic model. It is based on the assumption that each wave function describes a possible state of an individual quantum system and that the state of an individual system is described by some wave function.

This model creates many problems

- Bell's theorem implying the non-locality of QM – this creates the incompatibility with the relativity theory
- Leggett-Garg 's theorem implies the violation of the macro-realism – and this creates the incompatibility with the classical world
- The preceding two problems imply that the ontic model of the quantum world is incompatible with the classical world[1]
- The measurement problem
- The problem of definite outcomes of a measurement
- The problem that the measurement process is treated on the axiomatic level but not as a real physical process
- The instantaneous collapse of the state of an individual system during the measurement is un-physical

The general idea to solve these problems is to arrive at the thin film compromise:

- To have not many individual states (the individual state = the state of an individual system) so that the no-go theorems cannot be derived
- To have an enough number of individual states to be able to realize the standard measurements.

There is a unique possible solution to this problem: only one base of individual states for one system - and this is exactly the hybrid model proposed in [1], [3] [2]. The hybrid-epistemic model is the generalization of the hybrid model [2]. The property-epistemic model is a special case of a hybrid-epistemic model which is completely opposite to the ontic model.

This is our solution to the 85-years old problem of the meaning of quantum state.

Our plan how to procced is the following.

We shall construct a new model for QM**, the hybrid-epistemic model**, which will have the following properties

- It will be empirically equivalent to the standard QM model
- In this model it will be impossible to derive the no-go theorems (Bell's theorem, Leggett-Garg's theorem and others)
- In this model the measurement process will be treated as other internal processes in QM and the concept of a measurement will not be mentioned among axioms
- Not only that the non-locality is un-proved and un-provable in this model but moreover the explicit local mechanism for the EPR correlations is presented

[1] The problem of the compatibility between the quantum world and the classical world must be solved before the possible solution of the unification of QM with the general relativity
[2] In [1] the hybrid model of QM was called the modified QM.

Our proposal:

- to abandon the standard Dirac – von Neumann's model [5]
- to use instead of it the new hybrid-epistemic model or the property-epistemic model.

In this way we obtain many benefits

- the solution of all above mentioned problems
- the model which is empirically equivalent to the previous one so that the empirical confirmation of the new model is the same as the confirmation of the standard model
- the compatibility between quantum world and the classical world
- the invalidity of no-go theorems

A new, hybrid-epistemic model combines the features of ontic and epistemic models. The special case of this model, the hybrid model was introduced in [1], while the more general form of the hybrid-epistemic model was introduced in [2].

A new, hybrid-epistemic model is not only a new model of QM but, moreover, a completely new type of a model of QM. The novity consists in the main property of this new model: the concept of a measurement does not make a part of axioms but in the hybrid-epistemic model the measurement process is considered as an internal process in QM – like other QM processes.

The general question studied in this paper is whether the wave function can describe the state of an individual system or it describes the state of an ensemble. Our answer is that the wave function describes in most situations the state of an ensemble but in some rather very special situations the wave function can be considered as the state of the individual system. In the subclass of hybrid-epistemic models, the property-epistemic models, the wave function cannot be considered as the state of an individual system and it can describe only a state of an ensemble.

On the other hand, individual systems are taken into account since they are able to have **properties**. The consideration of properties of individual systems is a **new feature** of our models. Our proposed hybrid-epistemic model can be considered as a compromise between the epistemic view of quantum state (quantum states of ensembles) and the partially ontic view (properties of individual systems).

The interplay between states of ensembles and properties of individual systems is the base of the hybrid-epistemic model of QM.

This study is the result of a long series of previous papers which contain some of the presented ideas in the rudimentary form (see e.g. [1], [2], [3]).

In a general sense this paper gives a possible solution to the question what is a quantum state and what are possible properties of individual quantum objects. Problems created by no-go theorems are simply solved by showing that these theorems cannot be proved in the hybrid-epistemic model.

The text is divided into four parts

- In the first part we introduce and study the new hybrid-epistemic model of QM (sect. 2 – 4)
- In the second part we compare the hybrid-epistemic model to epistemic and ontic models (sect. 5 – 7)
- The third part contains the proof that no-go theorems are unprovable in QM and possible inconsistences of the ontic model (sect. 8 – 10).
- In the fourth part we define and study the property-epistemic models, their properties and consequences (sect.11). Then we present a discussion, conclusions and appendices (sect. 12 – 13).

In appendices there are proofs of theorems stated in the text, our conjectures and the ontic model of the Brownian motion.

The organization of the text is the following.

In the first part we give in the sect. 1 the introduction and in the sect. 2 the axioms for the hybrid-epistemic model and basic properties of this model. Then in sect. 3 we describe the internal model of a measurement in full details. In sect. 4 we show that the EPR correlations are explicitly local in the hybrid-epistemic model and that EPR correlations are completely analogical to the classical local correlations.

In the second part we describe in sect. 5 the axioms for the standard epistemic model of QM and in sect. 6 axioms we have collected axioms for the ontic model for QM. Detailed axiomatic formulations are necessary for the comparison of these models with the hybrid-epistemic model. In sect. 7 we give a proof of the empirical equivalence of the hybrid-epistemic model to the epistemic model and to the ontic model.

In the third part in the sect. 8 we show that the no-go theorems cannot be proved in epistemic and hybrid-epistemic models. In sect. 9 we present arguments for the inconsistence of the ontic model of QM and the incompatibility of the ontic model of QM with the classical physics. In sect 10 we present the internal inconsistences of the ontic model.

Then in the fourth part in sect. 11 we present and study the property-epistemic model. In sect.12 we present a discussion of some related question and in sect. 13 we described our conclusions.

In Appendix A we have collected the proofs of theorems and propositions stated in the text. In Appendix B we have collected our conjectures, hypotheses, opinions and remarks related to the given problematic.

In the Appendix C we describe the ontic model for the Brownian motion We give an argument that the ontic model of QM, as an analogy of the ontic model of the Brownian motion, is absurd.

2. A new, hybrid-epistemic model for QM

A new property of this model is the fact that the description of the measurement process is not a part of axioms. This make this model different from previously considered models. In the hybrid-epistemic model the measurement process is described entirely inside of QM and there is no relict of this process among axioms.

In this sense, this proposed model is of a completely new type of models with respect to older models – it is possible to say that this is the first model of this new type[4]. Thus this is not only a new model but this is, moreover, a new type of a model.

The other new feature of the hybrid-epistemic model is the new concept of a quantum state.

In general we assume that the wave function is associated with the state of an ensemble (but in certain rather very special situations the wave function can be also connected with the state of an individual system – see this below in the case of hybrid systems).

Individual systems cannot be characterized by the state but they will be characterized by **properties**. The concept of a **property of a system** is one of the most general concepts in logic – it is, of course, more general than the concept of a state.

Usually properties are associated with the position of the system. The typical situation is the position of a particle.

In the quantum world ensembles have states while in the classical world individual systems have properties and the hybrid-epistemic model is the system which makes compatible these two worlds.

Definition.

(i) A *property* is a map v which to each individual system S associates an element from the two-element set B = {1, 0},[3] i.e. v(S) = 1 means that S has a property v while v = 0 means that S does not have a property v.

(ii) Two distinct properties v and w are *mutually exclusive* if for each S v(S) = 1 implies w(S) = 0 and vice versa (i.e. the situation where v(S) = w(S) = 1 is impossible).

(iii) A set $\mathbf{C}(S) = \mathbf{C}_S$, $|\mathbf{C}_S| \geq 2$, of mutually exclusive properties is called the *classification for S* if there exists $v \in \mathbf{C}_S$, v(S) = 1 .

[3] B is the set of truth-values.

(iv) The trivial *classification* $\boldsymbol{C}_S = 0$ is also an allowed classification.

Axiom HE1. (HE = hybrid-epistemic.)
We shall assume that to each system S there is associated a complex (at least two-dimensional) *Hilbert space* $\mathbf{H}(S) = \mathbf{H}_S$ and the *classification* $\mathbf{C}(S) = \mathbf{C}_S$.

Example. For the typical two level system we have $\mathbf{C}_S = \{v^{\text{spin up}}, v^{\text{spin down}}\}$ and $\mathbf{H}_S$ is a finite dimensional complex Hilbert space and dim $\mathbf{H}_S$ may be very large (as an idea let us imagine the Stern-Gerlach measuring apparatus). This is a typical hybrid-epistemic system. In the case of a hybrid system (its definition is given below) with the same $\mathbf{C}_S$ we shall have dim $\mathbf{H}_S = 2$.

Definition. An *ensemble* is a set of systems $\mathbf{E} = \{S_1, \ldots, S_N\}$ which are created by some preparation procedure and satisfy the following conditions

(i) $\mathbf{H}(S_1) = \ldots = \mathbf{H}(S_N) = \mathbf{H}(\mathbf{E})$

(ii) $\mathbf{C}(S_1) = \ldots = \mathbf{C}(S_N) = \mathbf{C}(\mathbf{E})$

(iii) Properties of systems S_i and S_j, are independent for all $i \neq j$ (these systems are created independently and they do not interact with each other).

Axiom HE2.

(i) For each system S there exists at least one ensemble **E** containing S as an element

(ii) The state of each ensemble **E** (in a given moment of time) is described by some element of $\mathbf{H}_\mathbf{E}$ – more precisely by a ray $[\psi]$ in $\mathbf{H}_\mathbf{E}$.[4]

(iii) The time evolution of the state $[\psi]$ is given by the unitary group $\{U_t\}$ in $\mathbf{H}_\mathbf{E}$ and we have $[\psi](t) = [U_t(\psi)]$.

Now we shall consider and define the basic (probabilistic) relation between properties of individual systems and states of ensembles.

Definition. Let $\mathbf{E} = \{S_1, .. , S_N\}$ is an ensemble in the state $[\psi]$. Let $v \in \mathbf{C}_\mathbf{E}$. Then by observing $S_1, .. , S_N$ we obtain values $v(S_1), .. , v(S_N) \in \{0, 1\}$. We shall consider the *sub-ensemble*

$$\mathbf{E}^{(v)} = \{S \in \mathbf{E} \mid v(S) = 1\}.$$

By the definition of the probability we have

$$p(v \mid \mathbf{E}) = \lim_{N\to\infty} N^{-1} \mid \mathbf{E}^{(v)} \mid ,$$ [5] [6]

Axiom HE3.
We shall assume that $p(v \mid \mathbf{E})$ will depend only on the state $[\psi]$ of the ensemble **E**, so that

$$p(v \mid \mathbf{E}) = p(v \mid [\psi]).$$

[4] In this paper we do not consider the mixed states since they are not necessary. A ray $[\psi]$ is defined by $[\psi] = \{\alpha\psi \mid |\alpha| = 1\}$, where $|\psi| = 1$.
[5] Here $|A|$ = the number of elements of A
[6] In this situation we shall assume the stabilization of relative frequencies.

The relation $p (v \mid [\psi])$ is the **basic relation** in our axiomatization: it relates properties of individual systems to states of ensembles.

Having this, we can define basic concepts of QM using both things: states of ensembles and properties of individual systems and we **do not need** the axiomatization of the formal definition of a measurement process.

To every ensemble **E** there corresponds a *probability distribution* $p (. \mid [\psi])$ on the set $\mathbf{C}(\mathbf{E})$ which depends on the state $[\psi]$ of an ensemble **E**.

Definition.

(i) Let $v \in \mathbf{C}(\mathbf{E})$. The state $[\psi] \in \mathbf{H}(\mathbf{E})$ is a *v-homogeneous state* if

$$p (v \mid [\psi]) = 1.$$

The state $[\psi]$ is *homogeneous* if $[\psi]$ is v-homogeneous for some $v \in \mathbf{C}(\mathbf{E})$. The set of all homogeneous states is denoted by *hom (**E**).*

(ii) For each $v \in \mathbf{C}(\mathbf{E})$ we define the set

$$\mathbf{L}^{(v)} = \{ \lambda\psi \in \mathbf{H}_{\mathbf{E}} \mid [\psi] \text{ is a v-homogeneous state, } \lambda \text{ is a complex number}\}.$$

We shall call any such $\mathbf{L}^{(v)}$, $v \in \mathbf{C}_{\mathbf{E}}$ a *homogeneous subspace* of $\mathbf{H}_{\mathbf{E}}$ corresponding to the property v. The set of all homogeneous subspaces will be denoted by *Hom (**E**).*

(iii) The property v is called the *individual property* if $\mathbf{L}^{(v)}$ is one-dimensional and it is called the *collective property* if $\mathbf{L}^{(v)}$ has dimension greater then 1. It is clear that the quantum state $[\psi] \in \mathbf{H}_{\mathbf{E}}$ can be attributed to the individual system S having the property v only if v is the individual property.

(iv) The quantum state $[\psi] \in \mathbf{H}_{\mathbf{E}}$ is called an individual state (or the quantum state of an individual system S) if and only if there exists an individual property v such that ψ is the generator of the (one-dimensional) space $\mathbf{L}^{(v)}$ and the system S has the property v .

Axiom HE4. The set $\mathbf{L}^{(v)}$ is a (complex) linear subspace of the Hilbert space $\mathbf{H}_{\mathbf{E}}$ for each $v \in \mathbf{C}(\mathbf{E})$

This axiom seems to be less intuitive than previous axioms. But it is possible to give to it a good motivation expressed in the following proposition.

Proposition 1. Let us assume that the probability p (v | [ψ]) depends quadratically[7] on the wave function . Then each homogeneous subspace $\mathbf{L}^{(v)}$ is a linear subspace of $\mathbf{H_E}$ **.**

The proof of this proposition can be found in the Appendix A.

Definition.

(i) Let the operator $P^{(v)}$ *be an orthogonal projection* from $\mathbf{H_E}$ onto $\mathbf{L}^{(v)}$ for v ∈ **C**(**E**)

(ii) The ensemble **E** is called a *balanced ensemble* if all its homogeneous subspaces have the same dimension

Axiom HE5. (The existence of measuring systems.)
For each n≥2 there exists at least one balanced ensemble **E** of measuring systems satisfying |**C**(**E**)| = n .

Remarks.

(i) The ensemble **E** is in a v-homogeneous state if v(S) = 1 for each S ∈ **E** and, equivalently, if w(S) = 0 for each w ≠ v and for each S ∈ **E** [8].

(ii) The ensemble **E** is in a homogeneous state if $v(S_1) = v(S_2)$ for each S_1, S_2 ∈ **E** and for each v. This means that elements of the homogeneous ensemble have the same properties (this is the intuitive meaning of homogeneity) .

(iii) For each homogeneous ensemble **E** in the state [ψ]. there exists a unique property $v^{[\psi]}$ such that p ($v^{[\psi]}$ | [ψ]) =1 and p ($v^{[\psi]}$ | [ϕ]) = 0 for each [ϕ] ≠ [ψ]. [9]

(iv) It is clear that in the hybrid-epistemic model the focus is moved from the measured system onto the measuring system.

Axiom HE6. (The existence of a composition of two systems.)
Let **E** = $\{S_1, \ldots, S_N\}$ and **F** = $\{T_1, \ldots, T_N\}$ be two independent ensembles.

(i) The composite ensemble is $\mathbf{E} \oplus \mathbf{F} = \{S_1 \oplus T_1, .. , S_N \oplus T_N\}$ [10]

(ii) We have $\mathbf{H_{E \oplus F}} = \mathbf{H_E} \otimes \mathbf{H_F}$

(iii) Let v ∈ **C**(**E**) and w ∈ **C**(**F**) are properties . The construction of **C**(**E** ⊕ **F**) depends on the situation

a. If |**C**(**E**)| ≥ 2 and |**C**(**F**)| ≥ 2 then **C**(**E** ⊕ **F**) = **C**(**E**) × **C**(**F**) . Moreover, we have $(v,w)\ (S_k \oplus T_k) = v(S_k)\ .\ w(T_k)$.

b. If |**C**(**E**)| ≥ 2 and |**C**(**F**)| = 0 then **C**(**E** ⊕ **F**) = **C**(**E**) . Moreover, we have $v(S_k)$ as a value

c. If |**C**(**E**)| = 0 and |**C**(**F**)| ≥ 2 then **C**(**E** ⊕ **F**) = **C**(**F**) . Moreover, we have $w(T_k)$ as a value

d. If |**C**(**E**)| = |**C**(**F**)| = 0 then **C**(**E** ⊕ **F**) = 0

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[7] In quantum theory the probability depends, in general, linearly on the density operator and consequently quadratically on the wave function.

