On pi : a definite integral

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abstract

This note presents a definite integral for pi

1. INTRODUCTION. The constant pi is defined by

$$\pi = 4\int_0^1 \frac{1}{1+x^2}\,dx = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.14159265... \qquad (1)$$

2. A DEFINITE INTEGRAL FOR PI.

$$\pi = \sqrt{3}\int_0^{2/3} \ln\left(\frac{1-2x^2+\sqrt{1+4x^2}+\sqrt{2}\sqrt{1-6x^4+\left(1-2x^2\right)\sqrt{1+4x^2}}}{4x^2}\right)dx \qquad (2)$$

$$\begin{aligned}\pi &= \frac{4}{\sqrt{3}}\left(1-2\ln 2+\ln 3\right)+ \\ &+\sqrt{3}\int_0^{2/3} \ln\left(1-2x^2+\sqrt{1+4x^2}+\sqrt{2}\sqrt{1-6x^4+\left(1-2x^2\right)\sqrt{1+4x^2}}\right)dx\end{aligned} \qquad (3)$$

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