[8] We assume that events with the zero probability never happen.

[9] Let S_0 ∈ **E.** There exists a property $v^{[\psi]}$ such that $v^{[\psi]}(S_0) = 1$ and then $v^{[\psi]}(S) = 1$ for each S ∈ **E** since **E** is homogeneous.

[10] The order of sequence is not important since by changing the order in the ensemble does not change the ensemble

(iv) Let us assume that the ensemble $\mathbf{E}$ is in the state $[\psi]$ and the ensemble $\mathbf{F}$ is in the state $[\phi]$ and that these ensembles are independent. Then the ensemble $\mathbf{E} \oplus \mathbf{F}$ will be in the state $[\psi \otimes \phi]$.

(v) Let P be a projection in the space $\mathbf{H_E}$ and $\mathrm{Id}(\mathbf{H_F})$ be an identity map in the space $\mathbf{H_F}$. Then the extension $P \otimes \mathrm{Id}(\mathbf{H_F})$ as a map in the space $\mathbf{H_E} \otimes \mathbf{H_F}$ is defined by

$$(P \otimes \mathrm{Id}(\mathbf{H_F}))\,(\psi \otimes \phi) = P(\psi\,) \otimes \phi\,.$$

Axiom HE7. (The Born rule.)
Let $\mathbf{E} = \mathbf{M}\oplus\mathbf{S} = \{M_1\oplus S_1, .. , M_N\oplus S_N\}$ be an ensemble of independent composite systems in the state $[\Psi] \in \mathbf{P_{M\oplus S}}$. Let $M_k\oplus S_k$ be an individual system composed from a measuring system M_k and from the measured system S_k and let $\mathbf{P}^{(v)}$ be a projection in $\mathbf{H_M}$ corresponding to the property $v \in \mathbf{C}(\mathbf{M})$.

Then the probability of observing the property v on the subsystem M_k is given by the formula

$$p\,(\,v \mid [\Psi]\,) = \|\,(\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S))\,(\Psi)\,\|^2\,, \quad \text{for each } v \in \mathbf{C}(\mathbf{M})\,.$$

Axiom HE8. (The up-dating rule.)
Let $\mathbf{E} = \mathbf{M}\oplus\mathbf{S} = \{M_1\oplus S_1, .. ,M_N\oplus S_N\}$ be an ensemble of independent composite systems in the state $[\Psi] \in \mathbf{P_{M\oplus S}}$. Let us assume that the individual subsystem M_k from the composite system $M_k\oplus S_k$ has been observed and it was found that the property v has occurred, $v(M_k) = 1$. Then the individual system $M_k\oplus S_k$ will be an element of an up-dated ensemble

$$\mathbf{E}^{(v)} = \{M_k\oplus S_k \mid v(M_k) = 1\,\}$$

which is in the state

$$\|\,(\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S))\,(\Psi)\,\|^{-1}\,.\ (\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S))\,(\Psi)\,.$$

Definition .
Axioms **HE1 – HE8** define the *hybrid-epistemic model* of QM.

There are two basic theorems in the hybrid-epistemic model.

Theorem 1.
Let $v,\ w \in \mathbf{C}(\mathbf{E})$ and let $v \neq w$. Then homogeneous subspaces $\mathbf{L}^{(v)}$ and $\mathbf{L}^{(w)}$ are orthogonal.

Theorem 2.
Let us assume that $\mathbf{C}(\mathbf{E}) \neq 0$. Then the set Hom $(\mathbf{E})$ of all homogeneous subspaces is the complete orthogonal decomposition of the state space $\mathbf{H_E}$.

Proofs of both theorems can be found in the Appendix A.

It is useful to introduce certain classification into the set of possible systems.

Definition.

(i) The system S is an *epistemic system* if $\mathbf{C}(S) = 0$.

(ii) The system S is the *hybrid-epistemic system* if $|\mathbf{C}(S)| \geq 2$.

(iii) The system S is *hybrid* if S is the hybrid-epistemic system and if dim $\mathbf{L}^{(v)} = 1$ for each $v \in \mathbf{C_S}$

(iv) The system S is *property-epistemic system* if it is hybrid-epistemic system and if dim $\mathbf{L}^{(v)} > 1$ for each $v \in \mathbf{C_S}$

The important special subclass of hybrid-epistemic models are hybrid models of QM. Originally the hybrid model was called the modified QM (see papers [1], [3]).

Definition .

(i) The hybrid-epistemic model is called a *hybrid model* if it contains only hybrid systems. .

(ii) The hybrid-epistemic model is called a *property-epistemic model* if it contains only epistemic systems or property -epistemic systems .

A typical ensemble **G** of hybrid–epistemic systems $\{S_k\}$ is the composite of the hybrid system M_k and the epistemic system E_k where $S_k = M_k \oplus E_k$, $\mathbf{H_G} = \mathbf{H_M} \otimes \mathbf{H_E}$.

As a consequence we obtain the following simple theorem (its proof is very simple).

Theorem 3.

(i) In the hybrid-epistemic model only hybrid-epistemic or epistemic systems can exist.

(ii) Some hybrid-epistemic systems must exist – see Axiom **HE5**.

(iii) The situation where only hybrid systems exist is possible – this will be the hybrid model

(iv) Each measuring system must be hybrid or hybrid-epistemic.

Superposition among homogeneous states are impossible or limited. Superposition of individual states are impossible in the hybrid-epistemic model.

Theorem 4.

(i) In the hybrid-epistemic model the non-trivial superposition of two homogeneous states is a homogeneous state if and only if these homogeneous states are elements of the same homogeneous subspace.

(ii) If $[\psi]$ and $[\varphi]$ are two different individual states of the system S then their non-trivial superposition $[\alpha\psi + \beta\varphi]$ is not an individual state. This means that in the hybrid-epistemic model for QM the individual anti-superposition principle holds (see [1]). [11]

[11] The more general form is the following. If $[\psi]$ and $[\varphi]$ are two different homogeneous states of the system S then their non-trivial superposition $[\Psi] = [\alpha\psi + \beta\varphi]$ is not an individual state. The proof is simple. If $[\psi]$

(iii) A v-homogenous state is an individual state only if the corresponding homogenous subspace $\mathbf{L}^{(v)}$ is one-dimensional.

The proof of this theorem can be found in Appendix A.

3. The internal description of the measurement process in the hybrid-epistemic model of QM

Now we shall describe the general schema of the *measurement process* as an internal process inside of QM.

Up to now, our axiomatization was occupied only with the concept of an observation of properties of a measuring system and the concept of a measurement was not mentioned.

Now we shall show that the process of a measurement can be defined and studied as an internal phenomenon of QM. Up to now we have been interested only in the "measuring systems" but not in "measured systems". It is exactly opposite to standard QM where the main interest is focused on measured systems and the measuring systems are generally not considered.

The true measurement process needs both systems, the measured system and the measuring system since the measurement process is based on the interaction between these two systems.

Let us assume that the *measuring ensemble* $\mathbf{M} = \{M_1, \dots , M_N\}$ is a balanced ensemble and that its orthogonal decomposition into homogeneous subspaces is $\mathbf{H}_M = \mathbf{L}^{(1)} + \dots + \mathbf{L}^{(n)}$, where these homogeneous subspaces correspond to properties $v_1, \dots, v_n$ (all homogeneous subspaces have the same dimension). Projections onto $\mathbf{L}^{(i)}$ is denoted by $\mathbf{P}^{(i)}$.

Definition.

(i) The *measurement* is parametrized by the orthogonal decomposition $\mathbf{H}_S = \mathbf{K}_1 + \dots + \mathbf{K}_n$ or by the projectors $\mathbf{R}_1, .. , \mathbf{R}_n$ which are projectors onto these subspaces $\mathbf{R}_i : \mathbf{H}_S \rightarrow \mathbf{K}_i$.

and [φ] are not elements of the same $\mathbf{L}^{(v)}$ then [Ψ] is not a homogeneous state, thus not an individual state. If [ψ] and [φ] are elements of the same $\mathbf{L}^{(v)}$ then [Ψ] is also element of $\mathbf{L}^{(v)}$ and $\mathbf{L}^{(v)}$ cannot be one-dimensional.

(ii) The measurement is called *non-degenerate* if all subspaces $\mathbf{K}_1, \dots, \mathbf{K}_n$ are 1-dimensional.

The composite ensemble $\mathbf{G} = \mathbf{M} \oplus \mathbf{S}$ will be based on the Hilbert space $\mathbf{H_G} = \mathbf{H_M} \otimes \mathbf{H_S}$.

Definition.
The *measuring transformation* is a unitary transformation $\mathbf{U}$ in the space $\mathbf{H_G}$ satisfying the following conditions

(i) $\mathbf{U}\,(\mathbf{L}^{(1)} \otimes \mathbf{K}_i) = \mathbf{L}^{(i)} \otimes \mathbf{K}_i$, for i = 1, … , n .

(ii) There exist unitary maps $\mathbf{V}_i$ from $\mathbf{L}^{(1)}$ onto $\mathbf{L}^{(i)}$, for i = 1, … , n , [12] such that $\mathbf{U}\,(\phi\otimes\psi) = \mathbf{V}_i\,(\phi) \otimes \psi$ for each $\phi \in \mathbf{L}^{(1)}$, $\psi \in \mathbf{K}_i$, i = 1, … , n .

The input part of the *measurement process* consists in the following:

(i) The *ensemble of measured systems* $\mathbf{S} = \{S_1, \dots, S_N\}$ in the state $\psi \in \mathbf{H_S}$.

(ii) The *measurement* defined by the orthogonal decomposition $\mathbf{H_S} = \mathbf{K}_1 + \dots + \mathbf{K}_n$ and by corresponding projections $\mathbf{R}_1, .., \mathbf{R}_n$

(iii) The *ensemble of measuring systems* $\mathbf{M} = \{M_1, \dots, M_N\}$ in the state $\phi \in \mathbf{L}^{(1)}$ together with the decomposition of $\mathbf{H_F}$ into homogeneous subspaces $\mathbf{H_F} = \mathbf{L}^{(1)} + .. + \mathbf{L}^{(n)}$ and corresponding projections $\mathbf{P}^{(1)}, .., \mathbf{P}^{(n)}$. (This decomposition depends on the set of properties $\mathbf{C(F)}$.)

(iv) The *ensemble of composite systems* $\mathbf{G} = \{M_1\oplus S_1, \dots, M_N\oplus S_N\}$ in the state $\phi\otimes\psi$.

Then the state ψ can be decomposed with respect to the decomposition $\mathbf{H}_S = \mathbf{K}_1 + \dots + \mathbf{K}_n$ into $\psi = \alpha_1\psi_1 + \dots + \alpha_n\psi_n$.

Here $\psi_i = \|\mathbf{R}_i\,\psi\|^{-1}\,\mathbf{R}_i\,\psi \in \mathbf{K}_i$, $\alpha_i = \|\mathbf{R}_i\,\psi\|$, if $\|\mathbf{R}_i\,\psi\| > 0$, while for $\|\mathbf{R}_i\,\psi\| = 0$ we take any unit vector $\psi_i \in \mathbf{K}_i$ and $\alpha_i = 0$.

The output part of the *measurement process* consists in the following:

(i) To the ensemble $\mathbf{G}$ in the state $\phi\otimes\psi$ the measuring transformation $\mathbf{U}$ is applied and we obtain an *ensemble* $\mathbf{G'}$ in the state $\mathbf{U}(\phi\otimes\psi)$. We obtain

$$\mathbf{U}(\phi\otimes\psi) = \alpha_1\,\mathbf{U}(\phi\otimes\psi_1) + \dots + \alpha_n\,\mathbf{U}(\phi\otimes\psi_n) = \Sigma\,\alpha_i\,\mathbf{V}_i\,(\phi) \otimes \psi_i\,.$$

(ii) Let v be a property of the system M_k . Then the *sub-ensemble*

$$\mathbf{G'}^{(v)} = \{\, M_k\oplus S_k \mid v\,(M_k) = 1 \,,\, k = 1, \dots, N \,\}\,.$$

---

[12] The unitary map $F : H \to H'$ satisfies $(F(x) \mid F(y)) = (x \mid y)$ for each $x, y \in H$

is defined.

Theorem 5. Let us assume the situation described above.

(i) If $v = v_j$ then the relative frequency of $\mathbf{G'}^{(v)}$ in the ensemble $\mathbf{G'}$ is

$$N^{-1} \mid \mathbf{G'}^{(v)} \mid = p\ (\ v_j \mid \mathbf{U}(\phi \otimes \psi)\ ) = \mid \alpha_j \mid^2 .$$

(ii) Let us assume that after the measurement the individual system M_k was observed and it was found that M_k has the property $v = v_j$, $v(M_k) = 1$, Then $M_k \oplus S_k$ is an element of the ensemble $\mathbf{G'}^{(v)}$ and the state of $\mathbf{G'}^{(v)}$ is

$$\mathbf{V}_j(\phi) \otimes \psi_j \ \text{ where } \psi_j = \ \|\mathbf{R}_j \psi\|^{-1} R_j \psi \in \mathbf{K}_j \ \text{ if } \ \|\mathbf{R}_j \psi\| > 0$$

while if $\|\mathbf{R}_j \psi\| = 0$ then the probability to find M_k with $v(M_k) = 1$ is zero and $\mathbf{G'}^{(v)}$ is empty. This means that after the measurement the sub-systems $\{S_k\}$ from $\mathbf{G'}^{(v)}$ will be in the state ψ_j while the sub-systems $\{M_k\}$ from $\mathbf{G'}^{(v)}$ will be in the state $\mathbf{V}_j(\phi)$.

(iii) For each ensemble of measured systems $\mathbf{S} = \{S_1, \dots , S_N\}$ there exist an ensemble of measuring systems $\mathbf{M} = \{M_1, \dots , M_N\}$ and the measuring transformation $\mathbf{U}$ satisfying all required properties.

Proofs can be found in Appendix A.

The epistemic or ontic model of QM cannot be a model of the hybrid-epistemic model of QM since these models of QM require the measurement concept as a part of axioms. We have done another thing – we have represented the measurement process as an internal process in QM and we shall show (in the next section) that all empirical predictions of the epistemic or ontic models of QM are contained among empirical predictions of the hybrid-epistemic model of QM.

4. The local nature of EPR correlations in the hybrid-epistemic model

Usually it is supposed that the well-known EPR anti-correlations of two spin ½ particles in the entangled state is the phenomenon showing the non-locality of QM. This is true in the ontic model of QM but we shall show here that this is **not true** in the hybrid-epistemic model. We shall describe the **local** mechanism underlying the EPR correlations in the hybrid-epistemic model.

This will proceeds in steps.

(i) We shall consider two ensembles of measured systems $\mathbf{S}^1$ and $\mathbf{S}^2$,[13] each two dimensional, dim $\mathbf{H}(\mathbf{S}^1)$ = dim $\mathbf{H}(\mathbf{S}^2) = 2$. We shall assume that $S^1_k \in \mathbf{S}^1$ and $S^2_k \in \mathbf{S}^2$ are epistemic systems, k = 1, .. , N, i.e. $\mathbf{C}(\mathbf{S}^1) = \mathbf{C}(\mathbf{S}^2) = 0$.
Then we shall consider two ensembles $\mathbf{M}^1$ and $\mathbf{M}^2$ of measuring systems M^1_k and M^2_k which are hybrid-epistemic satisfying $|\mathbf{C}(\mathbf{M}^1)| = |\mathbf{C}(\mathbf{M}^2)| = 2$. The system M^1 "will measure" S^1 and M^2 "will measure" S^2.

(ii) The corresponding classifications of measuring systems will be $\mathbf{C}(\mathbf{M}^1) = \{v^1_1, v^1_2\}$, $\mathbf{C}(\mathbf{M}^2) = \{v^2_1, v^2_2\}$. The dimensions dim $\mathbf{H}(\mathbf{M}^1)$, dim $\mathbf{H}(\mathbf{M}^2)$ can be arbitrary (possibly very large). Thus we shall consider the ensemble of systems

$$\mathbf{G} = \{M^1_k \oplus S^1_k \oplus S^2_k \oplus M^2_k \mid k = 1, .. , N\} .$$

Let the base $\{e^1_1, e^1_2\}$ in $\mathbf{H}(\mathbf{S}^1)$ be a z-spin base for S^1_k, k = 1, .. , N and analogously $\{e^2_1, e^2_2\}$ in $\mathbf{H}(\mathbf{S}^2)$ be a z-spin base for S^2_k, k = 1, .. , N .

(iii) The starting state will be following

$$\Psi = 2^{-1/2}\ \Phi^1 \otimes (e^1_1 \otimes e^2_2 - e^1_2 \otimes e^2_1) \otimes \Phi^2$$

where $\Phi^1 \in \mathbf{L}^{1(1)}$ and $\Phi^2 \in \mathbf{L}^{2(1)}$ are arbitrarily chosen states of homogeneous subspaces of ensembles $\mathbf{M}^1$ and $\mathbf{M}^2$.

This formula can be written more compactly using the totally anti-symmetric tensor $\boldsymbol{\varepsilon}$ defined by $\boldsymbol{\varepsilon}_{11} = \boldsymbol{\varepsilon}_{22} = 0$ **,** $\boldsymbol{\varepsilon}_{12} = 1$ **,** $\boldsymbol{\varepsilon}_{21} = -1$. Then we can write

$$\Psi = (½)^{-1}\ \Phi^1 \otimes (\Sigma\, \boldsymbol{\varepsilon}_{ij}\, e^1_i \otimes e^2_j) \otimes \Phi^2 .$$

(iv) Now we define the possible measurement.
The measuring apparatuses will be oriented at Alice and at Bob in the same direction. This means that the decompositions $\mathbf{H}(\mathbf{S}^1) = \mathbf{K}^1_1 + \mathbf{K}^1_2$, $\mathbf{H}(\mathbf{S}^2) = \mathbf{K}^2_1 + \mathbf{K}^2_2$ will be isomorphic. We can choose the generating vectors of these 1-dimensional subspaces: $\psi^i_j \in \mathbf{K}^i_j$, i, j = 1, 2 . It is possible to choose the generating vectors in such a way that there exists a unitary map U' of e's onto ψ's satisfying: $U'(e^i_j) = \psi^i_j$, i, j = 1, 2 .

The important fact is that this map is the same for S^1 and S^2 – this means that the same measurement is applied to S^1 and to S^2. For us the more important map will be the inverse map $U = U'^{-1}$ satisfying $U(\psi^i_j) = e^i_j$. In detail this means that $e^i_j = \Sigma_n U_{jn}\, \psi^i_n$, i, j = 1, 2 .

Inserting this into the formula for Ψ we obtain

[13] Upper indexes will indicate systems 1, 2.

$\Sigma \boldsymbol{\varepsilon}_{ij}\, e^1_i \otimes e^2_j = \Sigma \boldsymbol{\varepsilon}_{ij} U_{in} \psi^1_j \otimes U_{jm} \psi^2_j = \Sigma (U^T_{ni}\, \boldsymbol{\varepsilon}_{ij}\, U_{jm}) \psi^1_n \otimes \psi^2_m = \Sigma (U^T \boldsymbol{\varepsilon}\, U)_{nm}\, \psi^1_n \otimes \psi^2_m$[14].

(v) Now we shall use the simple but important technical lemma.

Lemma 1. Let A be any 2 by 2 matrix. Then $A\, \boldsymbol{\varepsilon}\, A^T = \det A\, \boldsymbol{\varepsilon}$ [15]**.**
The proof can be done by calculation.

(vi) Using this lemma we obtain $(U^T \boldsymbol{\varepsilon}\, U) = \boldsymbol{\varepsilon}$ since det U = 1 and the state of the ensemble **G** will be

$$\Psi = 2^{-1/2}\, \Phi^1 \otimes (\Sigma\, \boldsymbol{\varepsilon}_{nm}\, \psi^1_n \otimes \psi^2_m) \otimes \Phi^2\ .$$

We can see that the anti-symmetry is conserved in the transition from the base of e's to the new base of ψ's. This means that the choice of the measurement basis (the orientation of measurement apparatuses) does not change the anti-symmetry of the state.

(vii) Let us now assume that Alice has made a measurement on the sub-system $M^1_k \oplus S^1_k$. This implies the application of the measurement transformation on the Alice's side

$$\Psi \rightarrow \Psi' = (½)^{-1}\, \Sigma\, \mathbf{V}^1_n(\Phi^1) \otimes (\, \boldsymbol{\varepsilon}_{nm} \psi^1_n \otimes \psi^2_m) \otimes \Phi^2)\ .$$

(viii) Let us now assume that Alice has observed the subsystem M^1_k of the system $M^1_k \oplus S^1_k \oplus S^2_k \oplus M^2_k$, k = 1, .. , N (in the k-th round of an experiment) and she has found that this system has a property (for example) v^1_2 , i.e. that $v^1_2(M^1_k) = 1$.

Let us denote $v = v^1_2$. Following the Axiom **HE8** the individual system $M^1_k \oplus S^1_k \oplus S^2_k \oplus M^2_k$ will be an element of an up-dated ensemble

$$\mathbf{G}^{(v)} = \{\, M^1_k \oplus S^1_k \oplus S^2_k \oplus M^2_k \mid v(M^1_k\} = 1\}$$

which is in the state

$$\Psi'' = N^{-1}\, [\mathbf{P}^{1(v)} \otimes \mathrm{Id}(\mathbf{H}(\mathbf{E}^1) \otimes \mathbf{H}(\mathbf{E}^2) \otimes \mathbf{H}(\mathbf{F}^2))]\ (\Psi')\ ,$$

where

$$N = \|[\mathbf{P}^{1(v)} \otimes \mathrm{Id}(\mathbf{H}(\mathbf{E}^1) \otimes \mathbf{H}(\mathbf{E}^2) \otimes \mathbf{H}(\mathbf{F}^2))]\ (\Psi')\|$$

Since $\mathbf{P}^{(1v)}$ projects $\mathbf{H}(\mathbf{F}^1)$ onto $\mathbf{L}^{1(2)}$, only the term with $\mathbf{V}^1_n(\Phi^1)$, n = 2 from Ψ' will be relevant and we obtain

$$\Psi'' = N^{-1}\, 2^{-1/2}\, \Sigma\, \mathbf{V}^1_2(\Phi^1) \otimes (\, \boldsymbol{\varepsilon}_{2m} \psi^1_2 \otimes \psi^2_m) \otimes \Phi^2)$$

---

[14] U^T denotes the transposed matrix of U.
[15] A^T denotes the transposed matrix of A.

$$= N^{-1}2^{-1/2}\, \mathbf{V}^1{}_2(\Phi^1)\otimes\boldsymbol{\varepsilon}_{21}\psi^1{}_2\otimes\psi^2{}_1\otimes\Phi^2 = -\,\mathbf{V}^1{}_2(\Phi^1)\otimes\psi^1{}_2\otimes\psi^2{}_1\otimes\Phi^2 .$$

(ix) The application of the measurement transformation on the Bob's side gives the following state

$$\Psi''' = -\,\mathbf{V}^1{}_2(\Phi^1)\otimes\psi^1{}_2\otimes\psi^2{}_1\otimes\mathbf{V}^2{}_1(\Phi^2) .$$

Let us denote $w = v^2{}_1$. Using **HE7,** the probability of observing the property $v^2{}_1$ on the subsystem M^2 will be given by

$$p\,(v^2{}_1 \mid \Psi''') = \|(\mathrm{Id}(\mathbf{H}(\mathbf{F}^1)\otimes\mathbf{H}(\mathbf{E}^1)\otimes\mathbf{H}(\mathbf{E}^2))\otimes\, \mathbf{P}^{2(w)})\,(\Psi''')\,\|^2 = 1$$

since $\mathbf{P}^{2(w)}$ projects $\mathbf{H}(\mathbf{M}^2)$ onto $\mathbf{L}^{2(1)}$ we obtain $\mathbf{P}^{2(w)}\,(\mathbf{V}^2{}_1(\Phi^2)) = \mathbf{V}^2{}_1(\Phi^2)$ having $\|\mathbf{V}^2{}_1(\Phi^2)\| = 1$.

Thus the probability $p\,(v^2{}_1 \mid \Psi''') = 1$, i.e. certainty and this means that Bob will observe $v^2{}_1\,(M^2{}_k) = 1$ with certainty.

We have obtain that values of properties $v^1{}_i(M^1{}_k)$ and $v^2{}_j(M^2{}_k)$ are anti-correlated for each k.
(The situation when Alice finds $v^1{}_1(M^1{}_k) = 1$ is analogical.)

The simple explanation is the following. Once we obtained the formula in (v) where $\psi^1{}_n$ and $\psi^2{}_m$ are anti-correlated, then the anti-correlation between $v^1{}_i(M^1{}_k)$ and $v^2{}_j(M^2{}_k)$ is a simple consequence of this fact since (in the measurement processes) $v^1{}_i$ is correlated with $\psi^1{}_i$ and $v^2{}_j$ is correlated with $\psi^2{}_j$. The main step is Lemma 1, since this allows to transfer the anti-correlation between $e^1{}_i$ and $e^2{}_j$ to the anti-correlation between $\psi^1{}_i$ and $\psi^2{}_j$.

The main features of this situation in the hybrid-epistemic model is following:

- Individual measuring systems are anti-correlated individually (in the sense of properties)
- Measured systems $S^1{}_k$ and $S^2{}_k$ are anti-correlated collectively, i.e. as an ensemble. They cannot be correlated individually since they are epistemic systems and they have no individual properties and no individual states.
- Thus the individual information on $M^1{}_k$, say $v^1{}_2(M^1{}_k) = 1$ is transferred to Bob as $v^2{}_1(M^2{}_k) = 1$ through the **ensemble** of measured systems which is in the anti-correlated state.
- In the ontic model there exist also individual states of $S^1{}_k$ and $S^2{}_k$ which are anti-correlated, so that the explanation of the anti-correlation between $M^1{}_k$ and $M^2{}_k$ is trivial. (But the price for the simplicity of the ontic model are no-go theorems!)
- The fact that in the hybrid-epistemic model we have the individual anti-correlation only between measuring systems while in the ontic model there exists also the individual anti-correlation between measured systems creates the **fundamental difference** between these two models.

In fact, the EPR correlations are not different from classical correlations. We shall show an example considering pairs of gloves.

Let us consider the ensemble $\mathbf{G}_0 = \{G^1{}_k \oplus G^2{}_k \mid k = 1, .. , N\}$ of pairs of gloves. Each pair is (randomly) divided into two parts and one part $G^1{}_k$ is sent to Alice and second part $G^2{}_k$ is sent to Bob. Alice has an ensemble of measuring systems $\{M^A{}_k \mid k = 1, .. , N\}$. each measuring system is able to decide whether the glove $G^1{}_k$ is left-handed or right-handed. Bob has the analogical ensemble $\{M^B{}_k \mid k = 1, .. , N\}$. We can considered the complete ensemble

$$\mathbf{G} = \{M^A{}_k \oplus G^1{}_k \oplus G^2{}_k \oplus M^B{}_k \mid k = 1, .. , N\}.$$

Let us assumed that $M^A{}_k$ has found that the glove $G^1{}_k$ is left-handed. In this moment Bob obtains the information that his glove $M^B{}_k$ is right-handed.

Can this process be considered as a superluminal transport of the information? We are sure that there is no super-luminal transport of any information. It is clear that this correlation is created by the preparation of the ensemble $\mathbf{G}_0$ based on the interaction in the past.

In the classical situation gloves are individually anti-correlated but in the hybrid-epistemic model S^1's and S^2's are only collectively anti-correlated and this is the basic difference. It is clear that the collective (and not individual) anti-correlation between S^1's and S^2's makes the no-go theorems un-provable.

There may exist an impression that the up-dating axiom **HE8** is global, i.e. it is non-local and that this axiom is the kernel of the non-locality of QM. But the same situation is in the example of pairs of gloves.

All this can be seen from another perspective. We can always define the sub-ensemble

$$\mathbf{G}^{(A,\,\mathrm{left})} = \{ M^A{}_k \oplus G^1{}_k \oplus G^2{}_k \oplus M^B{}_k \mid v^{(\mathrm{left})}(M^A{}_k) = 1, k = 1, .. , N\}$$

This definition cannot be a non-local process since the definition of a new ensemble is the mental process but it is not the physical process.

For each $M^A{}_k \oplus G^1{}_k \oplus G^2{}_k \oplus M^B{}_k \in \mathbf{G}^{(\mathrm{left})}$ we have clearly that $v^{(B,\,\mathrm{right})}(M^B{}_k) = 1$. Thus through this ensemble the information $v^{(\mathrm{left})}(M^A{}_k) = 1$ is transposed to Bob's information $v^{(\mathrm{right})}(M^B{}_k) = 1$.

But there is nothing non-local in this classical example: the state of an ensemble is based on the correlation in the past and this does not create any problems.

We have shown that in the situation in QM after applying Lemma 1 is completely analogous to the classical case and thus the EPR correlations can be considered as a consequence of the correlation in past and as such they are local.

We have shown that the EPR correlations are completely analogical to the standard local classical correlations.

5. The standard epistemic model for QM

We have to give the axiomatic definition of the epistemic and of the ontic models since only then it will be possible to prove the theorem that all three models considered here are empirically equivalent.

The basic idea of the epistemic model is an idea of a particle stream which can be also considered as an **ensemble**. We shall be interested only in ensembles and their states. In the epistemic model the individual system has no state and no properties, so that individual systems cannot be directly considered.

Axiom SE1.

(i) To each ensemble **E** of systems there is associated a complex Hilbert space $\mathbf{H_E}$ and rays $[\psi]$ in this space are the possible pure states of **E**.[16] The mixed states of **E** are described by density operators in $\mathbf{H_E}$. At each given time t to each ensemble **E** there corresponds a density operator $St(\mathbf{E}; t)$ in $\mathbf{H_E}$ as a state of **E**.

(ii) Each ensemble is a result of certain preparation procedure. In a given time a given system can participate in only one preparation procedure.

(iii) In a given time ensembles are mutually disjoint. This is a consequence of (ii).

(iv) Each system is an element of some ensemble in a pure state.

(v) For each density operator ρ in $\mathbf{H_E}$ and for each time t there exists an ensemble **E** such that $St(\mathbf{E}; t) = \rho$.

Axiom SE2. Let $\mathbf{F} = \{M_1, .. , M_N\}$, $\mathbf{E} = \{S_1, .. , S_N\}$ be two ensembles. Then their composite $\mathbf{F}\oplus\mathbf{E}$ is given by $\mathbf{F}\oplus\mathbf{E} = \{M_1\oplus S_1, .. , M_N\oplus S_N\}$ and $\mathbf{H_{F\oplus E}} = \mathbf{H_F} \otimes \mathbf{H_E}$ **.**

Axiom SE3. For each ensemble **E** there is given a unitary group $\{U_t\}_{t\in R}$ in $\mathbf{H_E}$ which describes the evolution of the state of an ensemble **E** by

$$\text{state}\,(\mathbf{E}\,;t) = U_t\,(\,\text{state}\,(\mathbf{E}\,;0)\,)\,,\quad \text{i.e. } \psi(t) = U_t\,\psi(0)\,.$$

For mixed states ρ we have $\rho(t) = U_t\,\rho(0)\,U_t^*$.

[16] A ray $[\psi]$ is defined by $[\psi] = \{\alpha\psi \mid |\alpha| = 1\}$, where $|\psi| = 1$.

Axiom SE4. To each observable A of the system there exists a bounded self-adjoint operator A in the space $\mathbf{H}_E$ such that its spectral decomposition is

$$A = \Sigma_{\lambda \in \Lambda} \lambda P_\lambda \ , \quad \Lambda \subset (0, \infty) \ ,$$

where Λ is the spectrum of A and P_λ is the projection onto an eigenspace corresponding to the eigenvalue λ.

Definition (Born's formula).
For a given ensemble $\mathbf{E}$ in the state $[\psi] \in \mathbf{H}_\mathbf{E}$ and a given eigenvalue λ of A we define

$$p (\lambda \mid [\psi] ; A) = \mathrm{tr}\, P_\lambda (\psi \otimes \psi^*) = (\psi \mid P_\lambda\psi) = \|P_\lambda\psi \|^2 \ .$$

Definition (The transformation map).
The transformation map corresponding to an eigenvalue λ is defined by

$$T_\lambda ([\psi]) = [\| P_\lambda \psi \|^{-1} P_\lambda \psi] \ \text{ for each } \psi \in \mathbf{H}_\mathbf{E} , \ \| P_\lambda \psi \| > 0 \ .$$

Axiom SE5.
The measurement of the observable A is an operation transforming the ensemble $\mathbf{E}$ in a state $[\psi]$ onto a set of new ensembles

$$\{ \mathbf{E}^\lambda \mid \lambda \in \Lambda_0 (\psi) \} , \ \text{ where } \ \Lambda_0 (\psi) = \{ \lambda \in \Lambda \mid P_\lambda \psi \neq 0 \}$$

where $\mathbf{E}^\lambda$ and $\mathbf{E}^\mu$ are disjoint for $\lambda \neq \mu$ and $\mathbf{E} = \cup \{ \mathbf{E}^\lambda \mid \lambda \in \Lambda_0 (\psi) \}$,
The state of $\mathbf{E}^\lambda$ is $T_\lambda ([\psi])$.

This operation can be considered as a filtration F^A : from an ensemble $\mathbf{E}$ it creates a set of ensembles

$$F^A : \ \mathbf{E} \ \to \{ \mathbf{E}^\lambda \mid \lambda \in \Lambda_0 (\psi) \} \ .$$

What is interesting is the fact that there are fixed points of this map.

Definition. An ensemble $\mathbf{E}$ in the state $[\psi]$ is called λ-homogeneous if $F^A (\mathbf{E}) = \{\mathbf{E}^\lambda\}$, $\mathbf{E}^\lambda = \mathbf{E}$ and $\Lambda_0(\psi) = \{ \lambda \in \Lambda \mid P_\lambda \psi \neq 0 \} = \{\lambda\}$ (i.e. $P_\mu\psi = 0$ for each $\mu \neq \lambda$, or equivalently $P_\lambda\psi = \psi$) .

Definition. The output map $o : E \to \Lambda$ is defined by

$$o(S) = \lambda \ \text{ if and only if } \ S \in \mathbf{E}^\lambda .$$

Then the sub-ensembles $\mathbf{E}^\lambda$ can also be defined by $\mathbf{E}^\lambda = \{ S \in \mathbf{E} \mid o(S) = \lambda\}$ for each $\lambda \in \Lambda$.

Clearly, the filtration F^A is equivalent to the output map $o : E \to \Lambda$. The ensemble **E** is λ-homogeneous if and only if $o(S) = \lambda$ for each $S \in \mathbf{E}$.

Axiom SE6. (Born's rule.) The relative frequency of λ given by $f(\lambda) = N^{-1} \mid \mathbf{E}^\lambda \mid$ is equal to

$$f(\lambda) = p(\lambda \mid [\psi] ; A) .$$

Axiom SE7. After the measurement the ensemble **E** is replaced by $\mathbf{E}^\lambda$ in the situation when a value λ is obtained as a result of the measurement.
The ensemble $\mathbf{E}^\lambda$ will be in the state $T_\lambda([\psi])$.

Axiom SE8. For each system S and for each $\psi \in \mathbf{H}_S$ there exists an ensemble **E** in the state $[\psi]$ such that $S \in \mathbf{E}$.

This axiom is necessary for the comparison between epistemic and ontic models of QM. It is also quite natural. This means that the system S can be, in the sense $S \in \mathbf{E}$, for any state $[\psi]$.

The disadvantages of the standard epistemic model:

(i) In the standard epistemic model individual systems have no role, they have no states and no properties. The unique relation concerning the individual system is the fact that the individual system may be an element of a given ensemble.
(ii) If QM wants to describe the measurement process, the consideration of the individual systems is necessary since the basic step in this process consists in the observation of an individual measuring system.
(iii) The complete neglecting of individual systems is the main reason why the standard epistemic model is considered as insufficient and unacceptable.

6. The standard ontic model for QM

This is the standard Dirac-von Neumann's model of QM [5]. We describe its axiomatization in details since we make later the detailed comparison with other models. The main idea of this model is the requirement that **each wave function describes a possible state of an individual system** (and a state of each individual system is described by some wave function).

Axiom SO1. To each system S there corresponds a complex (finite dimensional) Hilbert space $\mathbf{H}(S) = \mathbf{H}_S$, $\dim \mathbf{H}_S \geq 2$ of states of the system S . In fact, the true states are rays $[\psi] = \{\alpha\psi \mid |\alpha| = 1\}$, $||\psi|| = 1$, $\mathbf{P}_S = \{[\psi] \mid \psi \in \mathbf{H}_S\}$. At a given time t the system S is in a state $St(S; t) \in \mathbf{P}_S$.

Axiom SO2. If M and S are two systems, their composite system $M \oplus S$ satisfies $\mathbf{H}_{M\oplus S} = \mathbf{H}_M \otimes \mathbf{H}_S$. If the system M is in the state $[\phi]$ and the system S is in the state $[\psi]$, and systems M and S are independent, then the system $M \oplus S$ will be in the state $[\phi \otimes \psi]$.

Axiom SO3. For each system S there exists an unitary group of transformations $\{U_t\}$ in the space $\mathbf{H}_S$ such that the evolution of the state of S is given by $[\psi(t)] = [U_t\, \psi(0)]$.

Axiom SO4. To each observable *A* of the system S there exists a bounded self-adjoint operator A in the space $\mathbf{H}_S$ such that its spectral decomposition is

$$A = \Sigma_{\lambda \in \Lambda}\, \lambda\, P_\lambda \ , \quad \Lambda \subset (0, \infty) \ ,$$

where Λ is the spectrum of A and P_λ is the projection onto eigenspace corresponding to the eigenvalue λ.

Below we shall consider the fixed observable A.

Definition (Born's formula). For a given system S in the state $[\psi]$ and a given eigenvalue $\lambda \in \Lambda$ we denote

$$p\,(\,\lambda \mid [\psi]\,;\, A\,) = \mathrm{tr}\, P_\lambda\, (\psi \otimes \psi^*) = (\psi \mid P_\lambda\, \psi) = \|\, P_\lambda\, \psi\, \|^2 \ .$$

Definition (the transformation map). The transformation map corresponding to $\lambda \in \Lambda$ is defined by

$$T_\lambda\,([\psi]) = [\, \|\, P_\lambda \psi\, \|^{-1}\, P_\lambda \psi\,] \ \text{ for each } \psi \in \mathbf{H}_S\, ,\ \|\, P_\lambda \psi\, \| > 0 \ .$$

Definition . The ensemble **E** in the state $[\psi]$ (at a given time t) is defined as a set of independent systems $\mathbf{E} = \{S_1, .. , S_N\}$ satisfying.

(i) $\mathbf{H}(S_1) = .. = \mathbf{H}(S_N) = \mathbf{H_E}$
(ii) Each system S from **E** is in the state $[\psi]$

Such an ensemble is called the homogeneous ensemble since all its elements are in the same state.[17]

Axiom SO5. (The measurement schema.)

[17] The concept of a homogeneous ensemble was introduced by von Neumann who postulated that each ensemble in the pure state is homogeneous.

Let us consider an ensemble **E** in the state [ψ] and let us consider a measurement of an observable A.
As a result of a measurement we obtain the "output" map $o : \mathbf{E} \to \Lambda$ which means that in the measurement of a system $S \in \mathbf{E}$ we obtain the output value $o(S) \in \Lambda$.

Then we can define a new ensemble

$$\mathbf{E}^{\lambda} = \{ S \in \mathbf{E} \mid o(S) = \lambda \} \text{ for each } \lambda \in \Lambda .$$

The relative frequency of the output value λ in the sequence $o(S_1), .., o(S_N)$ is given by

$$f(\lambda) = N^{-1} \mid \mathbf{E}^{\lambda} \mid$$

Axiom SO6. (Born's rule.) The Born's rule holds

$$f(\lambda) = p(\lambda \mid [\psi] ; A) .$$

Axiom SO7. (The collapse postulate.) After the measurement where the output value was λ , the state of the individual system S will be (immediately) changed

$$\text{from } [\psi] \text{ to } T_{\lambda}([\psi]).$$

Axiom SO8. For each system S and for each $\psi \in \mathbf{H}_S$ there exists an ensemble **E** in the state [ψ] such that $S \in \mathbf{E}$.[18]

This axiom is necessary for the comparison of epistemic and ontic models. It is also quite natural. It says that the ensemble **E** can be constructed as a set of systems in the individual state [ψ] .

The disadvantages of the standard ontic model:

(i) The main object of the criticism is the well-known Collapse postulate **SO7**[19]. This immediate change of the state of the individual system S is **completely un-physical**. There were proposed solutions to this problem, namely the so-called collapse theories [20] which have the main drawback of being non-linear and different from QM. But nevertheless the collapse postulate (as defined by von Neumann) is a **purely mathematical operation** without any physical meaning.

[18] This axiom is the same as axiom SE8.
[19] The collapse postulate was formulated by von Neumann in his famous book on QM [5].
[20] See e.g. [11].

(ii) The quantum state [ψ] is attributed to the individual system S, but there is no empirical procedure by which we could find what is a quantum state of a given individual system S. To find the quantum state [ψ] it is necessary to make (in fact more than one) tests on an ensemble of systems in the state [ψ] – to have only one copy of S is clearly insufficient.[21] This implies that empirically the state [ψ] can be attributed only to an ensemble of systems.
Thus the quantum state of an individual system is empirically un-defined.

(iii) In probability theory usually the state is an attribute of an ensemble[22]. In the standard ontic model the probability distribution (a wave function) is attributed to the individual system and this is, at least, very strange.[23]

7. The empirical equivalence of these three models

Here we shall show relations among empirical predictions of these three models. The experiments and the empirical predictions of QM corresponding to the experiment have in general the following structure:

Exp: preparation → evolution → measurement → registration → statistics.

Here the registration is a pair:

(the output value , the new state of the ensemble of measured systems)

and the statistics is a set of triplets:

{(the output value, the probability of the output value , the new state of the ensemble of measured systems)} .

Empirical predictions of QM can be tested only on an ensemble. (Also in the case where the prediction is sure, i.e. with probability one, it must be tested on an ensemble!)

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[21] This is same as for Brownian particle. Having only one copy of the Brownian particle it is impossible to find its probability distribution. To find the probability distribution it is necessary to have an ensemble of Brownian particles, to measure their positions and to calculate the relative frequencies. Only in this situation the probability distribution of the Brownian particle can be found. In fact, for the Brownian particle case one test is sufficient while in the quantum case more than one test is necessary.

[22] See for example the Brownian particle. An individual particle has a unique property – its position. The probability distribution is an attribute of an ensemble.

[23] This indicates the idea of the "individual probability theory" which, up to now, does not exist.

Thus the empirical predictions of QM are always related to ensembles.[24] This means that also in the ontic model we have to consider ensembles (which were defined in the section devoted to the ontic model of QM).

Let us assume that the set of possible outputs of an experiment is $\mathbf{O} = \{o_1, .., o_n\}$.

Definition.
The *standard QM prediction* is a set of triplets (which depend on the prepared state [ψ])

$$\mathbf{pred} = \{(o_1, p_1, \psi_1), .. , (o_n, p_n, \psi_n)\} ,$$

where to each possible output o_i there corresponds the probability of this output $p_i = p(o_i \mid [\psi])$ and the state of a new up-dated ensemble $\psi_i = T_i([\psi])$, $I = 1, .. , n$.

Definition.
The *standard QM experiment* is the process where

(i) The ensemble is prepared in the state [ψ]
(ii) The resulting standard QM prediction **pred** is tested

It is simple to recognize that the first two steps (preparation → evolution) are the same in all three models. The differences are only in the measurement part.

So we can assume that we have an ensemble $\mathbf{E} = \{S_1, \dots, S_N\}$ prepared in the state $\psi \in \mathbf{H_E}$.

Theorem 6. The empirical predictions of the epistemic model of QM and the hybrid-epistemic model of QM are the same.

The proof can be found in Appendix A.

Theorem 7. The empirical predictions of the epistemic model of QM and of the ontic model of QM are the same.

The proof can be found in Appendix A.

This is an interesting situation: we have a theory which has at least three different models with the same empirical content.[25] What means a truth in such a theory?

In mathematics, this is a quite common situation: we have non-standard models of Peano's arithmetic, of Zermelo-Fraenkel's set theory etc. The truth in these theories means statements which are true in all models.

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[24] We exclude in this paper the existence of something like "individual probability theory".

[25] Here we do not consider so-called Bohm mechanics since it is rather different in its structure from the standard QM.

In general, there may exist theorems which are provable in one model, but un-provable in another model. This is the case of QM: Bell's theorem and other no-go theorems can be proved in the ontic model but they cannot be proved in epistemic and hybrid-epistemic models (see sect. 8).

To find the true content of the Bell's theorem requires the more careful considerations. Bell's theorem asserts that the locality implies the Bell's inequality, and since Bell's inequality contradicts to QM the Bell's theorem asserts the non-locality of QM. This theorem can be proved only in the ontic model. In fact, **in all correct proofs** of the Bell's theorem the main (hidden) assumption is so-called "local **realism**" – and the realism means the assumption of the ontic model.

There is an apparent paradox that certain theorems can be proved in one model and not in other models. This seems to contradict to empirical equivalence of all three presented models of QM.

But there is no paradox: the point consists in the fact that Bell's theorem is not an empirical prediction – it has **no empirical content**. The experimental results on the empirical invalidity of Bell's inequality means only that the attempt to falsify QM was unsuccessful (i.e. the confirmation of QM). There are no known properties of this "non-locality" outside of the Bell's theorem.[26]

The distinction among these three models cannot be found on the base of the standard QM empirical predictions (and on the standard QM experiments). But there may exist statements (provable in some models) which do not have the form of a QM prediction and which cannot be tested in the standard QM experiment. The typical statement of this sort is the non-locality or the violation of the macro-realism discussed at length in the next section.

In each case, if we consider the principle that the correct consequences of QM are only such statements which are proved in all models of QM then **all known no-go theorems** in QM are not the **correct consequences of QM**.

The experimental test of the validity of Bell's inequality results in a negative answer and this is the standard QM experiment containing the standard QM prediction.

But the Bell's theorem (QM plus locality **plus the ontic model** implies the Bell's inequality) asserts that among assumptions {axioms of QM, the locality, the ontic model} some assumption must be false. But what is in the case of Bell's theorem the testable QM prediction? There does not exists any reasonable QM prediction and this show that Bell's theorem does not create any standard QM prediction and any standard QM experiment. The standard QM experiment on Bell's inequality tests the validity of QM and shows that Bell's inequality is false in the complete agreement with QM.

[26] In the section 4 it was proved that the famous EPR correlations ate perfectly local in the hybrid-epistemic model.

Assuming a priori validity of QM and the ontic model of QM we obtain as a result the non-locality of QM (the proof is done by contradiction, i.e. only non-constructive version is available). But what is the experimental proof of non-locality? Only one proof is available: the invalidity of Bell's inequality. In fact, there is no experimental confirmation of the non-locality outside of the negative test of Bell's inequality.

This is a serious problem. There is **no direct experimental manifestation** of so-called nonlocality of QM.

This shows that Bell's theorem does not give the standard QM prediction and that Bell's theorem belong to a new class of possible consequences of QM different from the standard QM predictions.

The question if in QM there can exist some consequences which cannot be represented in the form of the standard QM prediction is very important. At present, it seems that in the ontic model there exist such consequences and that they are the no-go theorems. Similar consequences are not known in epistemic and hybrid-epistemic models of QM.

We think that the explanations given here can explain the apparent contradiction between the empirical equivalence of the three proposed models and the different sets of theorems provable in these models.

This argument does not want to say that there is no content of Bell's theorem but that its content is different from the standard QM predictions[27].

8. The invalidity of no-go theorems in epistemic and hybrid-epistemic models

At first we shall consider the possible proof of Bell's theorem [6] in the epistemic model.

Let us state what we mean under Bell's theorem . This is the assertion

$$\text{locality} + \text{ontic model} \Rightarrow \text{Bell's inequality.}$$

The Bell's inequality contradicts to QM and since we assume the validity of QM, so that

[27] Our position is clear: the Bell's theorem is not a QM theorem but a meta-theorem proving the inner contradiction of the ontic model of QM. I.e. the proof of the inner inconsistency of the ontic model. It is possible to escape from this conclusion by assuming the non-locality of QM but an equally possible is to escape from this situation by using other model of QM different from the ontic model.

$$\text{ontic model} \Rightarrow \text{non-locality of QM} .$$

and equivalently

$$\text{the locality of QM} \Rightarrow \text{the ontic model is invalid.}$$

Let us immediately note that the assumption of the ontic model is necessary since we shall show below that in other two models of QM the Bell's theorem is **not** proved (and probably cannot be proved).

This means that sometimes used statements: QM + locality ⇒ Bell's inequality and QM ⇒ non-locality are false.

The right assumption is the local **realism** where realism means the assumption of the ontic model.

Each proof of Bell's theorem is based on the analysis of individual states (i.e. states attributable to individual systems). Already the principle of the proof of any Bell's theorem is to consider the Alice's system in some bases and the Bob's system in some bases. Then using the analysis of a certain inequality for individual systems this inequality can be integrated over the ensemble of such systems and the final Bell's inequality is obtained.

The origin of the proof is always based on the properties of states of individual systems. It is automatically supposed that each possible state of an individual system is described by a wave function.

Since in the epistemic model individual systems have no states and no properties, this proof cannot go through.

At the second step we shall consider (for the simplicity) the case of a hybrid model. In the hybrid model to each system there corresponds one orthogonal base of homogeneous states – other states are not homogeneous.

We can simply see that any proof of Bell's theorem requires either at least two different individual bases on the Alice's side or at least two individual bases on the Bob's side. Under the notion "individual base" we mean the base composed exclusively from individual states. The lowest number of bases is required in the original Bell's proof, where there must exist two individual bases at the Alice's side (A, A') and one individual base at the Bob's side (B).

In the standard proof of the CHSH inequality there are required two individual bases at Alice (A, A') and two individual bases at Bob (B, B').

In the well-known Mermin's proof there are required three different bases at Alice's side and three different bases at Bob's side.

The spirit of the Bell's proof consists in the fact that more than one individual base is used for some system. But exactly this is impossible in the hybrid model of QM where only one individual base is available on the Alice's side and only one individual base on the

Bob's side. Thus in the hybrid model of QM the standard proof of Bell's theorem cannot be proved.

Now we shall consider the case of a hybrid-epistemic model of QM. In some sense, the hybrid-epistemic model lies between hybrid model and the epistemic model and one could expect that the conclusion obtained for both hybrid and epistemic models will be true also for hybrid-epistemic model. But we shall consider the situation in more details.

For the hybrid-epistemic system there is nothing like a state of an individual system – in this sense the hybrid-epistemic system is similar to the epistemic system. Nevertheless for the hybrid-epistemic system there exist properties $v_1, .. , v_n$ which have a value defined on the individual system. But the corresponding states can came only from the homogeneous subspaces which create the orthogonal decomposition $\mathbf{L}^{(1)} + .. + \mathbf{L}^{(n)}$ of the state space and there is only one such decomposition. Thus the previous argument used for the hybrid system can be applied also to the hybrid-epistemic system.

We have proved the following theorem

Theorem 8.
In the epistemic and the hybrid-epistemic models the standard proof of Bell's theorem cannot be applied. Thus the Bell's theorem is not proved in these models.

Remark.
Bell's proof depends crucially on the concept of a state of an individual system. The calculation is at first done in the situation concerning an individual system and their possible states. This can be done only in the ontic model. (More exactly, more than one individual base must exist.) Usually this requirement is taken into account in the terms of so-called realism (realism = ontic model) but this is sometimes hidden in the term "local realism".

For example in the following formulation: "the local realism implies the non-locality of QM". This argument is not true. The falsity of local realism implies either that locality is false or that realism (= ontic model of QM) is false.

The assumption of realism is often hidden in this formulation but it means that the ontic model is assumed.

We have shown that the standard proof of Bell's theorem cannot be used in the hybrid-epistemic (or epistemic) model.

This does not exclude that some other proof could be possible. But this is extremely improbable since the basic idea of the Bell's proof is to make the calculation with individual systems and this is in principle impossible in the hybrid-epistemic model.

We can make the conclusion that Bell's theorem is proved in the ontic model but it is **not proved** in the epistemic and hybrid-epistemic models.

Now we shall consider the Leggett-Garg theorem [8] which is the "time" analog of the Bell's theorem.

The Leggett-Garg theorem says: macro-realism + ontic model ⇒ Leggett-Garg inequality. But this inequality is in contradiction with QM . Thus we obtain

ontic model ⇒ the falsity of the macro-realism .

The sometimes used simplified formulation: QM ⇒ the macro-realism is invalid since we shall show below that the assumption of the ontic model is necessary.

In the Leggett-Garg inequality there is considered an quantity Q at three different times $t_1 < t_2 < t_3$.

Let us consider the hybrid system S with the individual base $\{\psi_1, .., \psi_n\}$ given by the eigenvectors of $Q(t_1)$ and let us assume that $\{\phi_1, .., \phi_n\}$ is the base of eigenvectors for $Q(t_2)$. Let U be an evolution map from t_1 to t_2. We have ontic states $\psi_1, .., \psi_n$ at t_1 and $\phi_1, .., \phi_n$ at t_2. Using U we can obtain ontic states $U(\psi_1), .., U(\psi_n), \phi_1, .., \phi_n$ at the time t_2.

But in the hybrid system all individual (=homogeneous) states must be orthogonal.

This implies that $\phi_1 = U(\psi_{\pi(1)}) , .. , \phi_n = U(\psi_{\pi(n)})$ where π is a permutation. Thus the map U must be identity (after possible renaming of $\psi_1, .., \psi_n$) and we obtain that $Q(t_2) = Q(t_1)$ and similarly $Q(t_3) = Q(t_2)$. Then the Leggett-Garg inequality becomes trivial.

Thus the argument is very similar to the argument from the invalidity of the Bell's theorem in the hybrid model: at a given time only one orthogonal individual base can exist for the hybrid system so that the evolution map must transform the individual base at t_1 onto the individual base at t_2 etc. and this makes the Leggett-Garg theorem trivial.

In the case of the hybrid-epistemic systems the situation is analogical.

The Leggett-Garg theorem (which can be proved only in the ontic model) asserts that the macro-realism implies the Leggett-Garg inequality (which contradicts to QM for some evolution operator U).

Thus the consequence of the Leggett-Garg theorem is the statement that the macro-realism is not valid.

We have arrived at the following theorem

Theorem 9.

In the epistemic and the hybrid-epistemic models the standard proof of Leggett-Garg theorem cannot be applied. Thus the Leggett-Garg theorem is **not proved** in these models.

Since all no-go theorems are based on the investigation of states of individual system (for example GHZ tests and similar tests) we can expect that the situation with other no-go theorems will be similar: that they are proved in the ontic model of QM but that they are no provable in epistemic and hybrid-epistemic models.

Quite recently there were discovered new no-go theorems ([10], [11]). In both papers authors assume that QM is a valid theory but, in fact, they consider the ontic model of QM (they use the concept of quantum states of individual systems). Thus these no-go theorems can be proved only in the ontic model of QM.

Theorem 10.
The no-go theorems are proved in the ontic model of QM but they are not proved (and probably not provable) in both epistemic and hybrid-epistemic models.

9. The possible inconsistences of the ontic model of QM

We shall collect here arguments against the consistency of the ontic model of QM. We shall present a number of arguments:

- The incompatibility of the ontic model with the macro-realism of the classical world
- The Schrodinger cat paradox
- The incompatibility of the ontic model with the special and general relativity
- The collapse postulate (Axiom **SO7**)
- The postulate of definite outcomes

We shall consider these arguments step by step.

(i) The incompatibility of the ontic model with the macro-realism of the classical world – i.e. the incompatibility of the quantum world with the classical world.

In the ontic model it is possible to prove the Leggett-Garg theorem asserting that macro-realism is false. But in the classical world the macro-realism is true. Up to now there was no experiment proving the falsity of macro-realism in the classical world. (The proof of the falsity of the Leggett-Garg inequality does not imply the falsity of macro-realism since there is a **hidden assumption** of the ontic model – the possible proof of the falsity of the macro-realism cannot be based on the use of the Leggett-Garg theorem.)

The question of the compatibility of the quantum world with the classical world is of extreme importance. (This was the first Bohr's requirement that it is necessary to make quantum mechanics compatible with the classical world.) The well-known problem of the compatibility between General relativity and Quantum mechanics has a necessary prerequisite in the compatibility between QM and the classical world view.

Thus we consider the necessity of the compatibility between QM and the classical world view as a fundamental requirement of the consistency of physics (and a necessary condition for the possible unification of general relativity with QM).

In general, the compatibility between the quantum world and the classical world is principal requirement of the consistency in physics. Without this compatibility any interpretation of the world is impossible. This compatibility is more important than the standard requirement of the compatibility between General relativity and QM.

This compatibility (between QM and the classical world) is completely necessary. This requirement implies the rejection of the ontic model of QM as incompatible with the rest of physics.

(ii) The Schrodinger cat paradox

We can consider the individual state of a cat or the state of an ensemble of cats. We shall consider only the ensemble of cats in the Schrodinger cat state

$$\psi_{cat} = (2^{-1/2})\ (\psi_{alive} + \psi_{dead}).$$ [28]

As a state of an ensemble this can be considered as a possible state – there is an open question if an ensemble in such a state can exist. But in all of our three models we suppose that the ensemble in such a state exists. In fact, experimentally this seems to be an open question whether the ensemble in such "cat state" can be really created.

In an ontic model there is a problem. If there exists an ensemble **E** which is in the state ψ_{cat}, then it is necessary that any cat $S \in \mathbf{E}$ is in an individual state ψ_{cat}. But the situation when the **individual** cat is in a state ψ_{cat} is considered as absurd, at least by Schrodinger.

In this way the Schrodinger cat paradox **rejects** the possibility of the ontic model of QM.

For other models, epistemic and hybrid-epistemic there is no problem: the individual cat in the Schrodinger cat state need not exist. In fact, in epistemic model no individual states exist in general and in the hybrid-epistemic model there could exist individual states but as a maximum one orthogonal base for a system and it is insufficient for the Schrodinger cat paradox.

[28] The ψ_{cat} state was meant by Schrodinger ironically, of course.

This shows the absurdity of the individual Schrodinger cat state, and as a consequence, the absurdity of the ontic model of QM.

(iii) The incompatibility of the ontic model with the special and general relativity.

The ontic model implies the non-locality of QM. This is in the clear contradiction with the theory of relativity and this is serious.

But this clear contradiction has no concrete consequences since the non-locality of QM has no concrete manifestations.
But in principle this incompatibility cannot be accepted.

(iv) The collapse postulate (in the ontic model - Axiom **SO7**)

The von Neumann's collapse postulate (for the state of an individual system!) is a pure non-sense from the physical point of view. Any physical effect must have a clear physical origin. [29]

This was noticed by most physicists (notably this was the starting point of so-called many world interpretation of QM). The basic principles of QM are clearly the linearity and the unitarity and both these principles are violated in the process of collapse.

The collapse postulate as stated by von Neumann is a mathematical rule which is physically unacceptable. The von Neumann's postulate of collapse cannot be explained as a physical process. This indicates that the ontic model of QM (in the form of the von Neumann's axiomatization) is inconsistent or at least incompatible with the rest of world.

(v) The postulate of definite outcomes.

This postulate says that each (individual) experiment must have a uniquely defined and identifiable output. (This requirement is often related to the measurement problem.) This postulate is contained in our axiomatization of the hybrid-epistemic model of QM in the form of the definition what is a property: we required that the property v is a function which associated to each system S a number $v(S)$ which can have one of only two possible values 1 or 0. In Axiom **HE1** it is required that to each system there is associated a classification $\mathbf{C}_S$ which is a set of exclusive properties (but the requirement of exclusivity does not create any serious limitations).

[29] They are so-called collapse theories which explain the collapse of the state of an individual system but these theories are non-linear and thus cannot be interpreted inside QM.

10. The internal inconsistence of the ontic model of QM

In the ontic model the set of pure states is considered as the set of elementary states in the meaning of the probability theory. These states are considered as states of individual systems.

Thus the set $\mathbf{P}_S = \{ [\psi] \mid \|\psi\| = 1 \}$[30] is the analog of the set of possible positions of a Brownian particle (i.e. R^3).

Each statistical mixture A can be defined as a pair $A = \{spt_A , w_A\}$ where spt_A is a finite (or countably infinite) subset of $\mathbf{P}_S$ and the weight function w_A is a function from spt_A into positive numbers satisfying $\Sigma \{ w_A([\psi]) \mid [\psi] \in spt_A \} = 1$.

Two mixtures A and B are **equal** if and only if $spt_A = spt_B$ and $w_A = w_B$ – this is a consequence of the concept of wave functions (i.e. $\mathbf{P}_S$) as elementary events. This is a basic principle of the probability theory.

We can consider a simplest case $A = \{spt_A , w_A\}$, where $spt_A = \{[\psi]\}$, $w_A \equiv 1$, $B = \{spt_B , w_B\}$, where $spt_B = \{[\phi]\}$, $w_B \equiv 1$, $[\psi] \neq [\phi]$. Then clearly $A \neq B$.

The next most simple case is the following $A = \{spt_A , w_A\}$, $spt_A = \{[\psi_1], [\psi_2]\}$, $w_A \equiv ½$, $B = \{spt_B , w_B\}$, $spt_B = \{[\phi_1], [\phi_2]\}$, $w_B \equiv ½$, where $[\psi_1] \neq [\psi_2]$, $[\phi_1] \neq [\phi_2]$, $spt_A \cap spt_B = 0$. Then also clearly $A \neq B$. These two mixtures must be different since they are constructed from completely different elementary states.

But in the ontic model the concept of a statistical mixture is treated in a completely different way. Each pure state $[\psi]$ can be represented by its density operator $\psi\otimes\psi^*$. To each statistical mixture A there is associated the density operator defined as $\rho_A = \Sigma \{ w([\psi])\, \psi\otimes\psi^* \mid [\psi] \in spt_A \}$.

Then it is postulated that mixtures A and B are in the same state if $\rho_A = \rho_B$ (i.e. that all physical properties of a mixture A depend only on ρ_A)[31].

Now we shall show that there are infinitely many situations where different mixtures have the same density operator.[32] We shall consider the 2-dimensional Hilbert space with the orthogonal base $\{e_1, e_2\}$. Let φ and θ be two free parameters and we shall assume that $a^2 + b^2 = 1$. We define two vectors

$$u = \alpha^{-1} (a.\cos\varphi, -b.\sin\varphi.e^{i\theta}) , v = \beta^{-1} (a.\sin\varphi.e^{-i\theta}, b.\cos\varphi.) ,$$

where $\alpha = (a^2 \cos^2\varphi + b^2 \sin^2\varphi)^{½}$, $\beta = (a^2 \sin^2\varphi + b^2 \cos^2\varphi)^{½}$.

[30] $[\psi] = \{e^{i\alpha} \psi \mid \alpha \in R\}$

[31] QM predicts only probabilities and they depend (Born's rule) only on the density operator.

[32] This paradox of a duplicity of mixtures is generally known for a long time.

Then we obtain (by the numerical calculation) that

$$\alpha^2\, u\otimes u^* + \beta^2\, v\otimes v^* = a^2\, e_1\otimes e_1^* + b^2\, e_2\otimes e_2^* = \mathrm{diag}(a^2, b^2)\,.$$

Then we can define two mixtures A and B by

$spt_A = \{[e_1], [e_2]\}$, $w_A([e_1]) = a^2$, $w_A([e_2]) = b^2$,
$spt_B = \{[u], [v]\}$, $w_B([u]) = \alpha^2$, $w_B([v]) = \beta^2$,

For the corresponding density operators we have $\rho_A = \rho_B$. For the mixture A (parametrized by a, b satisfying $a^2 + b^2 = 1$) we have infinitely many (with free parameters φ and θ) mixtures B's which give the same density matrix as the density matrix ρ_A .

In fact, such multiplicity exists for each density operator ρ . Each ρ can be written in the form $\rho = a^2\, e_1\otimes e_1^* + b^2\, e_2\otimes e_2^*$ with some orthogonal base $\{e_1, e_2\}$[33] . Then the construction of B can be applied as well.

Thus for each mixture A we have at least a 2-dimensional variety of mixtures B such that $\rho_A = \rho_B$. Thus we can assert that the phenomenon of the infinite multiplicity of mixtures having the same density operator as a given mixture A is completely general.

Now we shal consider the particular case of this phenomenon of multiplicity.
Let us set $a^2 = b^2 = ½$ and $\theta = 0$. Then we have

$$u = (\cos\varphi\,, -\sin\varphi)\,,\ v = (\sin\varphi\,, \cos\varphi)\,,\ e_1 = (1, 0)\,,\ e_1 = (1, 0)\ \text{ and}$$
$$\rho_B = ½(u\otimes u^* + v\otimes v^*) = \rho_A = ½(e_1\otimes e_1^* + e_2\otimes e_2^*)\,.$$

We can obtain that for any orthogonal base (u, v) the corresponding mixture with the weight function $w \equiv ½$ we obtain the same density operator $\rho = \mathrm{diag}(½,½)$.

But these mixtures are constructed from completely different individual states. The supports $spt_A = \{e_1, e_2\}$ and $spt_B = \{u, v\}$ have nothing in common.

This situation is very strange and contradicts to the spirit of probability theory.
In the probability theory if $spt_A \cap spt_B = 0$ the mixtures A and B are surely physically different while in QM they can be equal. This implies that not all pure states can be considered as an individual states and this means that the ontic model of QM is not consistent.

This can be expressed in the form that the system of elementary states in the ontic model of QM is over-determined.

[33] We assume that ρ is a non-pure state.

In our hybrid model of QM for each system there exists exactly one orthogonal base composed from individual states. In the property -epistemic model there are no individual states for any system.

The one proposed solution is to consider mixtures A and B with $\rho_A = \rho_B$ as physically equal. But such an assumption clearly contradicts to the principles of the ontic model. e_1, e_2. u, v are different individual states. The ensemble $\mathbf{E}_A = \{S_1, .., S_{2N}\}$ is such that systems $S_1, .., S_N$ are in the individual states $[e_1]$ and systems $S_{N+1}, .., S_{2N}$ are in the individual state $[e_2]$ (the order of systems S_i is not important). The ensemble $\mathbf{E}_B = \{T_1, .., T_{2N}\}$ is such that systems $T_1, .., T_N$ are in the individual states $[u]$ and systems $T_{N+1}, .., T_{2N}$ are in the individual state $[v]$. It is evident that these two ensembles are physically different and thus they are in different states. And this contradicts to the ontic model.

In this situation we can conclude that **this inconsistence consisting in the multiplicity of mixtures is the internal inconsistence of the ontic model of QM.**

Quite recently, in papers [10] and [11] there were proved new no-go theorems where the consistency of QM is questioned. Authors use in these papers the ontic model of QM (to each system at a given time there is associated a wave function as its state). We interpret the results of these two papers as the strong indication that the ontic model of QM is internally inconsistent.

11. The property-epistemic model of QM

The property-epistemic model was defined at the end of the Sect. 2 by the assumption that each system S in the model **M** is property-epistemic or epistemic. The property-epistemic system is defined by the condition that each homogeneous subspace $\mathbf{L}^{(v)}$ for each S is at least two-dimensional for each condition $v \in \mathbf{C}_S$.

This conditions can be expressed in a very simple and expressive way.

We shall call this condition the **Einstein's QM principle** since in [9] he advocated that "… **the ψ-function is to be understood as the description not of a single system but of an ensemble of systems**." This means that the wave function cannot be associated with an individual system.

Axiom PE1. The hybrid-epistemic model of QM is the property-epistemic model of QM if the quantum state (i.e. wave function) can be attributed only to ensembles and cannot be attributed to individual systems.

In fact, in the hybrid-epistemic model the quantum state can be attributed to the state of an individual system only in the situation where for some property $v \in \mathbf{C}_S$ the corresponding homogeneous subspace $\mathbf{L}^{(v)}$ is one-dimensional.

Axiom **PE1** means that in the property-epistemic model there can exist only collective properties.

Thus in the property-epistemic model the situation is very clear: the quantum state (wave function or density operator) can be attributed only to ensembles, while properties can be attributed only to individual systems.

The Einstein's principle gives the simple and very clear solution to the famous **Bohr – Einstein debate** of the problem if the wave function is the complete description of the state of an individual system. The solution is really simple: the wave function cannot be considered as a state of an individual system so that the question about its completeness is meaningless.

The statement that the quantum state is attributable only to ensembles is the firm basis of the epistemic model. In the property-epistemic model we extend the epistemic model by the introduction of the concept of properties of individual systems.

In the **property-epistemic model of QM** many properties, statements and proofs are **extremely** simplified with respect to the hybrid-epistemic models of QM.

(i) The concept of an individual state[34] used in [1], [3] cannot be applied in the property-epistemic model since all systems in this model are either property-epistemic systems or epistemic systems.

(ii) The superposition principle for individual states (the individual state = the quantum state of an individual system) is meaningless in the property-epistemic models since there are no individual states.[35]

(iii) In the property-epistemic models individual systems can have properties and in this way the basic objection against the epistemic model is solved – individual systems have their real status. The basic objection to the epistemic model (that the epistemic model is unable to describe individual systems) is in this way solved. In the property-epistemic models the individual systems are described by properties.[36]

[34] I.e. the state where $\dim \mathbf{L}^{(v)} = 1$ and this means the associated homogeneous space to v is one-dimensional.
[35] The superposition principle for the states of ensembles is true but it is trivial since possible states belong to the linear Hilbert space.
[36] The epistemic systems (like a spin) can have no properties but there must exist sufficient number of property-epistemic systems - see axiom HE5.

(iv) The no-go theorems (Bell's, Leggett-Garg's and similar theorems) are based on the concept of an individual state (i.e. the quantum state of an individual system) and this concept is vacuous in the property-epistemic model. Thus these no-go theorems cannot be proved in the property-epistemic models.

(v) The property-epistemic models are in the direct opposition to the ontic models. In the ontic model each pure quantum state is a possible state of some individual system (and to each individual system there is attributed its pure quantum state) while in the property-epistemic model no quantum state can be attributed to an individual system.

(vi) In general, it can be assumed that the properties of individual system will be related to the position properties. These can be considered as particle properties. Thus one could assume that individual systems' properties will be of the kind of particle properties. We shall assume this. Then we can assume that individual systems have no wave properties, only particle properties.

(vii) In this way we obtain a simple solution to the problem of the **wave-particle duality**:

a. Wave properties can be attributed only to the ensembles – i.e. individual systems **have no wave properties** [37] [38] [39]

b. Particle properties can be attributed only to the individual systems[40].

c. The solution is then quite simple: the particle and wave properties are attributed to the different objects – to individual systems resp. to ensembles, so that there is no contradiction.

d. The Bohr's **complementarity principle** is superfluous, not necessary, meaningless and should be abolished.

(viii) The collapse problem cannot be solved in the standard ontic model since the immediate change of the state of an individual system during the collapse is un-physical.

[37] It is well-known that the interference picture ("demonstrating the wave nature of quantum objects") can be built up only by doing the experiment with a large number of photons, i.e. with an ensemble of photons.
[38] With one photon the interference property cannot be demonstrated – we obtain a point on the screen and where is the superposition picture? This implies that the interference phenomenon is a statistical phenomenon and not the physical phenomenon in each individual case. I.e. that the interference phenomenon can be attributed only to ensembles and not to individual systems.
[39] The fact that the interference phenomenon can be a statistical property is a consequence of the fact that in QM the quadratic probability theory must be used instead of the standard Kolmogorov probability theory [1].
[40] Of course, there are quantum states which have particle properties – consider the eigenstates of the position operator – but these are only exceptions of the general rule

In the epistemic model the collapse problem has a trivial solution – the update of the state of an ensemble after obtaining new information. The same solution of the collapse problem can be applied in the property-epistemic model.[41]

Remarks.

- It was clear that the no-go theorems cannot be proved in the epistemic model (the non-existence of individual states) but the epistemic model was a priori excluded as a possible solution of foundational QM problems.
- Our solution – to extend the epistemic model into the property-epistemic model solves the critique of the epistemic model but it retains the impossibility of proving no-go theorems.
- The main principle of the property-epistemic model is the non-existence of individual states (i.e. quantum states of individual systems). This goes directly against any intuition from the standard QM. It is not simple to take into account this new ideology and not to fall into the previous picture – the concept of the quantum state (i.e. ψ) of an individual system is meaningless. This is the main change.
- In Sect. 7 it was proved that the empirical predictions of the hybrid-epistemic model of QM and the empirical predictions of the ontic model of QM are the same. The property-epistemic models are the special sub-class of the hybrid-epistemic models and it can be simply shown that these two models generate the same empirical predictions. Thus the standard ontic model and the property-epistemic model are empirically equivalent.
- In the hybrid-epistemic models the situation is the same as in the property-epistemic models but proofs are more complicated. In the property-epistemic models the situation is completely clear and simple and proofs are almost trivial.
- There is some surprising similarity between the property-epistemic models of QM and the Bohr's concept of quantum mechanics.
    - The basic difference is clear: Bohr assumes that the quantum state (wave function) is attributed to the individual quantum system while we assume exactly the opposite.
    - Bohr assumes that microscopic systems are directly un-observable, only (classical) measuring systems are observable. The same is true for our epistemic systems (they have no properties), while the property-epistemic systems are individually observable through their properties.
    - Bohr assumed that the world is not monistic, i.e. that it is not true that all systems are of the same kind (he made difference between quantum systems and classical systems). The analogical (but surely not the same) difference exists between epistemic systems and property-epistemic systems. But our difference between epistemic and property-epistemic systems is systematic (i.e. well defined) and it does not use the concept of a macroscopic system.
    - As a consequence we see that the property-epistemic model is dualistic and that this dualistic approach corresponds properly to the character of QM. There are systems like the spin of a particle which have no (directly) observable properties and there are systems like the Stern-Gerlach

[41] It can be seen that many good features of an epistemic model can be extended onto property-epistemic models.

apparatuses which have observable properties (spin-up, spin-down). The difference between these two classes of systems objectively exists.
   - In fact, there does not exist an argument that QM must be monistic. This is rather the philosophical prejudice. QM can be dualistic and all experience shows that this is the real case.

Main consequences of the property-epistemic models of QM

- In the property-epistemic model of QM the Bell's theorem cannot be proved (this is a rather trivial consequence of the non-existence of individual states) and thus, for example, the **non-locality** of QM cannot be proved.
- This means that all the "industry of quantum non-locality" is out of a reality.
- This means that the "industry of Bell's inequalities" is out of a reality
- The same is true for the Leggett-Garg's theorem and the possible violation of macro-realism.
- No-go theorems are, in general, un-provable in the property-epistemic models of QM since the basic assumption of the existence of individual quantum states (i.e. quantum states of individual systems) is false in the property-epistemic models.

At the end we want to mention the important meta-argument

- In the standard classical probability theory (for example the description of the Brownian motion) the basic assumption is that the probability distribution describes a state of an ensemble. It is true that there are some exceptions: the ensemble of Brownian particles with the same position is the ensemble which can be associated with the state of individual Brownian particle. But this exception is rather random and un-important. Thus, in general, the probability distribution does not describe the state of an individual system.[42]
- In QM the wave function defines the resulting probabilities and thus it should be considered as an analog of the probability distribution. Thus it should describe the state of an ensemble and not the state of an individual system.
- As a consequence we obtain the statement that the wave function describes the possible state of an ensemble and that the wave function **does not** describe the state of an individual system. There may exist exceptions to this statement but they are not essential. We obtain the basic assumption of epistemic and property-epistemic models that wave function **cannot** be attributed to the individual system.
- The difference between epistemic and property-epistemic systems lies in the fact that in the epistemic model the individual systems have no status (they have no features) while in the property-epistemic model individual systems can have the proper objective existence since they can have properties.

The attribution of the quantum state (i.e. the wave function) to the individual quantum system is absurd in the same way as the attribution of the probability distribution to the individual Brownian particle is absurd. Both are non-sense.

[42] The probability distribution is the non-negative function on R^3 while the state of the individual Brownian particle is defined by the position $x \in R^3$ (at a given time). The exceptional probability distribution which can be associated with the individual state is the well-known delta-function $\delta(x)$.

This is our main conclusion: the attribution of the quantum state to the individual quantum system is a non-sense. Individual quantum systems cannot be described by quantum states.

12. The discussion

In this section we shall discuss some typical arguments in this problematic.

Argument 1. The experiments on the Leggett-Garg inequality proves that the macro-realism is false.

Answer 1. There are many counter-arguments to this statement.

(i) This experiment cannot prove anything without the assumption of the validity of QM – in fact, without assuming and using QM this experiment cannot be described and interpreted. The statement that this experiment alone proves something is false. This experiment only confirms the validity of QM (or that the possible refutation of QM was un-successful) and nothing else.

(ii) What is really important is the theoretical fact that Leggett-Garg inequality contradicts to QM - possible experiments are, in fact, irrelevant (they are relevant only for the validity of QM).

(iii) The most important counter-argument considers what is the true content of the Leggett-Garg theorem. This theorem says inaccurately that the macro-realism ⇒ the Leggett-Garg inequality or, equivalently QM ⇒ the violation of the macro-realism. But the true content of this theorem expresses equivalent statements:
- Macro-realism **+ the ontic model** ⇒ Leggett-Garg inequality
- The ontic model ⇒ the violation of macro-realism
- The macro-realism ⇒ the invalidity of the ontic model
- The existence of the classical world ⇒ the invalidity of the ontic model.

(iv) The Argument 1. is false since there exists the hybrid-epistemic model in which the Leggett-Garg theorem cannot be proved (at least it is not proved).

Argument 2. The superposition principle is the base of QM (confirmed by experiments) and it cannot be attacked.

Answer 2. There are many counter-arguments.

(i) The superposition principle is generally assumed for states of ensembles where it is a consequence of basic axioms of all three models considered here.

(ii) The superposition principle is problematic for states of individual systems - let us call it the individual superposition principle. In the ontic model each (pure) state of an ensemble is also a possible state of an individual system, so that in this case the individual superposition principle is a trivial consequence of axioms for the ontic model.
(iii) In the epistemic model the individual superposition principle has no sense.
(iv) In the property-epistemic model also the individual superposition principle has no sense since there are, in general, no states of individual systems (individual systems can have only properties but no states). For details see Theorem 4.
(v) The individual superposition principle contradicts to the classical world.
(vi) The individual superposition principle is in the core of paradoxes and contradictions of the ontic model.
(vii) In the hybrid-epistemic model the superposition principle is false for individual states.

Argument 3. The QM is evidently the valid theory (i.e. experimentally confirmed) and thus the superposition principle must be true also for macroscopic systems.

Answer 3. Individual superposition principle is a consequence of QM only in the ontic model of QM. In the hybrid-epistemic model of QM the individual superposition principle is not consequence of axioms. It follows that the individual superposition principle **is not** a consequence of QM. The situation with the superposition principle is following: for ensembles it is trivial and for individual systems it is false.

Argument 4. The macroscopic superposition must exist.

Answer 4. There are some counter-arguments.
(i) Up to now nobody has observed such macroscopic states
(ii) The question must be specified: does there exist ensembles (of macroscopic systems) which are in the cat states? The existence of such ensembles was not proved. But in all three models considered here it follows from axioms the necessary existence of such states.
(iii) The alternative question: does there exist individual systems in such states? This is completely different question from the preceding question.

We think that the question (ii) could be answered (in the future) perhaps positively but the question (iii) cannot have a positive answer.

People often believe in some "absurd" consequences of QM (like the existence of "cat states") since they argue that QM is evidently true. But also in the case when QM is true, the argument is false since these "absurd" consequences of QM are not consequences of QM but of QM **+ the ontic model**. This means that these "absurd" consequences are, in fact, consequences of the ontic model but there exists also the hybrid-epistemic models where such consequences are not possible. Thus the argument is false.

The solution of above mentioned problems is simple: to take into account the existence of other models of QM than the ontic model.

Argument 5. The Bell's theorem was considered by many scientists as "the most important discovery of science". After the proof of its invalidity what is then its meaning?

Answer 5. The new meaning of the Bell's theorem is equally important.

The Bell's theorem must be considered as the first clear indication (almost a proof) that the ontic model of QM is **inconsistent**. The Bell's theorem rejects the local realism but this statement was generally interpreted (erroneously) as a rejection of the locality.

But the right interpretation of the Bell's inequality (see this paper) implies that the realism (= the ontic model of QM) must be rejected.

Thus what Bell discovered with his inequality is **the internal inconsistence of the standard ontic model of QM** introduced by von Neumann (1932). This opened the way to the other models of QM considered here.

13. Conclusions

Our main conclusion is:

Ensembles have states while (some) individual systems have properties. This is our answer to the question about the nature of quantum states.

Quantum states are attributable only to ensembles (with a few exceptions) and to individual systems quantum states cannot be attributed.

In this sense we have solved in the hybrid-epistemic model the basic argument against the epistemic model: the zero status of individual systems. We have extended the epistemic models of QM onto hybrid-epistemic models (resp. property-epistemic models), where individual systems can have properties.

The ontic model of QM is inconsistent with the classical world on many levels. Very probably it is inconsistent also internally.

Fortunately there exists a substitute – the hybrid-epistemic model which has the same empirical content as the ontic model and does not have any problems created by the ontic model.

What we have obtained in this paper

- The axiomatic definition of a new model for QM, the hybrid-epistemic model
- The basic properties of the hybrid-epistemic model
- In the hybrid-epistemic model states can be attributed only to ensembles (there are only very rear exceptions from this rule)
- Individual systems may be characterized by properties (individual hybrid-epistemic systems can have properties while individual epistemic systems do not have any properties)
- The proof that this new model and the standard QM model give the same empirical predictions - i.e. they are empirically equivalent (this is true also for the epistemic model)
- **Since the valid statements of QM must be valid in all models of QM, we have shown that no-go theorems are not valid statements in QM (they are valid statements only in the ontic model of QM)**
- The proof that the ontic model of QM is incompatible with the classical physics and, in general, the incompatibility of the quantum world with the classical world in the ontic model of QM
- There are arguments indicating that the ontic model of QM is internally inconsistent

Merits of the hybrid-epistemic model

- The **impossibility** to prove no-go theorems
- In the hybrid-epistemic model there exists a mechanism of EPR correlations which is explicitly local and this together with the impossibility to derive Bell's theorem gives **the locality of QM**
- The locality of QM in the hybrid-epistemic model is a **great achievement** since the presumed non-locality of QM would have great consequences not only in physics but in the general natural philosophy and in the general understanding of the world.
- The deliverance from the **non-locality prison –** the world is now more or less standard and without any (unclear) non-local phenomena (this non-locality prison started in 1935 with the famous EPR paper and continued up to now)
- The model of an internal measurement process, i.e. the possibility to consider the measurement process as the standard QM process

Merits of the **property-epistemic** models of QM

- The property-epistemic models are the sub-class of hybrid-epistemic models thus they inherit all properties of hybrid-epistemic models
- The almost trivial proof of the impossibility to prove no-go theorems
- No problems with the collapse of individual states and with the other foundational problems

- The realistic view of quantum systems

The conclusions which follow from these findings:

- The standard ontic model of QM should be abandoned since it creates too many problems and inconsistences and since the ontic model contradicts to the reality of the classical world
- Instead of the ontic model the new hybrid-epistemic (or property-epistemic) models should be accepted as a better solution to foundational problems which is, moreover, **empirically equivalent** to the ontic model
- The hybrid-epistemic model with states of ensembles and properties of individual systems describes better the reality of the quantum world than the ontic model. The property-epistemic models could do this still better.

This shows that the main obstacle in the solution of foundational problems of QM is the concept of an individual quantum state (= the quantum state of an individual system).

But the concept of an individual state is the basis of the ontic model and of the standard QM and this was the real obstacle in the development of the true understanding of QM in the past 85 years.

Our conjectures, opinions and remarks can be found in Appendix B.

Appendix A: proofs.

In this part we collect proofs of theorems and propositions.

The proof of Proposition 1.
We can assume that there exist a Hermitian operator K, such that $p (v \mid [\psi]) = (\psi \mid K\psi)$ for each $\psi \in \mathbf{H}_S$, $\|\psi\| = 1$. Let $K = \lambda_1 P_1 + .. + \lambda_m P_m$, $\lambda_1 > .. > \lambda_m$ is the spectral decomposition of K. We have $(\psi \mid K\psi) = \lambda_1 \|P_1\psi\|^2 + .. + \lambda_m \|P_m\psi\|^2$ and from $0 \le p (v \mid [\psi]) \le 1$ we obtain $1 \ge \lambda_1 > .. > \lambda_m \ge 0$. Then $p (v \mid [\psi]) = (\psi \mid K\psi) = 1$ is possible if and only if $\lambda_1 = 1$ and $\psi \in P_1(\mathbf{H}_S)$.

The proof of Theorem 1.
Let us consider an ensemble $\mathbf{E} = \{S_1, \dots ,S_N\}$ in the state $[\psi]$, $\psi \in \mathbf{L}^{(w)}$. From this we obtain that for values $\{w(S_1), \dots, w(S_N)\}$ we have $\lim_{N\to\infty} N^{-1} \mid E^{(w)} \mid = 1$. Since properties v and w are exclusive (i.e. they cannot be verified simultaneously) we have

$$\lim_{N\to\infty} N^{-1} \mid E^{(v)} \mid = 0.$$

Then using Axiom **HE7** (with $\mathbf{H}_S$ trivial) we obtain
$\| \mathbf{P}^{(v)} \psi \|^2 = p (v \mid [\psi]) = \lim_{N\to\infty} N^{-1} \mid E^{(v)} \mid = 0$.
From this it simply follows that ψ is orthogonal to the subspace $\mathbf{L}^{(v)}$.

The proof of Theorem 2.
We have already shown the orthogonality of homogeneous subspaces. Let us assume that Hom(S) is not a complete orthogonal decomposition of $\mathbf{H}_S$. Then there exists a state $[\psi] \in \mathbf{P}_S$ which is orthogonal to all elements of Hom (S). Using Axiom **HE7** we obtain

$$p (v \mid [\psi]) = \| \mathbf{P}^{(v)} (\psi) \|^2 = 0, \text{ for each } v \in \mathbf{C}(S).$$

This means that the function $p (. \mid [\psi])$ is trivially equal to 0 and this is impossible since this function is a probability distribution.

The proof of Theorem 4.

(i) If two homogeneous states belong to two distinct homogeneous subspaces, then their non-trivial superposition cannot be an element of some homogeneous subspace since these subspaces are orthogonal. But only an element of some homogeneous subspace is a homogeneous state. On the other hand if two homogeneous states belong to the same homogeneous subspace, then any superposition off these two states belongs to the same homogeneous subspace and thus this superposition is a homogeneous state.

(ii) If $[\psi] \in \mathbf{L}^{(v)}$, $[\varphi] \in \mathbf{L}^{(w)}$, $\mathbf{L}^{(v)}$ and $\mathbf{L}^{(w)}$ are orthogonal then the non-trivial superposition $[\alpha\psi + \beta\varphi]$ cannot be orthogonal to both $\mathbf{L}^{(v)}$ and $\mathbf{L}^{(w)}$ thus $[\alpha\psi + \beta\varphi]$ cannot be an individual state.

The proof of Theorem 5.

(i) Let $v = v_j$. Then $(\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S)) (\mathbf{U}(\phi\otimes\psi)) = \Sigma_i \alpha_i (\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S)) (\mathbf{U}(\phi\otimes\psi_i))$.
Since $\mathbf{U}(\phi\otimes\psi_i) \in \mathbf{L}^{(i)} \otimes \mathbf{K}_j$ for $i \neq j$ we obtain $(\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S)) (\mathbf{U}(\phi\otimes\psi_i)) = 0$.
Thus we have to assume that i = j and then we obtain

$$(\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S)) (\mathbf{U}(\phi\otimes\psi)) = \alpha_j (\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S)) (\mathbf{U}(\phi\otimes\psi_j)) = \alpha_j \mathbf{U}(\phi\otimes\psi_j)$$

since $\mathbf{U}(\phi\otimes\psi_j) \in \mathbf{L}^{(j)} \otimes \mathbf{K}_j$ and $\mathbf{P}^{(v)}$ is identity on $\mathbf{L}^{(j)}$.

By **HE7** we obtain
$p (v \mid [\mathbf{U}(\phi\otimes\psi)]) = \|(\mathbf{P}^{(v)} \otimes \mathrm{Id}(\mathbf{H}_S)) (\mathbf{U}(\phi\otimes\psi))\|^2 = \|\alpha_j \mathbf{U}(\phi\otimes\psi_j)\|^2 = |\alpha_j|^2$.

(ii) We have $\mathbf{U}(\phi\otimes\psi) = \Sigma \alpha_j \mathbf{U}(\phi\otimes\psi_j) = \Sigma \alpha_i \mathbf{V}_i(\phi) \otimes\psi_i$.
From $v_j(M_k) = 1$ we obtain $\mathbf{U}(\phi\otimes\psi) = \mathbf{V}_j(\phi) \otimes\psi_j$ since only $\mathbf{V}_j(\phi) \in \mathbf{L}^{(j)}$ and $\mathbf{V}_{j'}(\phi)$, $i' \neq i$, is orthogonal to $\mathbf{L}^{(i)}$.

(iii) For given $n \geq 2$ there exists a balanced observable system satisfying $|\mathbf{C}(\mathbf{E})| = n$ by the axiom **HE5**.

Up to now the map $\mathbf{U}$ was defined only on $\mathbf{L}^{(1)} \otimes \mathbf{H}_S$ by $\mathbf{U}(\phi\otimes\psi) = \mathbf{V}_i(\phi) \otimes\psi$ for $\psi \in \mathbf{K}_i$.
If $\phi \in \mathbf{L}^{(j)}$ and $\psi \in \mathbf{K}_i$ we set $\mathbf{U}(\phi\otimes\psi) = \mathbf{V}_{i+j-1}(\phi) \otimes\psi$ for $i+j-1 \leq n$ and $\mathbf{U}(\phi\otimes\psi) = \mathbf{V}_{i+j-1-n}(\phi) \otimes\psi$ for $i+j-1>n$.

The proof of Theorem 6. It will be divided into two parts.

(i) We shall start with the -experiment described in the epistemic model which produces the prediction **pred** and we shall show that the same prediction can be obtained in the hybrid-epistemic model.
In the preparation (and evolution) parts the ensemble $\mathbf{E} = \{S_1, .. , S_N\}$ in the state ψ is produced. The specification of the measurement is given by the observable A with the spectral decomposition $A = \Sigma\, a_i \mathbf{R}_i$. We shall define corresponding eigenspaces by $\mathbf{K}_i = \mathbf{R}i(\mathbf{H_E})$. The eigenvalue a_i will be the output value.

The corresponding prediction will be

$$\mathbf{pred} = \{(a_i, p_i. \psi_i) \mid i=1, .. , n\} \text{ where } p_i = \|\mathbf{R}_i(\psi)\|^2 ,\ .\ \psi_i = \|\mathbf{R}_i(\psi)\|^{-1} \mathbf{R}_i(\psi) .$$

In the hybrid-epistemic model we shall consider the same ensemble $\mathbf{E} = \{S_1, .. , S_N\}$ of measured systems in the state ψ. The measurement will be defined by the orthogonal decomposition $\mathbf{H_E} = \mathbf{K}_1 + \dots + \mathbf{K}_n$ and the corresponding projections will be denoted by $\mathbf{R}_i$.

The state ψ can be decomposed $\psi = \alpha_1\psi_1 + \dots + \alpha_n\psi_n$ where $\psi_i = \|\mathbf{R}_i \psi\|^{-1} \mathbf{R}_i \psi \in \mathbf{K}_i$, $\alpha_i = \|\mathbf{R}_i \psi\|$, if $\|\mathbf{R}_i \psi\| > 0$, while for $\|\mathbf{R}_i \psi\| = 0$ we take any unit vector $\psi_i \in \mathbf{K}_i$ and $\alpha_i = 0$.

By the Axiom **HE5** there exists at least one balanced observable ensemble $\mathbf{F} = \{M_1, \dots , M_N\}$ of measuring systems satisfying $|\mathbf{C}(\mathbf{F})| = n$. The decomposition into homogeneous subspaces will be denoted $\mathbf{H_F} = \mathbf{L}^{(1)} + .. + \mathbf{L}^{(n)}$ and corresponding projections will be denoted by $\mathbf{P}^{(1)}, .. , \mathbf{P}^{(n)}$. We shall suppose that the state of $\mathbf{F}$ will be $\phi \in \mathbf{L}^{(1)}$. The ensemble of composite systems $\mathbf{G} = \{M_1 \oplus S_1 , \dots , M_N \oplus S_N\}$ will be in the state $\phi\otimes\psi$. These will be the input data for the measurement process.

After applying the measuring transformation $\mathbf{U}$ we obtain the output data $\mathbf{U}(\phi\otimes\psi) = \Sigma\, \alpha_i V_i(\phi) \otimes\psi_i$ and by Theorem 5. (i) that $p(v_i \mid \mathbf{U}(\phi\otimes\psi)) = |\alpha_i|^2$.
By the same theorem (ii) we obtain that the updated state will be $\mathbf{V}_i(\phi) \otimes\psi_i$.
Thus the state of the subsystem S will be ψ_i.

This result is equal to the prediction **pred** calculated above in the epistemic model.

(ii) We shall start with the measurement in the hybrid-epistemic model described in the Sect. 5. by considering the ensemble of measuring systems $\mathbf{F} = \{M_1, \dots, M_N\}$ in the state ϕ and the ensemble of measured systems $\mathbf{E} = \{S_1, \dots, S_N\}$ in the state ψ .

We shall use freely the notation from the Sect. 5. The measurement will be defined by the orthogonal decomposition $\mathbf{H_E} = \mathbf{K}_1 + \dots + \mathbf{K}_n$ and the corresponding projections will be denoted by $\mathbf{R}_i$ and the state ψ will be decomposed $\psi = \alpha_1\psi_1 + \dots + \alpha_n\psi_n$.

Then by the Theorem 5. we obtain the output data from the measurement: $p_i = |\alpha_i|^2$ and the updated state of an ensemble $\mathbf{V}_i(\phi) \otimes \psi_i$.

Now we shall consider the same experiment in the epistemic model. We shall consider the composite ensemble $\mathbf{G} = \{M_1 \oplus S_1, \dots, M_N \oplus S_N\}$ in $\mathbf{H_G}$ in the state $\Psi' = \Sigma_i \mathbf{V}_i(\phi) \otimes \psi$. We shall consider the observable A in the space $\mathbf{H_G}$ with the spectral decomposition $A = \Sigma\, i\, \mathbf{L}^{(i)} \otimes \mathbf{R}_i$ and we shall consider the measurement of this observable (in the epistemic model). The resulting prediction will be

$$\mathbf{pred} = \{(i, p_i. \Psi_i) \text{ where}$$

$$p_i = \|(\mathbf{L}^{(i)} \otimes \mathbf{R}_i)(\Sigma_j \mathbf{V}_j(\phi) \otimes \psi)\|^2 = \| \mathbf{V}_i(\phi) \otimes \mathbf{R}_i(\psi)\|^2 = \|\mathbf{R}_i(\psi)\|^2 = |\alpha_i|^2 \text{ since}$$
$$\mathbf{L}^{(i)} (\Sigma_j \mathbf{V}_j(\phi)) = \mathbf{L}^{(i)} (\mathbf{V}_i(\phi)) = \mathbf{V}_i(\phi) , \|\mathbf{V}_i(\phi)\| = 1$$

$$\Psi_i = \|(\mathbf{L}^{(i)} \otimes \mathbf{R}_i)(\Sigma_j \mathbf{V}_j(\phi) \otimes \psi)\|^{-1} ((\mathbf{L}^{(i)} \otimes \mathbf{R}_i)(\Sigma_j \mathbf{V}_j(\phi) \otimes \psi))$$
$$= \|\mathbf{V}_i(\phi)) \otimes \mathbf{R}_i(\psi)\|^{-1} \mathbf{V}_i(\phi)) \otimes \mathbf{R}_i(\psi) = |\alpha_i|^{-1} \mathbf{V}_i(\phi)) \otimes \mathbf{R}_i(\psi) = \mathbf{V}_i(\phi)) \otimes \psi_i$$

This prediction $(|\alpha_i|^2 , \mathbf{V}_i(\phi)) \otimes \psi_i)$ coincides with the prediction from the hybrid-epistemic model.

The proof of Theorem 7. It will be divided into two parts. Let us assume that there is given an observable A with the spectral decomposition $A = \Sigma\, a_i\, \mathbf{R}_i$.

(i) In the epistemic model the preparation and evolution parts produce the ensemble $\mathbf{E} = \{S_1, .., S_N\}$ in the state ψ. The corresponding prediction will be

pred $= \{(a_i, p_i. \psi_i) \mid i=1, .., n\}$ where $p_i = \|\mathbf{R}_i(\psi)\|^2$, $\psi_i = \|\mathbf{R}_i(\psi)\|^{-1} \mathbf{R}_i(\psi)$.

In the ontic model we shall consider systems $S_1', .., S_N'$ in the (individual) state $[\psi]$ and the ensemble $\mathbf{E}' = \{ S_1', .., S_N'\}$ in the state $[\psi]$. The observable $A = \Sigma a_i \mathbf{R}_i$ will be measured on the system S_k'. Using the output map o we obtain the output value $o(S_k') \in \{a_1, .., a_n\}$.

The Axiom **SO6** says that the relative frequency of the value a_i will be $\|\mathbf{R}_i(\psi)\|^2$.

The axiom **SO7** says that the collapsed state will be $\|\mathbf{R}_i(\psi)\|^{-1}\,\mathbf{R}_i(\psi)$. Thus we see that the corresponding prediction in the ontic model will be the same as in the epistemic model.

(ii) Let us consider measurement in the ontic model. We shall consider systems $S_1', .. , S_N'$ in the (individual) state $[\psi]$ and the ensemble $\mathbf{E}' = \{ S_1', .. , S_N'\}$ in the state $[\psi]$. The observable $A = \Sigma a_i\mathbf{R}_i$ will be measured on the system S_k'.
Using the output map o we obtain the output value $o(S_k') \in \{a_1, .. , a_n\}$.

Using Axioms **SO6** and **SO7** we shall obtain the prediction

$$\mathbf{pred} = \{(a_i, p_i. \psi_i)\} \text{ where } p_i = \|\mathbf{R}_i(\psi)\|^2 , \quad \psi_i = \|\mathbf{R}_i(\psi)\|^{-1}\,\mathbf{R}_i(\psi) .$$

In the epistemic model we shall consider the corresponding ensemble $\mathbf{E} = \{ S_1, .. , S_N\}$ in the state $[\psi]$ and the same observable A . Using Axioms **SE6** and **SE7** we obtain corresponding prediction which will be identical to the prediction **pred** from the ontic model.

Appendix B: conjectures and opinions.

Up to now all statements were proved. Now we would like, in this part, to express our conjectures, hypotheses , opinions and remarks:

- The ontic model of QM is **wrong** and probably also internally inconsistent (and thus **provably** wrong)
- The concept of the ontic model was a **big error** from the beginning of QM (von Neumann, 1932) and the source of most misunderstandings in QM (the Schrodinger cat paradox, the collapse rule for individual systems, the role of an observer in experiments and others)
- No-go theorems are not the empirical statements but the proofs of an internal **contradictions** in the ontic model of QM (see [7])
- The ontic model of QM contains internal contradictions and has to be abandoned on the base of its logical inconsistence (but the ontic model cannot be rejected on the experimental ground since it is empirically equivalent to the other models of QM)
- The principle of superposition is one of the **biggest errors** in the history of physics: either it is generally assumed (for ensembles) or it is wrong (for individual

systems)[43] [44]. It is also the source of most misunderstandings, absurdities and problems in QM.

- QM is basically the applied probability theory but based on the new "quadratic" probability theory introduced in [1] (and not on the standard Kolmogorov's probability theory): i.e. **QM is the standard Markov theory of time-reversible processes in the quadratic probability theory** (completed with a certain complex symmetry) [1].
- The hybrid-epistemic model of QM is able to solve many foundational problems of QM (all of problems mentioned above)
- The property-epistemic model of QM solves the same problems as the hybrid-epistemic model but in a simpler and more transparent way. We prefer the property-epistemic model of QM as the best solution to most (but not all) foundational problems of QM.
- But notice that not all foundational problems can be solved in the hybrid-epistemic model (e.g. the problem of the role of the information in physical processes, the delayed choice experiments etc.) – these other problems require the full use of the quadratic probability theory.
- The simplest and probably the best solution to foundational problems of QM is given by the property-epistemic model
    - the principle of superposition holds only for states of ensembles
    - the no-go theorems are almost trivially non-provable
    - the solution to the wave-particle duality problem
    - this is an epistemic model combined with the properties of individual systems
    - the analogy with the classical probability (the Brownian motion) is a good motivation
    - the elegance and simplicity of the property-epistemic model is extraordinary
- We would like to express our opinion that the **property-epistemic model** of QM is the most elegant, the most simple, the most pure and the most powerful solution to the foundational problems of QM. The main advantage of this model is the fact that in this model the individual states (i.e. the quantum states of individual systems) do not exist.

[43] The superposition principle $|\Psi\rangle = \alpha|\psi\rangle + \beta|\phi\rangle$ is formally meaningless. In our notation we have $[\Psi] = \alpha[\psi] + \beta[\phi]$ and already the expression $\alpha[\psi]$ is meaningless since the space of pure states $\mathbf{P}_S$ has no linear structure. The definition $\alpha[\psi] + \beta[\phi] = [\alpha\psi + \beta\phi]$ is also meaningless because it does not define a unique state – the result depends on the choice of representatives $\psi \in [\psi]$, $\phi \in [\phi]$. Clearly, Dirac assumed (erroneously) that ψ describes the quantum state directly without considering $[\psi]$ as a state.

[44] In the case of hybrid-epistemic models which contain no hybrid systems there do not exist any individual states (i.e. states of individual systems) and thus the individual superposition principle has no meaning. Such models are quite reasonable and they exist very often (it is rather difficult to find a realistic model containing at least one hybrid system). Models which do not contain any hybrid system are called the property-epistemic models and have been considered in Sect. 11.

- The problem of “wave” and “particle” properties of individual systems. Individual system’s properties in the hybrid-epistemic model can be usually interpreted as system’s positions and thus as a particle’s properties.
- Our solution to the wave-particle duality problem:
  - wave properties can be attributed **only** to ensembles – individual systems **cannot have any wave properties**. This means that the so-called wave properties are only statistical (in the sense of the quadratic probability theory) properties of ensembles
  - The interference picture needs an ensemble of photons – one photon cannot create an interference picture. Thus the interference is the (statistical) property of ensembles but not the property of individual systems.
  - Our solution is very clear: particle-like and wave-like properties can quietly coexist in the property-epistemic model since they are attributed to different objects – particle-like properties are properties of individual systems while wave-like properties are properties of ensembles. (This means that the so-called wave properties are statistical properties of ensembles.) This is the unique pure solution to this problem (known up to now).
  - Otherwise there can exist infinite discussions of the co-called principle of complementarity (which was never exactly formulated and which is probably meaningless and wrong).

Appendix C: the ontic model for the Brownian motion and its absurdity.

In this part we shall describe the analogue of an ontic model description for a Brownian motion with the aim to show clearly the **absurdity** of the ontic model of QM. We let on the reader to evaluate our arguments but nevertheless we think that this classical model is interesting.

We shall confront the ontic model of the Brownian motion with the ontic model of QM. We have considered above only finite dimensional quantum systems while the Brownian motion is described by the infinity dimensional model. But we think that this difference is not substantial for the understanding and validity of our arguments.

The ontic model of the Brownian motion of a Brownian particle can be defined by following axioms.

BMO1. To each system S (i.e. to each Brownian particle) there corresponds the Lebesgue L^1 space on the space R^3

$$\mathbf{L}(S) = \mathbf{L}_S = \{ f: R^3 \to R^1 \mid \int |f(x)|\, d^3x < \infty \}^{45} .$$

The space of possible states of S , i.e. the space of probability distributions is given by

$$\mathbf{P}(S) = \mathbf{P}_S = \{ f \in \mathbf{L}_S \mid f \geq 0 , \int f(x)\, d^3x = 1 \} .$$

To each system S (at a time t) there corresponds its state $St(S; t) \in \mathbf{P}_S$.

BMO2. For two systems S and M we have $\mathbf{L}_{M \oplus S} = \mathbf{L}_M \otimes \mathbf{L}_S$ and the corresponding $\mathbf{P}_{M \oplus S}$.

BMO3. The evolution of the system S is given by the semi-group $\{ U_t \mid t \geq 0 \}$ of stochastic operators, where $U_t : \mathbf{P}_S \to \mathbf{P}_S$, $t \geq 0$. This semi-group is generated by the heat equation.

BMO4. To each observable *F* there corresponds a bounded function $F : R^3 \to R^1$ which (for simplicity) attains only finite number of values, i.e. $| F(R^3) | < \infty$.

Definition.

(i) We set $\Lambda = F(R^3)$. Then for each $\lambda \in \Lambda$ we set $A_\lambda = F^{-1}(\lambda)$.

(ii) For each $\lambda \in \Lambda$ we define the projector

$$P_\lambda (F) = F . \chi(A_\lambda) \quad , \quad P_\lambda : \mathbf{L}_S \to \mathbf{L}_S$$

where $\chi(A)$ denotes the characteristic function of the set $A \subset R^3$.

(iii) For the observable F and for the probability distribution $f \in \mathbf{P}_S$ we set

$$p (\lambda \mid f ; F) = \int f . \chi(A_\lambda)\, d^3x = \int P_\lambda (f)\, d^3x$$

(iv) We set

$$T_{\lambda; F}(f) = (\int P_\lambda (f)\, d^3x)^{-1} . P_\lambda (f) \text{, assuming } \int P_\lambda (f)\, d^3x > 0 .$$

We clearly have $F = \sum_{\lambda \in \Lambda} P_\lambda (F)$.

Definition. The ensemble **E** in the state f is the set of independent systems

$$\mathbf{E} = \{ S_1, .. , S_N \} \quad \text{satisfying}$$

(i) $\mathbf{P}(S_1) = .. = \mathbf{P}(S_N)$ and we denote $\mathbf{P}(\mathbf{E}) = \mathbf{P}(S_1) = .. = \mathbf{P}(S_N)$

(ii) For each $S \in \mathbf{E}$ we have $St(S; t) = f$ at a time t

BMO5. (The measurement schema.)

[45] In fact, in the general case, the probability distribution can be a measure on R^3. We shall consider, for the simplicity, only probability densities. But we shall also consider the δ-functions $\delta_x(y) = \delta(x-y)$, x, y $\in R^3$.

Let us consider an ensemble **E** in the state f and let us consider a measurement of an observable F.
As a result of a measurement we obtain the "output" map $o : \mathbf{E} \to \Lambda$ which means that in the measurement of a system $S \in \mathbf{E}$ we obtain the output value $o(S) \in \Lambda$.

Then we can define a new ensemble

$$\mathbf{E}^{\lambda} = \{ S \in \mathbf{E} \mid o(S) = \lambda \} \text{ for each } \lambda \in \Lambda_0 .$$

The relative frequency of the output value λ in the sequence $o(S_1), .., o(S_N)$ is given by

$$g(\lambda) = N^{-1} \mid \mathbf{E}^{\lambda} \mid$$

BMO6. ("Born's rule".) The "Born's rule" holds

$$g(\lambda) = p(\lambda \mid f ; F) .$$

BMO7. (The collapse postulate.) After the measurement where the output value was λ, the state of the individual system S will be (immediately) changed

$$\text{from } f \text{ to } T_{\lambda}(f) .$$

BMO8. For each system S and for each $f \in \mathbf{P}_S$ there exists an ensemble **E** in the state f such that $S \in \mathbf{E}$**.**

Axioms **BMO**1 – **BMO**8 define the ontic model of the Brownian motion.

Main principles of the **standard** model of the Brownian motion:

- **BMS1.** The individual state of a Brownian particle at time t is given by its position

$$x(t) \in R^3$$

- **BMS2.** The state of an ensemble is given by the probability density $f \in \mathbf{P_S}$
- **BMS3.** The probability that the Brownian particle is found in the set $A \subset R^3$ is given by

$$\text{prob}(x(t) \in A) = \int f \,.\, \chi(A)\, d^3x$$

- **BMS4.** The evolution of the probability density is given by the semi-group $\{U_t\}_{t>0}$ as in **BMO4**.

The standard model describes the well-known physical reality.

It is clear that the ontic model of the Brownian motion is from the physical point of view a **complete non-sense**.

Nevertheless, there are interesting observations:

- The ontic model of the Brownian motion is from the mathematical point of view (axioms BMO1 – BMO8) correct – i.e. consistent
- Empirical predictions of the ontic model are equivalent to the empirical predictions of the standard model of the Brownian motion.

This means that it is possible to transform the correct standard model of the Brownian motion into the non-standard ontic model which is mathematically and empirically equivalent to the standard model but which is **physically in-correct**.

How to evaluate the present situation?

- To attribute the probability density to an individual Brownian particle (as in the ontic model) is a pure non-sense – it is evident that the local position $x(t)$ of an individual Brownian particle cannot have nothing in common with the non-local probability density $f : R^3 \to R^1$.
- It is clear that there can exist a completely artificial mathematical model which is equivalent to the standard model but which has no physical meaning.
- The ontic model of the Brownian motion is evidently absurd.
- The collapse postulate **BMO7** is, of course, the physical absurdity.

We can consider the ontic model of the Brownian motion as an analog of the ontic model of a QM particle. This implies (heuristically) the **absurdity** of the ontic model of QM.

Inside of these arguments there is the **very important internal argument**.

- In any probability theory the level of complexity of the description of individual states must be substantially lower than the lever of complexity of the description of an ensemble (in the case of the Brownian motion this is the description $x(t)$ of an individual Brownian particle with respect to the description $f : R^3 \to R^1$ of an ensemble of Brownian particles)
- It is clear that the description of an ensemble must be substantially more complex that the description of an individual particle since already the ensemble as an object (containing potentially infinite number of particles) is more complex than one particle.
- But this is **not true** in the ontic model of QM: the level of complexity of the description of an individual system (the wave function $\psi(x)$) is of the same order of complexity as the description of an ensemble (the density operator $\rho(x, y)$).
- This means that in any probability theory the complexity of an individual system must be substantially lower than the complexity of an ensemble. Exactly this

requirement **is not satisfied** in the ontic model of QM but it is satisfied in the hybrid-epistemic model of QM.

We consider this argument against the ontic model of QM as a **deepest** argument which disqualifies the ontic model of QM since this argument is very general.

The idea that the state of an individual Brownian particle can be described by the probability distribution f is evidently absurd.

But why the idea that the state of an individual electron can be described by the wave function ψ is not considered as **equally absurd**?

The idea of the ontic model was formulated at the beginning of QM (von Neumann, 1932) and from this time it was considered as the sacral dogma (completely frozen by the so-called Bohr's victory in the Bohr – Einstein dispute, 1935) .

A unique person who has protested against the ontic model of QM was A. Einstein in [9]. We have shown that he was right.

The rejection of the ontic model of QM has, of course, important consequences: no no-go theorems, no no-locality of QM, no Bell's inequalities, the vanished role of an observer, no wave properties of individual systems etc.

But after this the world will be more standard and less extravagant.

We believe that after some time it will be generally accepted that the idea to associate the wave function to an individual quantum particle is **completely absurd** (as in the case of a Brownian particle) and that this fact will be considered as evident.

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